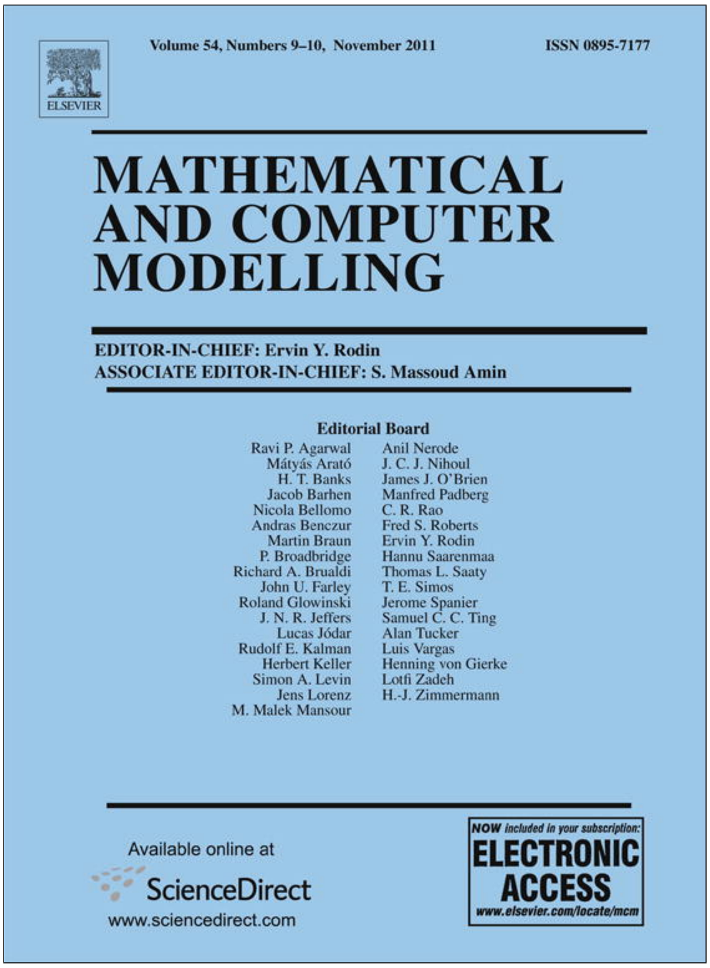

Volume 54, Numbers 9–10, November 2011 ISSN 0895-7177

ELSEVIER

MATHEMATICAL AND COMPUTER MODELLING

Mathematical and Computer Modelling 54 (2011) 2087–2103

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

Mathematical modeling of quality in a medical structure: A case study

M. Lo Schiavo [a,*], B. Prinari [b,c], A.V. Serio [d,e]

[a] Università di Roma "Sapienza", Dipartimento di Scienze di Base e Applicate per l'Ingegneria, Roma, Italy
[b] University of Colorado at Colorado Springs, Department of Mathematics, Colorado Springs, CO, United States
[c] Università del Salento, Dipartimento di Fisica and Sezione INFN, Lecce, Italy
[d] Università del Salento, Dipartimento di Scienze Pedagogiche, Psicologiche e Didattiche, Lecce, Italy
[e] Dipartimento di Salute Mentale ASL - BR, Italy

ARTICLE INFO

Article history:
Received 14 March 2011
Received in revised form 5 May 2011
Accepted 8 May 2011

Keywords:
Generalized kinetic theories
Mathematical models
Social sciences
Quality of health care
Complex systems

ABSTRACT

A mathematical model, based on a statistical system approach, has been implemented and tested on the basis of a four-year-long experimental data set, with the aim of analyzing the performance and clinical outcome of an existing medical ward, and predicting the effects that possible readjustments and/or interventions on the structure may produce on it. The dynamics of the system is assumed to be connected to a variable called "atmosphere" that refers to the perceived social and organizational climate, as well as the comfort and ease realized in the ward. In this context, the atmosphere is intuitively related to the "quality" that is (or is perceived as being) offered by the service, as it affects the ability to satisfy the patients' needs, to provide a livable environment for patients and medical staff, and to guarantee more efficient performances and a more complete professional development. Identifying variables, parameters and events that control the atmosphere is therefore of the deepest importance from a social and health-care point of view. The proposed interdisciplinary approach, referring to paradigms of physical and mathematical models integrated with theories and methods typical of social sciences, has chances of gaining the attention of the scientific community in both fields, and higher possibilities of obtaining appreciation and generalization.

1. Introduction

Over the past decade, there has been an increasing interest in mathematical models that describe and analyze complex structures and processes such as interacting systems of human beings. It is not infrequent that mathematical theories that are well known in some research areas strongly contribute to the creation of new methods and perspectives even in fields remote to those that motivated their introduction. This is the case of so-called generalized kinetic models, which proved to represent a fruitful predictive and descriptive tool not only in describing events of plasma physics, but also in the area of social sciences and even in highly organized human structures. Generalized kinetic models transfer the methodology developed for systems with a great number of interacting particles (such as Boltzmann and Vlasov equations, with direct interactions among the particles or mean field terms and external forces [1]) to various other fields of research, such as traffic dynamics (see, for instance, [2–5]), cellular dynamics (see, among others, [6–10]), social and population dynamics [11–16] and biological systems in general [17–19]. Various models of the classical mathematical kinetic theory have been investigated and generalized in several contexts, [20–22]. The interested reader can find thorough reviews of the theory and applications of generalized kinetic models in the monographs [23–25].

* Corresponding author. Tel.: +39 0649916633; fax: +39 064957647.
E-mail address: loschiavo@dmmm.uniroma1.it (M. Lo Schiavo).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.05.017

Table 1
The color code for the Ward Atmosphere.

■ green: Everything is fine in the ward and there are no negative emotions in the staff.
■ blue: The behavior of one or more patients makes the staff feel uneasy.
■ yellow: The ward feels crowded, or there are patients in a more acute state, or behaving somewhat violently. The staff feels worried.
■ orange: The ward feels very crowded and one or more patients in critical conditions become problematic or aggressive, requiring some exceptional intervention. The staff feels alerted (fearful, powerless, etc.).
■ red: It becomes necessary to restrain one or more patients, or to call for external help. The staff cannot withdraw.

Motivation. In Ref. [26], on the lines of the investigations started in [27,28], the authors propose a further extension of generalized kinetic theory to obtain the (statistical) description of the time evolution of a global variable – "atmosphere" – related to the quality of a complex system such as a medical ward. The work was originally motivated by the analysis of experimental data collected for almost 10 years in an acute psychiatric in-patient care unit (SPDC, located in San Pietro Vernotico, Brindisi, Italy) under the direct responsibility of one of the authors (Serio, MD), in the framework of a regional research project for monitoring and improving the quality of psychiatric wards. The ward has been considered as a closed system containing two populations: patients and staff (medical and nursing). The dynamics, and the mutual relations among the individuals, have been modeled as dependent at various degrees on the occurring (or perceived as occurring) atmosphere, and on the effects produced on each individual by the performance/behavior of all the others. The experimental data collected over the years are both quantitative and qualitative and include: 1. monitoring a global variable called Ward Atmosphere (or also "therapeutic atmosphere"); 2. presence and status of medical and nursing staff (on three shifts per day); 3. critical or sentinel events (such as episodes of aggressiveness or violence, accidents, restraints, escapes, etc.); 4. internal and external events both of positive and negative nature (such as visits by mental health community teams or relatives, social activities, leaves; uneasy admissions, such as patients on involuntary admission or at their first hospitalization); 5. ordinary flux data (daily admissions/discharges of patients).

The concept of climate or atmosphere is well known in the specialized medical and sociological literature since the 1950s. According to the World Health Organization [29], the most important single factor in the efficacy of the treatment given in a mental hospital appears to be an intangible element which can only be described as its atmosphere; and, in attempting to describe some of the influences which go to the creation of this atmosphere, the type of relationship between people that are found within it is of utmost relevance.

Climate can be thought of both as organizational and psychological nature. Organizational climate is a relatively stable feature that is experienced by the members of a given structure [30], possibly generated by the interactions among the members [31,32], and influencing their behavior. Psychosocial climate of in-patient units proved to be related to both patient satisfaction and clinical outcome [33]. Moreover, the working conditions of the entire staff are related to patient satisfaction and patients' perceptions of the treatment environment [34,35]. As a matter of fact, patients' perception of ward atmosphere is a clinically meaningful measure that appears to be a strong predictor of satisfaction, as well as a valid indicator of quality of care [36–38], and is tightly correlated to the values of the ward atmosphere as carefully measured by the staff.

As it is clearly understood, the problem of attributing the most conceivable value at each instant of time to the climate is a difficult and questionable matter. One of the most commonly used Social Climate Scales is the Ward Atmosphere Scale (WAS), an extensively researched instrument in clinical settings [39,40]. However, the WAS assesses the climate of an hospital-based psychiatric treatment as a continuation of a set of data localized in time: it assumes that they remain constant for a relatively long period of time, typically some months. It is the authors' opinion that in this way the peculiar property of the atmosphere, namely its stemming from a dynamic process of actions and of individual and collective decisions in a continuous stream of exchanges, may only scarcely be achieved, and that a series of answers to a single questionnaire may provide just one timed information: the WAS is likely to capture only static aspects of the atmosphere.

On the contrary, in the present study the atmosphere has been considered as a dynamical variable, directly dependent on the individual perceptions of all the surrounding (environment and other actors) and hence able to refer about the functioning and organization of the structure with a remarkable amount of data and acceptable adherence also on relatively short time scales as hereafter clarified. Indeed, the atmosphere has been evaluated by the staff 3 times a day (more precisely, near the end of each shift) since Jan 11, 2001, and filed according to a color code presented in Table 1. The table summarizes the criteria according to which the atmosphere has been evaluated, and the fact that the evaluation takes into account the state of both patients and staff.

In order to allow numerical and statistical investigations on the collected data, the color code has been (somewhat questionably) converted into the ordered set of five positive integers $\{2, 4, 6, 8, 10\}$, and hence on an equal-interval (ordinal Likert) scale, with 2 corresponding to *green* and 10 corresponding to *red*.

Whether individual Likert items can be considered as interval-level data, or whether they should be considered merely ordered-categorical data is a subject of discussion. Such items could be regarded only as ordinal data, because, especially when using only five levels, one cannot assume that all pairs of adjacent levels are perceived as equidistant. On the other hand, to treat it merely as an ordered set without specifying a distance would lose information. This issue will be further discussed in Section 4, when dealing with the statistical significance of the model at study.

