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Wave Soliton Solutions on a Generic Background for KPI Equation

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1. Formulation of the Problem

Our aim is to extend the spectral transform for the KPI equation

$$(u_t - 6uu_x + u_{xxx})_x = 3u_{yy}, \tag{1.1}$$

to the case of potentials nondecaying along a finite number of directions in the plane, that is we are interested in a potential u having nontrivial limits

$$u_{n,\pm}(x,y) = \lim_{y'\to\pm\infty} u\bigl(x + 2\mu_n(y-y'), y'\bigr), \quad n = 1, 2, \dots, N, \tag{1.2}$$

for N real constants μ_n and otherwise vanishing at large distances.

Such extension of the admissible potentials is of high mathematical relevance and, above all, is crucial in physical applications since solitons in two dimensions are in general not localized in space and also in the case of Davey Stewartson equation, for which localized solitons have been discovered, the dynamics is governed by the inverse scattering theory of the KPI equation.

Some general results were obtained in [**2, 3**] by using a new approach in the Inverse Scattering Method, called resolvent approach. The validity of our approach is general and in [**7, 8**] it was also tested in the discrete case.

Here we present the case of a potential obtained by superimposing a soliton to a generic rapidly decaying background via Bäcklund transformations. We are able to construct the resolvent, the Jost solutions and the spectral data and we present a complete study of their properties, showing that they meet completely our scheme. From one side this demonstrates that the theory is not void of interest since we found that it works well in a sufficiently generic case, and from the other side it allows us to get some experience with a case less involved from a mathematical point of view than the generic case.

2. The General Scheme of the Method

A large class of nonlinear equations can be linearized by using the well known Inverse Spectral (or Scattering) Transform technique (IST). KPI equation is integrable in the framework of IST method, since it can be associated to the linear

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This is the final form of the paper.