Methodology. The proposed model is of statistical nature: it is based on the time evolution equations for the population distributions over a "microscopic" state regarded as a random variable, which in the literature on generalized kinetic models is referred to as "activity". Evolution is driven both by the interactions among the system actors, i.e., patients and staff, and by various external and internal events. Obviously, the activity variable takes on different meaning when referred to individuals of different populations. In the case of patients, activity is related to their mentally disturbed behavior; for staff, it is related to the stress they are subject to in performing their tasks. Due to the peculiar kind of observed data, the statistical description of the two populations, patients and staff, has been assumed to be correctly acquired by discretizing their activities into two sets of five real values; correspondingly, two sets of five probabilities refer about the percentage of actors that at any instant of time are expected to be found in each of the five discrete states that correspond to the color code in Table 1. The reason to prefer this "kinetic" description to a microscopic, individual, deterministic one where each single actor determines and evolves according to his own activity value is twofold. On one hand, it is clear that such a microscopic dynamics can hardly be specified; on the other hand, in our approach individual interactions among the actors represent isolated though important events that can be singled out and differently treated with respect to the other kind of events that drive the dynamics, i.e. the global ones. A strictly "macroscopic", averaged, continuous model of the ward is, however, unjustifiable not only because of the small number of actors under exam, but also since in this case the model would acquire a totally phenomenological nature. Therefore, we developed a "mesoscopic" picture that takes advantage of a probability density function to account for the uncertainty due to the reactions of individuals both to what they perceive of the surrounding social climate, and to the effects of single (pairwise) interactions with the other individuals of the system. Hence, even though the number of actors is relatively small, in order to distinguish the contributions due to pairwise interactions from the mean field interactions, or, more generally, from those interactions that are not of pairwise nature, the model may treat the actors individually, which would be impossible in a purely continuous model. On the other hand, no 1–1 microscopic model is, to the best of our knowledge, currently available in the literature. As such, this picture proves to be able to refer about averaged measurable quantities, such as the ward atmosphere, seen as weighted moments, though subject to and driven by individual direct behaviors. This also allows the picture to be "nonlocal", in the sense that the dynamics of each probability function is driven by these averages performed over the whole system, and hence by all the values of the others probabilities.

Under these assumptions, the mathematical model reduces to a sequence of correlated initial value problems for a system of 10 coupled, nonlinear, nonlocal ordinary differential equations for the probability variables. The equations depend on a set of physical parameters that describe, at such mesoscopic scale, the nature and frequency of the direct interactions among the actors, as well as the effect of external positive or negative events, workload, and social terms (which specify how the individuals are affected by the overall ward state). Aim of the analysis is to make use of the history data set, and single out and characterize those physical parameters that control the dynamics. The ultimate goal is to predict, on a short time scale, the possible outbreak of a crisis, and, on a long time scale, the effects that specific planned or unplanned readjustments of the structure may produce on it and on patients and staff in terms of stress and satisfaction. To this aim, numerical simulations have been performed to provide the system evolution due to different initial and external conditions, both when these were the truly observed ones, and hence the results comparable with the collected data, and when they were merely simulated to analyze the single parameters influence on the system dynamics. The outputs of the numerical simulations will be discussed in the following and eventually compared with the actual data. At present, our principal goal has been the tuning of model parameters so as to best fit the experimental data. In the future, the model will be used as a predictive tool to help understanding which parameters/events most deeply influence the quality of the structure, and which adjustments, on different time scales, may help improving the ward performance.

The paper is organized as follows. In Section 2 we describe the mathematical model, by adapting the one proposed in [26] to the specific case under exam. In Section 3 we discuss the fine-tuning of the model parameters, and illustrate the output of the numerical simulations performed to solve the initial value problem. In Section 4 we compare the output of the numerical simulations with the experimental data and briefly discuss the statistical significance of the model. Section 5 is devoted to some concluding remarks.

2. Description of the specific model

To some extent, the general mathematical aspects of the model that will be discussed in what follows have already been presented in [26], which the interested reader is referred to. In the present article, we explicitly and minutely adapt that general construction to the specific case under exam. Therefore, for clarity reasons and easiness of exposition, here we shall use the terminology and properties that better describe this case, leaving all the possible generalizations to an easy guess. On the other hand, we all the same describe some parameter, or whole term in the equations, even when on the state of the facts it proved to be either useless, or at least less relevant than others.

As mentioned in the introduction, the model involves, on one hand, a statistical picture about a set of (macroscopic) variables that may be related to the quality of the system; on the other hand, as is typical of generalized kinetic models, it involves an individual picture about a (microscopic) variable, which is referred to as activity of the actors. Evolution equations are then obtained to describe the dynamics of the probability functions over the activities of each population.

The system consists of two different populations, P_1 and P_2: population P_1 is composed by $N_1(t)$ patients; population P_2 by $N_2(t)$ operators (staff members, i.e. nurses, medical doctors, social workers). As we will clarify later, the system is closed during each evolution interval; moreover, unlike other generalized kinetic models, here obviously no changes of populations are allowed. Individuals of the same population are identical, and only addressed to by the state variable denoting their activity: u_1 describes the psychotic behavior[1] of the patients; u_2 characterizes the stress level of the operators.

The system description requires a probability density function per each actor population over the corresponding state variable. On the lines of Table 1, in the present model each state variable u_i is assumed to vary on a finite discrete set of values $\mathbb{D}_i$ containing just five elements

$$u_1 \in \mathbb{D}_1 := \{u_{1,1}, u_{1,2}, \ldots, u_{1,5}\}, \qquad u_2 \in \mathbb{D}_2 := \{u_{2,1}, u_{2,2}, \ldots, u_{2,5}\}, \tag{2.1}$$

the value $u_{i,h}$ referring about the status (stress/psychotic behavior) felt by P_i-actors at level h. Therefore, each of the two densities reduces to five time-dependent probabilities. It is worth mentioning that, as it happens for the atmosphere, the activity variable also lacks of any a priori justifiable metric; the most notable consequence of this fact is that no *interaction distance* between the individuals is available. In what follows, the activities are represented by the values $\mathbb{D}_1 = \mathbb{D}_2 = \{.1, .3, .5, .7, .9\}$. Correspondingly, the sought for probability functions reduce to ten functions of time:

$$f_{i,h}(t) : [0, T] \subset \mathbb{R} \mapsto f_{i,h}(t) \in [0, 1], \quad i = 1, 2, \ h = 1, \ldots, 5 \tag{2.2}$$

$$\sum_{h=1}^{5} f_{i,h}(t) = 1 \quad \forall t \in [0, T]. \tag{2.3}$$

The value $f_{i,h}(t)$ represents the expected probability that a measurement (at time t) about the activity of all P_i-actors produces value $u_{i,h}$. The choice of 5 discrete states per population is the result of a balance between accuracy of the description, adherence to the physical system and feasibility of the analysis of the numerical simulations.

The evolution of the probability functions is defined along a sequence of unequal, adjacent time intervals. As a matter of fact, the intrinsic dynamics of the ward is based on three shifts per day, the first two are of length $\tau_2 = 7$ h, the night shift lasts $\tau_1 = 10$ h. Let M be the total number of days under exam (in our case, $M = 1428$), $m = 1, \ldots, M$ the ordinal of the mth day $\mathcal{I}_m$, and $n = 1, \ldots, 3M$ the ordinal of the nth shift I_n (note that these last ones are related only to the staff and not to the patient population, as we shall clarify below). Time is considered a continuous real variable defined on a finite interval $[0, T]$, which is subsequently divided into sub-intervals related to the in/outflow dynamics:

$$[0, T] = [0 =: t_0, t_1) \cup [t_1, t_2) \cup \cdots \cup [t_{3M-2}, t_{3M-1}) \cup [t_{3M-1}, t_{3M} := T], \tag{2.4}$$

$$t_n := \begin{cases} t_{n-1} + \tau_1 & \text{if } n = 3m, \ m = 1, 2, \ldots, M \in \mathbb{N}, \\ t_{n-1} + \tau_2 & \text{otherwise}, \ n = 1, 2, \ldots, 3M, \end{cases} \tag{2.5}$$

$$\mathrm{I}_n := [t_{n-1}, \ t_n), \qquad \mathcal{I}_m := [t_{3(m-1)}, \ t_{3m}). \tag{2.6}$$

The system is assumed to be completely specified by its state at each of the instants t_{n-1}, $n = 1, 2, \ldots, 3M$. Discontinuous changes in the composition of population P_2 are allowed at each of these instants; conversely, population P_1 is assumed to undergo discontinuous changes only at instants t_{3m-2} when patients are registered as discharged or admitted. At instants $t_{3(m-1)}$ specific events may initiate or terminate; at each t_{n-1} staff personnel turns over. The dynamics is therefore determined by a chain of $3M$ correlated initial value problems: given (according to the data, to the preceding dynamics, and partly guessed) the initial conditions, i.e., the values $f_{i,h}(t_{n-1})$, the model produces the time evolution $f_{i,h}(\cdot) : t \in [t_{n-1}, t_n) \mapsto f_{i,h}(t) \in \mathbb{R}$. Functions $f_{2,h}(t)$ are allowed to have 1st kind discontinuities at points t_{n-1}; functions $f_{1,h}(t)$ only at points t_{3m-2}, $m = 1, \ldots, M$; functions $N_i : t \in [0, T] \to N_i(t) \in \mathbb{N}$, denoting the number of actors of population P_i, are stepwise constant for $t - \tau_2 \in \mathcal{I}_m$ and right continuous. No other discontinuities are allowed.

At the macroscopic level, the system is described by conveniently chosen mean variables such as

$$U_i(t) = N_i(t) \sum_{h=1}^{5} u_{i,h} \, f_{i,h}(t), \quad i = 1, 2. \tag{2.7}$$

In conclusion, as far as the model equations are concerned, they could shortly be summarized as follows, with $(p_1, \ldots, p_5) := (f_{1,1}, \ldots, f_{1,5})$ and $(p_6, \ldots, p_{10}) := (f_{2,1}, \ldots, f_{2,5})$,

$$\frac{\mathrm{d}}{\mathrm{d}t} \, p_i = \sum_{k=1}^{10} \alpha_{i,k}(t) \, p_k + \sum_{h,k=1}^{10} \beta_{h,k}(t) \, p_h p_k, \quad i = 1, \ldots, 10, \tag{2.8}$$

where the coefficients α and β explicitly depend both on time and on appropriate ensemble means. In this concise way, however, the physical meaning of the various terms in the equations is very difficult to pursue and justify, even if only at a phenomenological level. Therefore, we follow the lines of a kinetic picture and, as already mentioned, to describe the various terms of the evolution equations we maintain the same terminology that is by now usual in the literature of generalized kinetic modeling. Although not necessary when dealing with a system of ordinary differential equations, all the same we

[1] For the sake of precision, we specify that here and in the following we use the expression ''psychotic'' behavior to briefly denote any altered or mentally disturbed behavior, without necessarily referring to the specific mental disturbance.

feel that this correspondence clarifies the meaning and the source of each term to be constructed. In this sense, the evolution equations about functions $\mathbf{f} := (f_{i,h})$ are *mass balance* equations in the state space. Specifically, the variation rate of each function $f_{i,h}$ is the sum of its direct variation with respect to time plus a *flow term* due to external actions and to internal actions of global nature; and the variation rate equals the balance between a *gain term* and a *loss term*, both referred to the specified probability function and due to internal *interactions*. Hence, the evolution equations may also be written in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t} f_{i,h} + \Phi_{i,h}[\mathbf{f}] = \mathcal{B}_{i,h}[\mathbf{f}] \quad i = 1, 2,\ h = 1, \ldots, 5 \tag{2.9}$$

where $\Phi_{i,h}[\mathbf{f}]$ has the meaning of a global or externally driven flow change, $\mathcal{B}_{i,h}[\mathbf{f}]$ refers about the balance of the internal direct interactions (of pairwise type) among the actors of the system, and where

$$\sum_{h=1}^{5} \Phi_{i,h}[\mathbf{f}](t) = \sum_{h=1}^{5} \mathcal{B}_{i,h}[\mathbf{f}](t) = 0, \quad \forall t \in [0, T],\ i = 1, 2.$$

In the following section, we minutely describe how the mean field term $\Phi_{i,h}[\mathbf{f}]$ and the interaction term $\mathcal{B}_{i,h}[\mathbf{f}]$ are modeled in the specific case study. In particular, to help the reader realize the proper order of magnitude of the various parameters involved in the numerical reconstruction of the experimental data, in Section 3 we specify the actual value of each coefficient used in the computations.

2.1. Modeling the field terms $\Phi_{i,h}[\mathbf{f}]$

In describing the field terms $\Phi_{i,h}$, which represent the effect of external and ensemble actions on the total dynamics, we refer to a hydrodynamic picture, and consider the present model as the discrete version of a "parent" continuous one, as described in Ref. [26]. If in Eq. (2.9) the only term that depends on interactions is the $\mathcal{B}_{i,h}$, and if the latter depends on interactions only, then the term $\Phi_{i,h}$ can be thought of as a flow. With this in mind, let subscript i address the population of actors being described: patients ($i = 1$) or staff ($i = 2$), let subscript h run over the possible state indices: $h = 1, \ldots, 5$, and let

$$\mathcal{K}_{i,h}[\mathbf{f}] : [0, T] \to \mathbb{R}, \quad i = 1, 2,\ h = 2, \ldots, 5$$

denote the functions that describe the convective speeds, or *social drifts*, experienced by individuals of population P_i between (adjacent) states $h-1$ and h. The terms $\Phi_{i,h}$ are set as follows. The states are thought of as elementary cells (contiguously) aligned according to the natural ordering, so that $\mathcal{K}_{i,h} > 0$ means that members of population P_i are (for various reasons that will be explained below) subject to a drift toward the next higher valued cell, namely those that are in state $h-1$ are pushed to state h, and the reverse if $\mathcal{K}_{i,h} < 0$. Let $\Phi^-_{i,h}$ represent the algebraic flow from $h-1$ to h (loosely speaking, the *mass* per unit time that enters the elementary cell h from cell $h-1$); then $\Phi^+_{i,h} := \Phi^-_{i,h+1}$ is the mass leaving cell h toward cell $h+1$, for $h = 2, \ldots, 4$. For completeness, let us also denote by $\Phi^-_{i,5}$ the external inflow to cell 1 and by $\Phi^+_{i,5} =: \Phi^-_{i,6}$ the external outflow from cell 5, both of which are assumed to be zero at each instant of time. Then we have:

$$\Phi_{i,h}[\mathbf{f}] = \Phi^-_{i,h+1}[\mathbf{f}] - \Phi^-_{i,h}[\mathbf{f}], \tag{2.10}$$

and

$$\Phi^-_{i,h}[\mathbf{f}] = \begin{cases} \mathcal{K}_{i,h}[\mathbf{f}]\, f_{i,h-1} & \text{if } \mathcal{K}_{i,h} > 0 \\ \mathcal{K}_{i,h}[\mathbf{f}]\, f_{i,h} & \text{otherwise,} \end{cases} \quad \text{for } h = 2, \ldots, 5, \qquad \Phi^-_{i,1} = \Phi^-_{i,6} = 0.$$

We assume that the drifts do not depend directly on the activity variable, i.e. that functions $\mathcal{K}_{i,h} =: \mathcal{K}_i$ do not vary with index h, and we set

$$\Phi_{i,h} = \begin{cases} \mathcal{K}_i\,(f_{i,h} - f_{i,h-1}) & \text{if } \mathcal{K}_i > 0, \\ \mathcal{K}_i\,(f_{i,h+1} - f_{i,h}) & \text{if } \mathcal{K}_i < 0, \\ 0 & \text{if } \mathcal{K}_i = 0, \end{cases} \tag{2.11}$$

or, shortly,

$$\Phi_{i,h} = |\mathcal{K}_i|\,(f_{i,h} - f_{i,j}), \quad \text{with } j = h - sgn(\mathcal{K}_i).$$

In other words, again following the lines of generalized kinetic theory, we assume that individuals undergo an *internal dynamics* together with a *social* one, and that interactions only affects the individual state variable u and with sharp changes on it, whereas the social field determines the *rate of change* of the activity by a flow term that may be seen as an average: $\int \mathcal{P}(t; u^*)\, f(t, u^*)\, \mathrm{d}u^*$ over the whole ensemble. The specific form of functions $\mathcal{K}_i[\mathbf{f}](t)$ that are used in the following is:

$$\mathcal{K}_i[\mathbf{f}](t) = [\beta_1\, \Delta U(t) + \beta_2\, \Delta C(t) - p_i\, CC(t) - r_i\, EE_i(t)\ + \upsilon_i]\, c_n(t), \tag{2.12}$$

where $\beta_i, p_i, r_i, \upsilon_i$, for $i = 1, 2$, are appropriate real parameters (discussed in Section 3), and where the function $c_n(t) \geq 0$ is properly set to account for the slowdown of the dynamics at night (see Eq. (2.4)) as follows:

$$c_n(t) = \begin{cases} \tilde{c}_n & \text{if } t \in [t_{3m-1}, t_{3m}),\ m = 1, \ldots, M \\ 1 & \text{otherwise,} \end{cases} \tag{2.13}$$

Table 2
External and sentinel events.

Positive external events:	
Visits, social activities, leaves	$e_1 = \epsilon_p$
Sentinel events:	
Restraint	$e_2 = -6$
Accident	$e_3 = -4$
Aggressiveness	$e_4 = -2$
Negative external events:	
Admission of a patient on involuntary treatment	$e_5 = -6\epsilon_b$
Admission of a patient at first hospitalization	$e_6 = -5\epsilon_b$
Admission of a patient from out of county	$e_7 = -4\epsilon_b$
Admission of a patient from outside catchment area	$e_8 = -3\epsilon_b$
Generic admission	$e_9 = -2\epsilon_b$
Generic discharge	$e_{10} = -1$

where $\tilde{c}_n = 0.4$. The meaning of the various terms in Eq. (2.12) (all of collective nature) is specified below, and the actual values that have been attributed to the corresponding parameters are discussed in Section 3.

Mean activity. The function

$$\Delta U(t) := \alpha(U_1 - U_1^c) + (1-\alpha)(U_2 - U_2^c), \qquad U_i^c := N_i(t)\, w_i \sum_{k=1}^{5} u_{i,k} \tag{2.14}$$

refers on how mean values U_i of the activities [see Eq. (2.7)] influence the evolution of the system. In particular, U_i acts on functions $\mathcal{K}_i$ with a positive (resp. negative) term depending on whether the sign of $U_i - U_i^c$ is positive (resp. negative). The critical values U_i^c have been defined in terms of two real, nonnegative weights w_1, w_2.

Climate field. The function

$$\Delta C := \gamma(C_1 - C_1^c) + (1-\gamma)(C_2 - C_2^c), \qquad C_i := \sum_{k \geq k_i^c} f_{i,k}(t) \tag{2.15}$$

directly accounts for the possible occurrence of a large fraction of the population being in highly negative states, specifically determined by the threshold levels k_i^c, and (real) parameters C_i^c.

The remaining terms in Eq. (2.12) are peculiar of our system in that they depend on the occurrence of certain events that are recorded as data, and listed in Table 2 according to their nature, i.e. positive external events, negative external events, and sentinel events.

External events. External events affect the system through the stepwise constant, real valued, global function $EE_i(t)$:

$$EE_i(t) = \sum_{m=1}^{M} \left(\sum_{q \in Q} e_q \nu_q(t) + e_0^{(m)}\, \delta_{i,2} \right). \tag{2.16}$$

Each event is identified by a real value e_q selected in a finite set $\mathbb{E} := \{e_{q_1}, e_{q_2}, \ldots, e_{q_E}\}$, $e_q \in \mathbb{R}$, and $q \in Q := \{q_1, q_2, \ldots, q_E\} \subset \mathbb{N}$, that collects all the possible, positive and negative, external events. The effects of an external event start at the beginning of an evolution interval $\mathcal{I}_m := [t_{3(m-1)}, t_{3m})$ and last for that interval only; the E-tuple of functions $(\nu_{q_1}, \nu_{q_2}, \ldots, \nu_{q_E})$ has values $\nu_q^{(m)} \in \{0, 1, 2, \ldots\}$, $q \in Q$, which specify the number $\nu_q^{(m)}$ of occurrences of qth event in interval $\mathcal{I}_m$, i.e. $\nu_q(t) = \nu_q^{(m)}$ for $t \in \mathcal{I}_m$, $m = 1, 2, \ldots, M$, and, in the case of positive events, the values e_q are reduced by a half during the night shifts, see Table 2. In addition, and only in the case of population 2 (the staff), a further term is added ($\delta_{i,2} = 1$ if $i = 2$, and zero otherwise) whose strength $e_0^{(m)}$ accounts for the duty overload on the ward operators, not only because negative events increase the need of attending to the patients, but also because new admissions, discharges, and overcrowding generate an extra duty. In fact, the workload of the operators is directly affected by the admissions/discharges of the day, and usually patients are admitted either at the end of the morning shift, or at the very beginning of the afternoon shift. This motivates the following logics that has been adopted in order to compute the workload for the day:

$$e_0^{(m)} = -\,\epsilon_0 \frac{\sum_{q=5}^{8} e_q\, \nu_q^{(m)}}{N_2(t_{3m-3}) + N_2(t_{3m-2}) + N_2(t_{3m-1})} - \epsilon_0 \frac{\sum_{q=9}^{10} e_q\, \nu_q^{(m)}}{N_2(t_{3m-3}) + N_2(t_{3m-2})} \tag{2.17}$$

for the morning shift, and

$$e_0^{(m)} = -\,\epsilon_0 \frac{\sum_{q=5}^{8} e_q\, \nu_q^{(m)}}{N_2(t_{3m-3}) + N_2(t_{3m-2}) + N_2(t_{3m-1})} \tag{2.18}$$

for the afternoon and night shifts. In the above equations, ϵ_0 is an overall reduction coefficient ($\epsilon_0 = 0.5$) in regard of the high professionalism of operators.

Sentinel events. Finally, and similarly to external events, sentinel events act on the social drift by means of the stepwise constant, real valued, global function $CC(t)$:

$$CC(t) = \sum_{m=1}^{M} \left(\sum_{c \in C} e_c \nu_c(t) + \sum_{n=1}^{3M-1} e_{cc}\, \delta_{cc}^{(n)}(t) \right). \tag{2.19}$$

Each event is identified by a real value e_c selected in a finite set $\{e_{c_1}, e_{c_2}, \ldots, e_{c_C}\}$, $e_c \in \mathbb{R}$, and $c \in C := \{c_1, c_2, \ldots, c_C\}$, that specifies the *sentinel events*, all of negative nature ($e_c < 0$), that happen inside the system. As for external events, the effects of a sentinel event start at the beginning of an evolution interval $\mathcal{I}_m := [t_{3(m-1)}, t_{3m})$ and last for that interval only; the E-tuple of functions $(\nu_{c_1}, \nu_{c_2}, \ldots, \nu_{q_C})$ has values $\nu_c^{(m)} \in \{0, 1, 2, \ldots\}$, $c \in C$, to testify the occurrence of cth event, in that $\nu_c(t) = \nu_c^{(m)}$ for $t \in \mathcal{I}_m$ and $m = 1, 2, \ldots, M$.

The second term of function $CC(t)$ is more subtle. It testifies the occurrence of facts of relevant stress/disorder observed inside the system. Specifically, a *critical regime* for the system is triggered if either a sentinel event occurs, or if it happens that $\Delta U_i := (U_i - U_i^c) > 0$ or that $\Delta C_i := (C_i - C_i^c) > 0$ for either $i = 1$ or $i = 2$. As long as a critical regime persists, the function $CC(t)$ is added with a weight e_{cc} median with respect to those of the sentinel events. This reflects the experimental observation that during a crisis certain actions are undertaken by the operators (patients can be sedated, or restrained, external help is sought for, etc.) in order to re-establish "equilibrium". Let n_c be the ordinal of the sub-interval $\mathrm{I}_{n_c} := [t_{n_c-1}, t_{n_c})$ (the shift) that saw the critical regime begin. Starting from I_{n_c} a whole period of time opens, addressed to as a *critical period*:

$$\mathcal{J}_c := \mathrm{I}_{n_c} \cup \mathrm{I}_{n_c+1} \cup \cdots \cup \mathrm{I}_{n_\ell}. \tag{2.20}$$

The duration of the critical period is specified by the integer $n_\ell = n_s + dn$, where n_s corresponds to the first interval I_{n_s} in which the said events disappear and the thresholds again respected, and dn is a delay that depends on the strength of the phenomenon. Experimental observations suggest that the delay dn must be increased by one unit per each further shift wherein the thresholds are not yet respected, and starts decreasing only after the n_sth shift. In summary, a critical period $\mathcal{J}_c$ starts on I_{n_c}, triggered either by a sentinel event, or by $\Delta U_i > 0$, or by $\Delta C_i > 0$, and lasts for at least $n_\ell - n_s$ shifts; with reference to Eq. (2.19), we have

$$\begin{cases} \delta_{cc}^{(n)}(t) = 1 & \text{if } t \in \mathrm{I}_c, \\ \delta_{cc}^{(n)}(t) = 0 & \text{otherwise;} \end{cases} \qquad \text{and} \qquad e_{cc} = \frac{1}{3} \sum_{c=2}^{4} e_c. \tag{2.21}$$

As described in the following, initial conditions are also influenced by a critical regime (cf. Section 2.3).

The specific set of external and sentinel events that has been considered, and the corresponding weights, are given in Table 2, where ϵ_b is an overall normalizing coefficient ($\epsilon_b = 0.7$), and $\epsilon_p = 1$ for the day shifts and $\epsilon_p = 0.5$ at night. All the other specific values adopted in the numerical simulations for the parameters introduced above are given in Section 3.

2.2. Modeling individual interactions

Direct interactions are events that, on account of the possible singleton, or double sets of actors, i.e. "candidate" and "field" ones, refer to output values of the state of the "test" actor. In what follows only these two sets of interactions are considered (no triple or higher ones), not only on account of the fact that they appear to be not so frequent or of deep relevance, but also because including these "rare" events in the picture would further complicate the model, by involved additional guess-work in modeling situations that are hardly observed.

Interactions are identified by a set of probabilities about the state changes, and by a set of rates about their occurrence. Namely, again with subscripts i, j referring to patients or staff ($i, j = 1$ or 2 respectively), and h, l, k running over the state indices, we have

$$\mathcal{B}_{i,h} = \left(\sum_{l=1}^{5} \eta_{i,l}[\psi_{h;i,l} - \delta_{h,l}] f_{i,l} + \sum_{j=1}^{2} \sum_{k=1}^{5} \sum_{l=1}^{5} \eta_{i,l,j,k}[\psi_{h;i,l,j,k} - \delta_{h,l}] f_{i,l} f_{j,k} \right) c_n(t)\, c_c(t), \tag{2.22}$$

where $c_n(t)$ is described in Eq. (2.13), and coefficient $c_c(t)$ is similarly defined to account for the fact that during a critical regime any kind of direct interaction on the evolution is drastically reduced, here by a factor $\tilde{c}_c = 0.6$. The other functions that appear in Eq. (2.22) are hereafter specified. They are to be interpreted as interaction probabilities and interaction rates, and consist of the following functions which depend on time not only explicitly but also through expectations over the densities $\mathbf{f}$; moreover, parametrical dependence on the activity values themselves may also occur.

- $\eta_{i,l}[\mathbf{f}] : [0, T] \to [0, \infty)$ rate of the events wherein an individual of population P_i in the state $u_{i,l}$ autonomously reflects about modifying his state.

- $\eta_{i,l,j,k}[\mathbf{f}] : [0, T] \to [0, \infty)$ rate of the events wherein an individual of population P_i in the state $u_{i,l}$ interacts with an individual of population P_j in the state $u_{j,k}$.
- $\psi_{h;i,l}[\mathbf{f}] : [0, T] \to [0, 1]$ probability that the state of an individual of population P_i changes from $u_{i,l}$ to $u_{i,h}$ because of an event wherein he autonomously reflects about modifying his initial state.
- $\psi_{h;i,l,j,k}[\mathbf{f}] : [0, T] \to [0, 1]$ probability that the state of a test individual of population P_i is $u_{i,h}$ at the end of an event wherein he is the candidate individual in the state $u_{i,l}$ and interacts with a field individual of population P_j in the state $u_{j,k}$.

For the sake of clarity and simplicity, in what follows we use the notation of a (corresponding) continuous model to describe frequencies and interaction probabilities, and refer about interaction rates and probability density distributions as functions of (activity) variables u_i defined on a real interval $\mathbb{D}$. In the numerical simulations, these functions have been straightforwardly (although drastically) discretized by keeping in mind that the allowed values for activities are specified by the discrete sets $\mathbb{D}_i$ defined in Eq. (2.1). Therefore $\eta_i(x)$, $\eta_{i,j}(x, y)$ are here described instead of $\eta_{i,h}$, $\eta_{i,h,j,k}$, and, respectively, $\psi_i(u; x)$, $\psi_{i,j}(u; x, y)$ instead of $\psi_{h;i,l}$, $\psi_{h;i,l,j,k}$. Moreover, probabilities are assumed to be normalized Gaussians with respect to the (outgoing) state variable u of the test individual, and implicit functions of the states of the other interacting individuals: the former state x of the same actor for $\psi_i(u; x)$, the candidate state x and field state y for $\psi_{i,j}(u; x, y)$. For instance:

$$\psi(u; \mu(x, y), \sigma(x, y)) = \exp[-(u-\mu)^2/2\sigma^2] \Big/ \int_{\mathbb{D}} \exp[-(u-\mu)^2/2\sigma^2]\mathrm{d}u, \tag{2.23}$$

so that only functions $\mu = \mu(x, y)$ and $\sigma = \sigma(x, y)$ are left to be defined. The advantage of this approach instead of straightforwardly defining a game matrix is twofold. Firstly, the physical interpretation of the expectation μ and deviation σ is a clear although possibly non-simple matter. Describing and motivating these two quantities, as well as adapting their description to the various possible interactions, is feasible also for non-mathematicians, whose input may hence give valuable and helpful suggestions. Secondly, probability ψ is significant in the whole range $\mathbb{D}$, which implies that substantial changes of states remain possible even though improbable.

Self-interaction frequency. In modeling self-interactions we adopted the point of view that the insight capabilities of patients with mental disorders are somewhat compromised. Nonetheless, it is reasonable to assume that a tendency exists in these patients aimed at autonomously exerting a certain level of self-control, in their own and others interest. On the contrary, staff members are constantly urged to exert self-control, and even more so when the situation is about to degenerate. Based on these considerations, self-interaction frequencies have been assumed to have the form:

$$\eta_1(x) = eo_1, \quad eo_1 \geq 0.0, \tag{2.24a}$$

$$\eta_2(x) = eo_3 + eo_2\, x, \quad eo_2,\ eo_3 \geq 0.0. \tag{2.24b}$$

The adopted values of the frequency parameters are given in Section 3.

Self-interaction probability. The model for this probability is meant to describe a natural homeostatic tendency of the system to readjust itself on a time scale that, as far as patients are concerned, is comparable with the one that corresponds to their average stay in the structure (which is around 2 weeks). Assuming that the probabilities are described by

$$\psi(u; \mu(x), \sigma(x)) = \exp[-(u-\mu(x))^2/2\sigma^2(x)] \Big/ \int_{\mathbb{D}} \exp[-(u-\mu(x))^2/2\sigma^2(x)]\mathrm{d}u, \tag{2.25}$$

we modeled the expected values $\mu = \mu_i(x)$ and deviations $\sigma = \sigma_i(x)$ per each population as follows:

$$\mu_i(x) = m_i\, x, \quad 0.0 \leq m_i \leq 1.0, \tag{2.26a}$$

$$\sigma_i(x) = \sigma_i, \quad 0.0 < \sigma_i \ll 1.0. \tag{2.26b}$$

As before, the adopted values for $m_1, m_2, \sigma_1, \sigma_2$ are specified in Section 3.

Pairwise interaction frequency. Pairwise interaction frequencies $\eta_{i,j}(x, y)$ refer about the rates of interactions between (different) individuals of the same or of different populations; for the specific physical problem at hand, we set for $i, j = 1, 2$:

$$\eta_{i,j}(x, y) = ez_{i,j} + e_{i,j}\,(1-x^2)(1-y^2), \quad ez_{i,j}, e_{i,j} \geq 0.0. \tag{2.27}$$

This assumption is meant to express the fact that the interactions among patients and operators have a base frequency $ez_{i,j}$, which is possibly modulated by a term that depends on the state variables in such a way that the number of interactions decreases when the state variable increases, i.e. when it refers to actors in more negative states (Fig. 1). Simulation values are given is Section 3; however it is appropriate anticipating here that those values are direct consequences of the normal life and daily procedures inside the ward, which effectively specifies their ranges.

We point out that all interaction rates necessarily depend on time: their values at night are drastically lower than their morning and afternoon ones, which motivates the overall factor $c_n(t)$ in Eq. (2.22).

Pairwise interaction probability. As mentioned in Eq. (2.23), we assume normalized Gaussian distributions with expectations $\mu = \mu_{i,j}(x, y)$ and deviations $\sigma = \sigma_{i,j}(x, y)$ specified below.

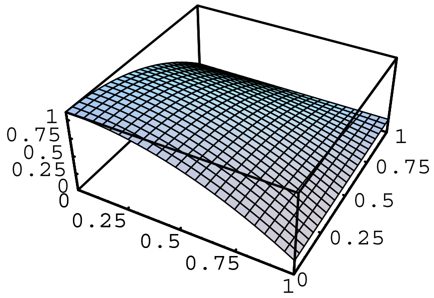

Fig. 1. Illustrative $\eta_{i,j}(x, y) = 0.2 + 0.9\,(1 - x^2)(1 - y^2)$.

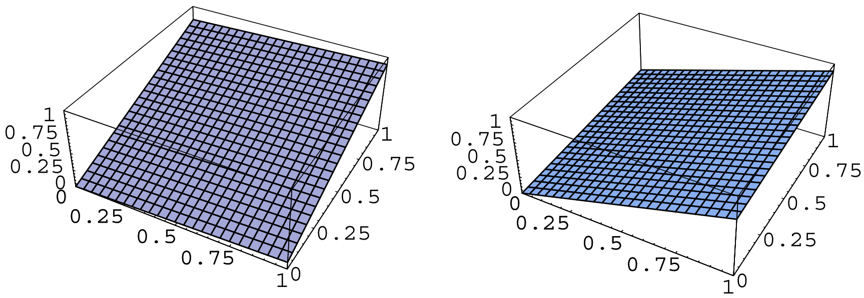

Fig. 2. Illustrative $\mu_{1,1}$ with $m_3 = 0.9$ (left) and $m_3 = 0.2$ (right).

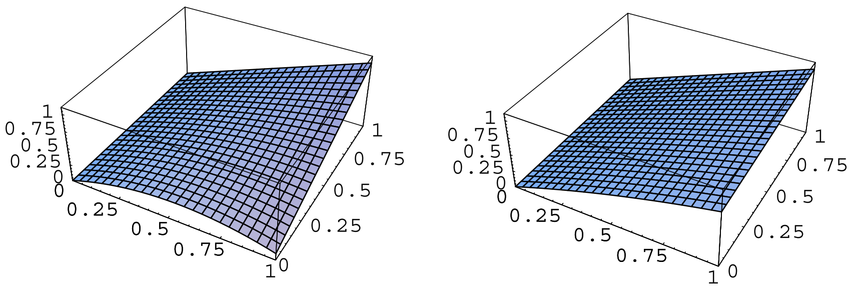

Fig. 3. Illustrative $\mu_{1,2}$ with $m_4 = 0.9$ (left) and $m_4 = 0.2$ (right).

For the interaction between two patients, the assumption is a simple altruistic model (Fig. 2):

$$\mu_{1,1}(x, y) = (1 - m_3)x + m_3 y, \quad \begin{array}{l} 0.0 \le m_3 \le 1.0, \\ \sigma_{1,1} =: \sigma_3 \ll 1.0. \end{array} \tag{2.28a}$$

Mean value μ aims at pointing out the expected state of the test actor after the interaction. Therefore, when it refers about an interaction of a patient with an operator,[2] it has to account for the therapeutical value of the interaction. The experimental observation that the improvement is usually found to be more dramatic the more negative the initial condition of the patient, and at the same time that it depends on the stress level of the operator, is expressed by (Fig. 3):

$$\mu_{1,2}(x, y) = x - m_4\, x^2(1 - y), \quad \begin{array}{l} 0.0 \le m_4 < 1.0, \\ \sigma_{1,2} =: \sigma_4 \ll 1.0. \end{array} \tag{2.28b}$$

In modeling the effect of operator–patient interactions on the operators, one obviously assumes that the operators are professionals, whose state is not altered by the interaction unless the patient is in critical conditions (Fig. 4):

$$\mu_{2,1}(x, y) = \begin{cases} x & \text{if } y \ge 0.5, \\ m_5\, x & \text{otherwise}, \end{cases} \quad \begin{array}{l} 0.0 \le m_5 < 1.0, \\ \sigma_{2,1} =: \sigma_5 \ll 1.0. \end{array} \tag{2.28c}$$

Finally, again taking into account the high professionalism of the operators, the interaction between two operators has been modeled as follows (Fig. 5):

$$\mu_{2,2}(x, y) = (1 - \tilde{m})\, x + \tilde{m}\, y, \quad \tilde{m} = \begin{cases} m_6\, x^* & \text{if } x \le x^* := 0.3, \\ m_6\, x & \text{otherwise} \end{cases} \tag{2.28d}$$
$$0.0 \le m_6 \le 1.0, \qquad \sigma_{2,2} =: \sigma_6 > 1.0.$$

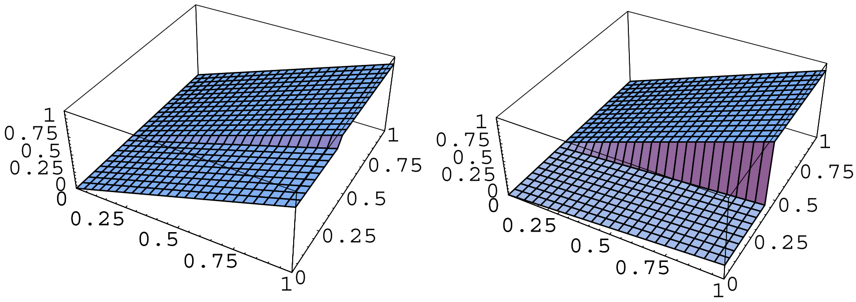

Fig. 4. Illustrative $\mu_{2,1}$ with $m_5 = 0.9$ (left) and $m_5 = 0.2$ (right).

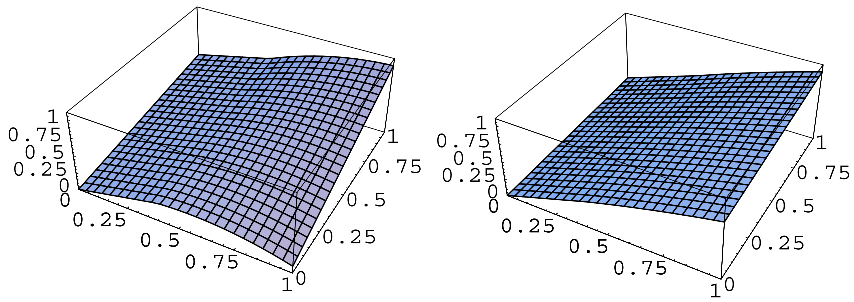

Fig. 5. Illustrative $\mu_{2,2}$ with $m_6 = 0.9$ (left) and $m_6 = 0.2$ (right).

The specific values adopted in the simulations are given in Section 3.

2.3. Modeling initial conditions

As explained above, the dynamics of the system develops, on a stepwise basis, over a sequence of consecutive time intervals $\mathrm{I}_n = [t_{n-1}, t_n)$, $n = 1, \ldots, 3M \in \mathbb{N}$. At each t_n staff turns over, and at each instant t_{3m-2}, $m = 1, 2, \ldots, M$, patients are registered as discharged or admitted. In order to determine the time evolution of the solutions $f_{i,h}(\cdot) : t \mapsto \mathrm{I}_n$ of the ODE system (2.9), one has to specify the necessary initial conditions, i.e. the values $f_{i,h}(t_{n-1})$ for all $n = 1, \ldots, 3M$. The assumptions on initial conditions are as follows.

Concerning the patients, admissions and dismissals happen during the morning shift of the day. Therefore the values $f_{1,h}(t_{3m-2})$ need to be defined, depending on those that pertain to the newly admitted actors and to those that already are inside the structure. Denoting by $g_{1,h}^{(m)}$ the distribution of the actors admitted in the morning shift $\mathrm{I}_{3m-2} = [t_{3(m-1)}, t_{3m-2})$ of the mth day $\mathcal{I}_m = [t_{3(m-1)}, t_{3m})$, we set

$$f_{1,h}(t_{3m-2}) = \frac{1}{N_1^{(m)}}[N_1^{(+,m)}\, g_{1,h}^{(m)} + (N_1^{(m-1)} - N_1^{(-,m)})\, f_{1,h}(t_{3m-2}^{(-)})] \tag{2.29}$$

where: $N_1^{(m)}$ denotes the number of patients that are eventually inside the structure on interval $\mathcal{I}_m$; $N_1^{(+,m)}$ and $N_1^{(-,m)}$ are the numbers of patients respectively admitted or discharged on interval I_{3m-2}; $f_{1,h}(t_{3m-2}^{(-)})$ denotes the distribution at the end of that same interval, namely: $\lim_{\epsilon\to 0^+} f_{1,h}(t_{3m-2} - \epsilon)$. Eq. (2.29) is motivated by the fact that the $N_1^{(-,m)}$ patients that leave the system during I_{3m-2} are assumed randomly extracted from those having distribution $f_{1,h}(t_{3m-2}^{(-)})$, while the remaining ones are in number of $(N_1^{(m-1)} - N_1^{(-,m)})$, and are similarly distributed. Clearly we have: $N_1^{(m)} = N_1^{(m-1)} - N_1^{(-,m)} + N_1^{(+,m)}$, and hence Eq. (2.29) may also be written as

$$f_{1,h}(t_{3m-2}) = \frac{N_1^{(+,m)}}{N_1^{(m)}}\, g_{1,h}^{(m)} + \left(1 - \frac{N_1^{(+,m)}}{N_1^{(m)}}\right) f_{1,h}(t_{3m-2}^{(-)}).$$

Further, the distribution of the $N_1^{(+,m)}$ individuals entering the system is specified by the maps $g_{1,h}^{(m)}$; these are again assumed according to Eq. (2.23), with a deviation of $\sigma = 0.2$ and with various possible expectations: $\hat{\mu}_1 = -0.1$, $\hat{\mu}_2 = 0.1$, $\hat{\mu}_3 = 0.5$,

[2] Here and in the following, we refer for shortness to any staff member as an "operator".

$\hat{\mu}_4 = 0.8$, $\hat{\mu}_5 = 1.1$, depending on the circumstances. On due account of the fact that patients that are newly admitted are certainly in a slightly negative state, in this model it is assumed that their distribution is roughly centered at $\hat{\mu}_2 = 0.1$, unless the evolution interval $\mathcal{I}_m$ is affected by a sentinel event (see Table 2), in which case this value is correspondingly higher. With the same notation as in Eq. (2.17) and with $[x]$ denoting the integer part of x, in this case the distribution $g_{1,h}^{(m)}$ is centered at $\hat{\mu}_\ell$ with ℓ given by

$$\ell = \left[\frac{\sum_{q=5}^{8} |e_q| \nu_q^{(m)}}{2\, N_1^{(+,m)}} \right].$$

On the other hand, as far as operators are concerned, a complete staff turn over occurs at each shift $\mathrm{I}_n = [t_{n-1}, t_n)$, $n = 1, 2, \ldots, 3M - 1$; hence, again unless the period $\mathcal{I}_m$ is affected by some sentinel event, we similarly guess that at each time: $t_{3m-3},\ t_{3m-2},\ t_{3m-1}$ the new personnel enters with a distribution centered at $\hat{\mu}_1$. Otherwise, i.e. if a sentinel event occurs on $\mathcal{I}_m$, on all the three shifts I_n for $n = 3m-2,\ 3m-1,\ 3m$, we set the expectations $\hat{\mu}_\ell$ of the distributions $f_{2,h}(t_{n-1})$ with ℓ given by

$$\ell = 1 + \left[\frac{\sum_{q=2}^{4} |e_q|\, \nu_q^{(n)}}{2\, N_2(t_{n-1})} \right].$$

The levels ℓ of expectations $\hat{\mu}_\ell$ of both patients and staff distributions are raised (up to saturation) of one level if a critical regime is already in place, and of two levels when this regime begins.

Finally, we conclude this section by presenting the complete input data matrix of our model. The matrix consists of $M = 1428$ rows, each relative to one day of the observation period. Using the notation introduced in Eqs. (2.16), (2.19) and (2.29), the mth row, $m = 1, \ldots, M$ of the input data matrix contains the following 16 integers:

$$N_1^{(m)}, N_1^{(-,m)}, N_1^{(+,m)}, N_2(t_{3m-3}), N_2(t_{3m-2}), N_2(t_{3m-1}), \nu_1^{(m)}, \nu_2^{(m)}, \ldots, \nu_{10}^{(m)}.$$

3. Numerical simulations

A numerical code has been implemented to set up and solve the chain of initial value problems for the system of ODEs (2.9), either when the input/initial value data are read from the historic series, or when they are randomly simulated via Monte Carlo processes.

In this section we discuss the fine-tuning of the (many) parameters of the model. A case period of 21 days, from May 27, 2001 to June 15, 2001, has been accurately selected, based on prototypical functioning of the medical service and of data collection procedures. The length of the case period has been suggested by the fact that the average stay of patients inside the structure is about 2 weeks, and by the fact that the number of autonomous evolution intervals in this period is acceptably high (3 shifts per turn, 3 turns per day, 21 days).

Analyzing a large number of numerical simulations, we have been able to identify those parameters whose variations the model is most sensitive to, and choose their optimal values, in comparison with those that, conversely, produce less important effects and can possibly be ignored. Obviously, the assumption that these values should be valid over extended periods of time, such as several months or even years, is questionable. Since the selected case period contains all kinds of significant events that characterize the system dynamics over the whole period of data collection (from Feb 1, 2001 to Dec 31, 2004), in this paper we present the results of the simulations over this extended period using the set of parameters tuned on the case period only. As a matter of fact, within the extended period several phases have been identified in the system, when the medical service exhibited distinct functioning patterns. These, in principle, might require different sets of parameter values, or at least a more specific and detailed analysis. However, it turns out that the observed phases can be put in correspondence with overall organizational changes in the service, and are evident only on a very long time scale. These issues will be briefly discussed in Section 4, and a thorough investigation is postponed to a future work.

Here it is more important to point out that, unlike mechanical models where the scales are dictated by clearly identifiable physical parameters, such as masses, lengths etc., in the system under study there are no obvious scales, neither for time nor for the activity variable (actually, too many for the first one and none for the latter). As well, no a priori structures or physical laws or conserved quantities are available that might indicate any kind of (microscopic, deterministic) dynamics or expected behavior. Therefore, the first important numerical simulations have been devoted to establish the order of magnitude of all the model parameters. Regardless of whether the latter are intrinsically of real or of integer type, or whether these properties were only assumed in developing the model, all the same they are to be considered as absolute, rather than rescalable or symbolic, quantities.

Once this task was accomplished, and the parameters which the model is more sensitive to were identified, then the response of the model to their variations within the established ranges has been analyzed. Different groups of relevant parameters have been contrasted and compared in the numerical simulations. In the following, we list the relevant parameters, the corresponding ranges of values where they have been varied, and the optimal choices found in connection with the aforementioned case period.

Social parameters. To begin with, the various parameters used to construct the phenomenological social (mean field) function are recalled; per each of them, the interval that has been explored, and the final value that proved to be the best choice in that interval (hence used in the simulations) is referred.

- α defines the balance between patients and staff, respectively, on the mean activity (cf. Eq. (2.14)); we have $\alpha = 0.5 \in [0.0,\ 1.0]$.
- β_1, β_2 weight the mean activity and climate field on the social drift (cf. Eq. (2.12)); β_1 larger than β_2 makes the drift depend more heavily on the activity than on the fraction of populations in highly negative states, and vice versa; we have $\beta_1 = 1.0 \in [0.0,\ 3.0]$, $\beta_2 = 2.0 \in [0.0,\ 3.0]$.
- γ defines the balance between patients and staff, respectively, on the climate field (cf. Eq. (2.12)); we have $\gamma = 0.5 \in [0.0,\ 1.0]$.
- p_1, p_2 weight the sentinel events field on the social drift respectively for patients and operators; we have $p_1 = 1.0 \in [-1.5,\ 3.0]$ and $p_2 = 0.0 \in [-1.5,\ 3.0]$.
- r_1, r_2 weight the external events field on the social drift respectively for patients and operators; we have $r_1 = 1.0 \in [0.5,\ 1.0]$ and $r_2 = 0.6 \in [0.5,\ 1.0]$.
- v_1, v_2 are constant shifts for the social drift; we have $v_1 = v_2 = 0.0 \in [-3.0,\ 6.0]$.
- w_1, w_2 define the reference thresholds on the mean activities (cf. Eq. (2.14)); we have $w_1 = 0.4 \in [0.0,\ 1.0]$ and $w_2 = 0.2 \in [0.0,\ 1.0]$.
- C_1^c, C_2^c are threshold values for the population fractions C_1 and C_2 in the climate field (cf. Eq. (2.15)); we have $C_1^c = 0.7 \in [0.4,\ 0.8]$ and $C_2^c = 0.7 \in [0.4,\ 0.8]$.
- k_1^c, k_2^c are threshold levels for patients/operators respectively, to define population fractions C_1 and C_2 in Eq. (2.15); we have $k_1^c = 4 \in [3,\ 6]$ and $k_2^c = 4 \in [3,\ 6]$.
- ϵ_b weights negative external events with respect to positive and sentinel ones (cf. Table 2); we have $\epsilon_b = 0.7 \in [0.0,\ 1.0]$.

Interaction frequency parameters. As already mentioned, the parameters that control the interaction frequencies in (2.24) and (2.27) play a fundamental role in the simulations. It has been estimated that, due to routine sanitary procedures, patients and operators interact among themselves roughly between 5 and 9 times a day; therefore, ultimately we set: $ez_{1,1} = ez_{2,2} = 5.0$, and $e_{1,1} = e_{2,2} = 4.0$. Interaction frequencies among operators and patients are slightly larger, on the average between 6 and 11 times a day; however, in these ranges of values the corresponding variations observed in the numerical simulations seemed to be negligible, and hence we set $ez_{1,2} = 5.0$, and $e_{1,2} = 4.0$ as well. Obviously interaction frequencies are symmetric, i.e. $ez_{1,2} = ez_{2,1}$, and $e_{1,2} = e_{2,1}$. For all of them the variation range has been [2.0, 6.0]. We have

$$\begin{aligned} &eo_1 = 2.0, \qquad eo_2 = 2.0, \qquad eo_3 = 2.0, \\ &ez_{1,1} = 5.0, \qquad ez_{1,2} = 5.0, \qquad ez_{2,2} = 5.0, \\ &e_{1,1} = 4.0, \qquad e_{1,2} = 4.0, \qquad e_{2,2} = 4.0. \end{aligned}$$

Interaction probability parameters. Interaction probability distributions have been modeled in terms of normalized Gaussian distributions, with expectations μ_j and $\mu_{i,j}$ and deviations σ_j and $\sigma_{i,j}$ [cf. (2.26) and (2.28)]. All deviations have been chosen to be constant, i.e., independent of the state variables, and eventually all set equal to 0.2. The parameters controlling the expectations $\mu_{i,j}$ have been varied in the range $[0.0,\ 1.0]$, and then chosen as follows:

$$\begin{aligned} &m_1 = 1.0, \qquad m_2 = 0.5, \qquad m_3 = 0.5, \\ &m_4 = 0.8, \qquad m_5 = 0.8, \qquad m_6 = 0.2. \end{aligned}$$

In Fig. 6 we show a typical graphic output produced by the simulation. The plots show, versus time, the five probability functions for patients populations (top half of the plot) and the corresponding staff ones (bottom half) relative to the period Sept 13, 2001–Oct 10, 2001. The colors used in the graphs refer to the activities that characterize each of the five levels, and are the same as those used for the Ward Atmosphere and explained in Table 1. The simulation shows critical regimes (the appearance of orange/red peaks), which indeed have a direct correspondence with what has been observed in the hospital in the period of time under consideration (cf. Fig. 8). In particular, one can observe how the first crisis, around the 2nd and 3rd day, is mostly due to patients, while the following ones on days 8th and 28th show both patients and staff in highly negative states. In fact, a large number of sentinel and negative external events has been recorded during each of the aforementioned critical periods. On the other hand, the rapidly varying behavior of the operator probability functions shows the effects of their work during each shift, whether it is a depletion due to high workload or an improvement due to professional gratification. Moreover, the different initial conditions accounts for their reaction on entering in an altered environment.

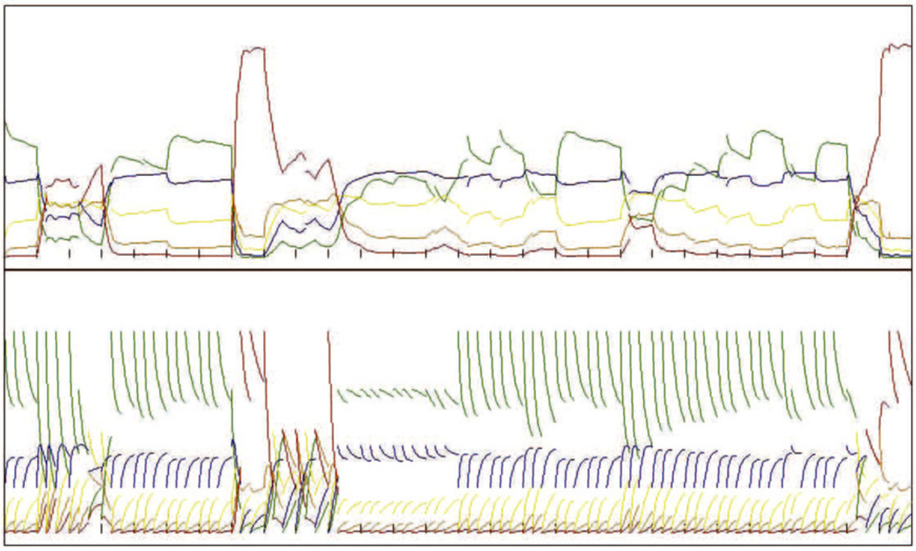

Fig. 6. Illustrative graphic output from the numerical simulations for the probability functions.

4. Comparison with experimental data and statistical significance

In order to compare the predictions of the model with the experimental data, it is necessary to convert the output of the numerical simulations, i.e., the 10 probability functions for patients and staff, into a single sequence of integer values in the set $\{2, 4, 6, 8, 10\}$ that represent the estimate for the ward atmosphere on each shift.

After several attempts, we managed to identify two indicators whose discretizations may be reasonably compared with the experimental data for the atmosphere. Specifically, we considered:

$$U = \alpha U_1 + (1-\alpha)U_2, \qquad U^c = \alpha U_1^c + (1-\alpha)U_2^c, \tag{4.1}$$

where U_i are defined in (2.7) and the thresholds U_i^c are specified in (2.14), and we introduced the rescaled variable

$$R = U/U^c. \tag{4.2}$$

Based on the variables U and R, we propose two different estimates for the ward atmosphere. The first estimate, based on the variable U, is given by:

$$\begin{array}{ll} U^* = 2 & \text{if } 0.0 \le U < 1.7 \\ U^* = 4 & \text{if } 1.7 \le U < 2.5 \\ U^* = 6 & \text{if } 2.5 \le U < 5.0 \\ U^* = 8 & \text{if } 5.0 \le U < 6.0 \\ U^* = 10 & \text{if } U \ge 6.0. \end{array} \tag{4.3a}$$

The second estimate, based on the variable R, is set to:

$$\begin{array}{ll} R^* = 2 & \text{if } 0.0 \le R < 0.4 \\ R^* = 4 & \text{if } 0.4 \le R < 0.6 \\ R^* = 6 & \text{if } 0.6 \le R < 0.8 \\ R^* = 8 & \text{if } 0.8 \le R < 0.9 \\ R^* = 10 & \text{if } R \ge 0.9. \end{array} \tag{4.3b}$$

In an attempt of enhancing the accuracy of the description, all intervals for the estimates have then been split into two, thus producing two new estimates:

$$\begin{array}{llll} bU = 2 & \text{if } 0.0 \le U < 1.7, & bR = 2 & \text{if } 0.00 \le R < 0.40, \\ bU = 3 & \text{if } 1.7 \le U < 2.0, & bR = 3 & \text{if } 0.40 \le R < 0.45, \\ bU = 4 & \text{if } 2.0 \le U < 2.5, & bR = 4 & \text{if } 0.45 \le R < 0.50, \\ bU = 5 & \text{if } 2.5 \le U < 3.0, & bR = 5 & \text{if } 0.50 \le R < 0.60, \\ bU = 6 & \text{if } 3.0 \le U < 4.0, & bR = 6 & \text{if } 0.60 \le R < 0.70, \\ bU = 7 & \text{if } 4.0 \le U < 5.0, & bR = 7 & \text{if } 0.70 \le R < 0.80, \\ bU = 8 & \text{if } 5.0 \le U < 6.0, & bR = 8 & \text{if } 0.80 \le R < 0.90, \\ bU = 9 & \text{if } 6.0 \le U < 7.0, & bR = 9 & \text{if } 0.90 \le R < 0.95, \\ bU = 10 & \text{if } U \ge 7.0, & bR = 10 & \text{if } R \ge 0.95. \end{array} \tag{4.3c}$$

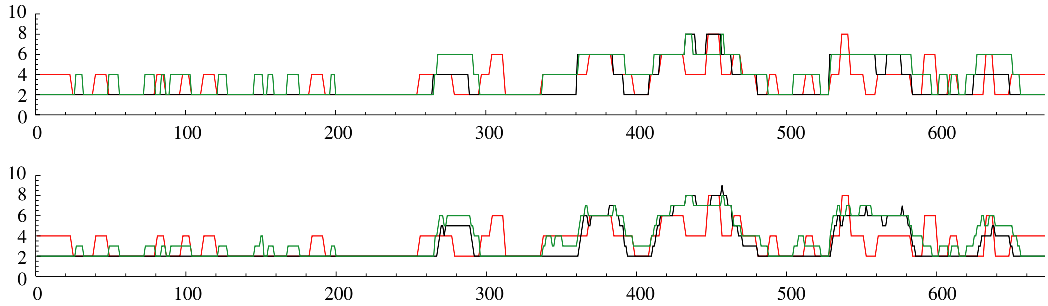

Fig. 7. Comparison among experimental and estimated atmospheres for the period May 24–Jun 20, 2001 (corresponding to 672 h on the *x*-axis). Top: Experimental (red/thin), R^* (black/solid), U^* (green/dashed). Bottom: Experimental (red/thin), *bR* (black/solid), *bU* (green/dashed).

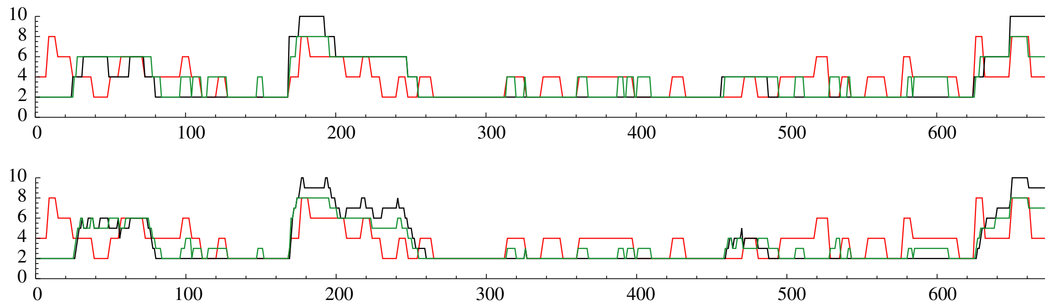

Fig. 8. Comparison among experimental and estimated atmospheres for the period Sept 13–Oct 10, 2001 (corresponding to 672 h on the *x*-axis). Top: Experimental (red/thin), R^* (black/solid), U^* (green/dashed). Bottom: Experimental (red/thin), *bR* (black/solid), *bU* (green/dashed).

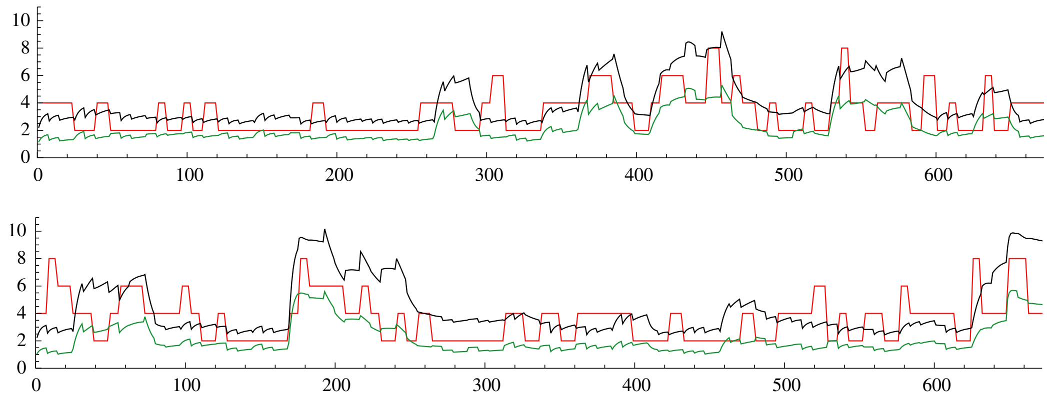

Fig. 9. Comparison among experimental and estimated atmospheres for the two periods May 24–Jun 20, 2001 (top), and Sept 13–October 10, 2001 (bottom), with Experimental (red/thin), U (green/dashed), $10R$ (black/solid).

A similar logics has been applied to the experimental data, for which a linear interpolation has been used to provide intermediate values for the atmosphere within each shift.

The threshold values for the discretizations specified above have been chosen after a number of attempts, as those that appeared to provide good approximations to the experimental values for the atmosphere.

It is clear that the quantitative results of these comparisons are severely biased by the choice of the thresholds in definitions (4.3) and that only a careful a posteriori test may provide the best choice for their values. All the same, if one is mainly interested in a qualitative estimate, then small differences in such choices do not play a significant role. In Fig. 7 we show a sample of the comparisons for two typical periods.

As a matter of fact, a qualitative comparison between the estimates and the experimental values for the atmosphere is easier and more informative if one looks directly at the "continuous" variables U, R, instead of using the discretizations introduced above. The comparison for the two periods indicated above is shown in Fig. 9.

On the other hand, in order to investigate the statistical significance of the model and to obtain a quantitative comparison, some global indicator has to be identified. To this aim, we first compute some straightforward mean square deviations between experimental and estimated values for the atmosphere. More specifically, the entire period of the analysis (Feb 1, 2001–Dec 31, 2004) has been divided into 51 periods of 28 days each. Then, we denote by $v_j, j = 1, \ldots, N$ the experimental value for the atmosphere at each hour j of the period, and by p_j the simulated value, according to one of the algorithms listed above for either U or R. Hence, $N = 672$ is the total number of evaluations per each of the periods under consideration. We

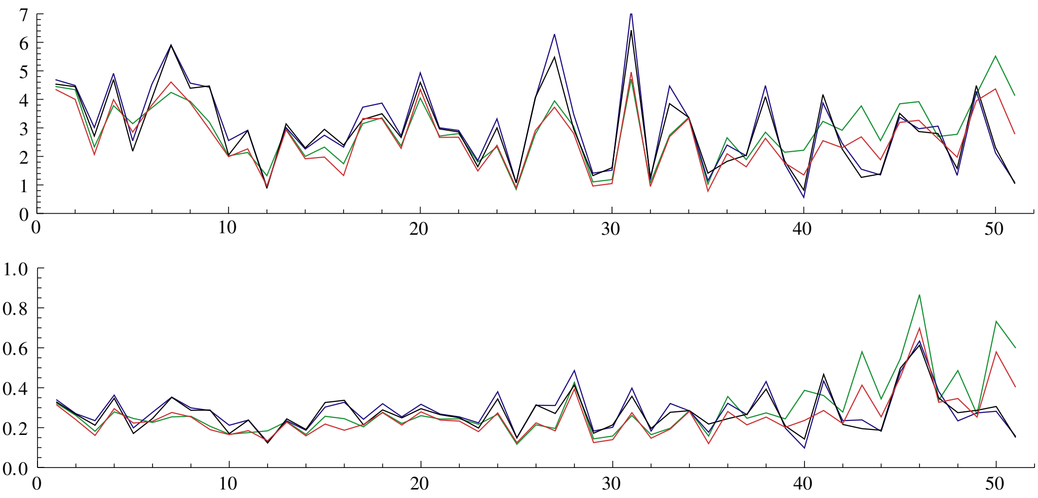

Fig. 10. Sd/N (top) and Sd/Sv (bottom), with U^* (green/dashed), R^* (black/solid), bR (blue/dot-dashed), bU (brown/dashed).

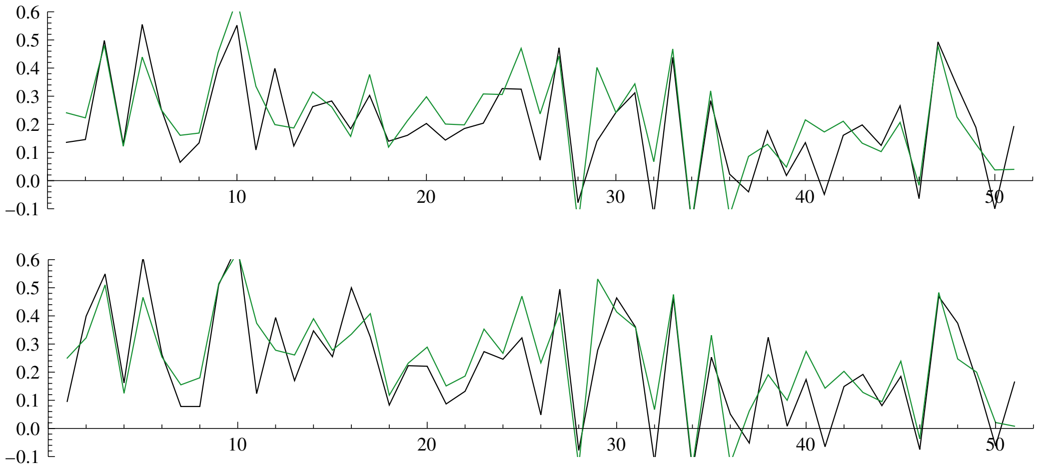

Fig. 11. Spearman's rho (top) and Pearson's correlation coefficient (bottom) — experimental Atm vs. U^* (green/dashed), R^* (black/solid).

introduce square deviations:

$$Sd = \sum_{j=1}^{N} (v_j - p_j)^2, \tag{4.4}$$

as well as the sums

$$Sv = \sum_{j=1}^{N} v_j^2. \tag{4.5}$$

The values of the deviations Sd/N and Sd/Sv are given by the polygonal lines in Fig. 10, where each point corresponds to one of the aforementioned 51 periods.

An alternative and somehow more sophisticated attempt of statistical analysis has been performed, in order to compare the experimental values for the atmosphere with the estimates from the numerical simulations by means of indicators that are well known in the specialized psychological and social literature. In particular, the values of the Pearson and Spearman correlation coefficients between the experimental data and two of the estimates above have been evaluated and given in Fig. 11. As before, each point in Fig. 11 corresponds to one of the aforementioned 51 periods.

Unlike Fig. 10, in Fig. 11, obviously, the agreement is expected to be better the larger the values of the correlation coefficients.

Even though it is questionable whether one can use linear statistics to compare these data, we observe that for most periods the 'small deviations' and 'large correlation' approaches tend to agree.

Also, it should be pointed out that for most of the periods that we have analyzed, the values obtained for the correlation coefficients, from the point of view of psychosocial statistics, are considered to indicate a remarkably good correlation among the data.

5. Concluding remarks

Even though we used generalized kinetic theories and the corresponding usual methodologies, our system is significantly different from those typically dealt with in the kinetic framework, and in particular from systems such as cells, or microorganisms. In fact, in our case: (1) the number of individuals is quite small; (2) we do not use mechanical variables; (3) macro- and micro-ensembles can be distinguished only by referring either to the type of effects (field or interactions) or to the type of variables (mean or individual variables). As a matter of fact, the assumption that the system is closed and that it consists of few individuals makes mechanical coordinates (position and velocity) totally inadequate to the description: neither frequency of interactions nor probability outputs can depend on the mechanics, i.e., on position and velocity of the individuals or of their interaction locations. Similarly, the mean field itself cannot be expressed in mechanical terms: it does not make sense to speak of short range or long range interactions, or of small or large scales. The only thing that can meaningfully be described are pairwise interactions between individuals/actors or mean field effects on each individual, and only in this sense the latter can be defined as "macroscopic effects".

Therefore, even methods and tools that have become standard in generalized kinetic theories must be suitably adapted: our model can at most be considered as analogue to a generalized kinetic model in which the populations are uniformly distributed both in space and with respect to the velocity field.

Since we deal with fully developed individuals, the table of the games itself cannot be simple: the factors that determine the behavior at the evolved human level are multiple and balance out in a complicated way. Quantities such as some of the variables in our system are already questionable with respect to their same definition, and even more so if one aims at investigating and predicting their quantitative aspect.

Moreover, on one hand the actors' perceptions create effects that are clearly defined and identifiable in the proper setting, and this happens without making use of any mechanical variable. On the other hand, there is no a priori dynamical law among the actors that one can identify and adopt. Therefore, the transition from a discrete (individual) to a continuous (social) setting poses obvious difficulties that we feel cannot be solved by recurring to standard renormalization procedures or to thermodynamic limits, and even more so on account of the small number of individuals. At most, one can look for and try to identify the series of rules that determine the effects of the single individual over the social environment, and of the social environment on the single individual.

It is in this context that our work can be of interest: although with some difficulties and uncertainties, in our opinion the present study showed, and can still show, that some of the predicted or possible behaviors (in our language, "parameters" or "events") are less relevant than others, which on the contrary are crucial, and also that certain groups of effects/contributions can cancel out, even though each of them cannot be separately neglected.

The problem of measuring the atmosphere is a serious one: the fact that it consists of only 5 levels, which are neither uniquely specified nor objectively measurable, suggested that in some cases the experimental datum was questionable, and even biased by factors external to the system. The assumption that the system is closed, and that it depends on the environment only through the annotations that compose the "historic series", is actually a very strong one under these respects. The fact that we are dealing with individuals and not with identical microorganisms generates an unavoidable series of factors that make them non-identical. In fact, their memories and opinions lead them to act in an autonomous and non-repeatable way. Therefore, even without explicit reasons that can justify unexpected behaviors, or on long time scales, the simulations may be considered more reliable, or at least more reasonable, than certain experimental data, which have then been marked as "errors".

A quantitative description is made even harder by those events that in the historic series appear as interactions with the external environment; indeed, these make any kind of asymptotic dynamics, even if just stationary, almost impossible. Admissions and discharges themselves, which are clearly an unavoidable aspect of the system, make the search for equilibria or asymptotic states almost meaningless.

In conclusion, the above remarks allow one to justify, at least partially, the use of a statistical model, i.e. a model that accounts for the uncertainty and indeterminacy intrinsic in the system. What we attempted here is to motivate this uncertainty by specific individual arguments, by others of a more social nature, and by letting the two aspects coexist and interact at an intermediate scale.

Given the intrinsic complexity of the system and of the research project, the results can be considered satisfactory. We realize that criticisms may come both from the world of mathematical–physical sciences and from that of medical, psychological and social ones. However, it is generally agreed that the use of quantitative and exact methods in the framework of social phenomena is more and more compelling. Moreover, interactions and communication between the two disciplines are required in order to develop new mathematical methods and new physical variables, justifying their definitions, interpretation and usage. This is precisely the gap that the present work has tried to explore and fill in.

Acknowledgments

MLS is grateful to UCCS for the kind hospitality during the completion of the final revision of the paper.

BP acknowledges partial support for this work by the National Science Foundation under Grant No. DMS-1009248 and by the UCCS Graduate School through a CRCW seed grant.

References

[1] C. Cercignani, The Boltzmann Equation and its Application, Springer, New York, 1988.
[2] M. Delitala, A. Tosin, Mathematical modelling of vehicular traffic: a discrete kinetic theory approach, Math. Models Methods Appl. Sci. 17 (2007) 901–932.
[3] A. Bellouquid, M. Delitala, Asymptotic limits of a discrete kinetic theory model of vehicular traffic, Appl. Math. Lett. 24 (2011) 672–678.
[4] M. Lo Schiavo, A personalized kinetic model of traffic flow, Math. Comput. Modelling 35 (2002) 607–622.
[5] D. Helbing, A. Hennecke, V. Shvetsov, M. Treiber, MASTER: macroscopic traffic simulation based on a gas-kinetic, non local traffic model, Transp. Res. B (2001) 183–211.
[6] S. De Lillo, M. Delitala, M.C. Salvatori, Modelling epidemics and virus mutations by methods of the mathematical kinetic theory for active particles, Math. Models Methods Appl. Sci. 19 (2009) 1405–1425.
[7] N. Bellomo, A. Bellouquid, J. Nieto, J. Soler, Complexity and mathematical tools toward the modelling of multicellular growing systems, Math. Comput. Modelling 51 (2010) 441–451.
[8] N. Bellomo, A. Bellouquid, J. Nieto, J. Soler, Multiscale biological tissue models and flux-limited chemotaxis for multicellular growing systems, Math. Models Methods Appl. Sci. 20 (2010) 1179–1207.
[9] A. Bellouquid, M. Delitala, Mathematical methods and tools of kinetic theory towards modelling complex biological systems, Math. Models Methods Appl. Sci. 15 (2005) 1639–1666.
[10] N. Bellomo, M. Delitala, From the mathematical kinetic, and stochastic game theory to modelling mutations, onset, progression and immune competition of cancer cells, Phys. Life Rev. 5 (2008) 183–206.
[11] E. Jager, L. Segel, On the distribution of dominance in a population of interacting anonymous organisms, SIAM J. Appl. Math. 52 (1992) 1442–1468.
[12] M.L. Bertotti, M. Delitala, On a discrete generalized kinetic approach for modelling persuaders influence in opinion formation processes, Math. Comput. Modelling 48 (2008) 1107–1121.
[13] M. Lo Schiavo, Population kinetic models for social dynamics: dependence on structural parameters, Comput. Math. Appl. 44 (2002) 1129–1146.
[14] M.L. Bertotti, M. Delitala, On the existence of limit cycles in opinion formation processes under time periodic influence of persuaders, Math. Models Methods Appl. Sci. 18 (2008) 913–934.
[15] M.L. Bertotti, M. Delitala, Conservation laws and asymptotic behavior of a model of social dynamics, Nonlinear Anal. RWA 9 (2008) 183–196.
[16] M.B. Gordon, J.-P. Nadal, D. Phan, V. Semeshenko, Discrete choices under social influence: generic properties, Math. Models Methods Appl. Sci. 19 (2009) 1441–1481.
[17] M.L. Bertotti, M. Delitala, From discrete kinetic and stochastic game theory to modeling complex systems in applied sciences, Math. Models Methods Appl. Sci. 14 (2004) 1061–1084.
[18] N. Bellomo, Modeling the hiding–learning dynamics in large living systems, Appl. Math. Lett. 23 (2010) 907–911.
[19] N. Bellomo, B. Carbonaro, L. Gramani, On the kinetic and stochastic games theory for active particles: some reasonings on open large living systems, Math. Comput. Modelling 48 (2008) 1047–1054.
[20] I. Gallo, A. Barra, P. Contucci, Parameter evaluation of a simple mean-field model of social interaction, Math. Models Methods Appl. Sci. 19 (2009) 1427–1439.
[21] G. Ajmone Marsan, N. Bellomo, M. Egidi, Towards a mathematical theory of complex socio-economical systems by functional subsystems representation, Kin. Rel. Mod. 1 (2008) 249–278.
[22] G. Ajmone Marsan, New paradigms towards the modelling of complex systems in behavioral economics, Math. Comput. Modelling 50 (2009) 584–597.
[23] I. Goryanin, O. Demin, Kinetic Modelling in Systems Biology, Chapman Hall/CRC, 2009.
[24] A. Bellouquid, M. Delitala, Modelling Complex Biological Systems A Kinetic Theory Approach, Birkhäuser, Boston, 2006.
[25] N. Bellomo, M. Lo Schiavo, Lectures Notes on the Generalized Boltzmann Equation, World Sci., London, Singapore, 2000.
[26] M. Lo Schiavo, B. Prinari, A.V. Serio, Analysing quality with generalized kinetic methods, Math. Comput. Modelling 47 (2008) 1150–1166.
[27] M. Lo Schiavo, The modelling of political dynamics by generalized kinetic (Boltzmann) models, Math. Comput. Modelling 37 (2003) 261–281.
[28] M. Lo Schiavo, Kinetic modelling and electoral competition, Math. Comput. Modelling 42 (2005) 1463–1486.
[29] World Health Organisation, Expert committee on mental health, Third report (Technical report series no. 73) Geneva, WHO, 1953.
[30] R. Tagiuri, G.H. Litwin, Organizational Climate: Exploration of a Concept, Harvard University Press, Boston, 1968.
[31] B. Schneider, Interactional psychology and organizational behavior, in: L.L. Cummings, B.M. Staw (Eds.), in: Research in Organizational Behavior, vol. 5, JAI Press, Greenwich, CT, 1983, pp. 1–31.
[32] L.R. James, L.A. James, D.K. Ashe, The meaning of organization: the role of cognition and values, in: B. Schneider (Ed.), Organizational Climate and Culture, Jossey-Bass, San Francisco, CA, 1990.
[33] J.I. Rossberg, I. Melle, S. Opjordsmoen, S. Friis, The relationship between staff members working conditions and patients perceptions of the treatment environment, Int. J. Soc. Psychiatr. 54 (5) (2008) 437–446.
[34] J.I. Rossberg, S. Friis, Patients' and staff's perceptions of the psychiatric ward environment, Psychiatr. Serv. 55 (2004) 798–803.
[35] I. Melle, S. Friis, E. Hauff, T.K. Island, S. Lorentzen, P. Vaglum, The importance of ward atmosphere in inpatient treatment of schizophrenia on short-term units, Psychiatr. Serv. 47 (1996) 721–726.
[36] T. Middelboe, T. Schjødt, K. Byrsting, A. Gjerris, Ward atmosphere in acute psychiatric in-patient care: patients' perceptions, ideals and satisfaction, Acta Psychiatr. Scand. 103 (2001) 212–219. doi:10.1034/j.1600-0447.2001.00102.x.
[37] B. Schneider, S. White, M.C. Paul, Linking service climate and customer perceptions of service quality: test of a causal model, J. Appl. Psychol. 83 (1998) 150–163.
[38] K. Shipley, B. Hilborn, A. Hansell, J. Tyrer, P. Tyrer, Patient satisfaction: a valid index of quality of care in psychiatric service, Acta Psychiatr. Scand. 101 (2000) 330–333.
[39] R. Moos, Evaluating Treatment Environments. A Social Ecological Approach, John Wiley, New York, 1974.
[40] S. Friis, Measurements of the perceived ward milieu: a reevaluation of the ward atmosphere scale, Acta Psychiatr. Scand. 73 (1986) 589–599. doi:10.1111/j.1600-0447.1986.tb02729.x.