Lamplighter groups and separated graphs

Pere Ara

Universitat Autònoma de Barcelona

Email: para@mat.uab.cat

Abstract: A separated graph is a pair (E, C), where E is a directed graph, $C = \bigsqcup_{v\in E^0} C_v$, and C_v is a partition of $r^{-1}(v)$ (into pairwise disjoint nonempty subsets) for every vertex v. (In case v is a source, we take C_v to be the empty family of subsets of $r^{-1}(v)$.) Leavitt path algebras $L_K(E, C)$ of separated graphs have been recently defined by Goodearl and the presenter [1]. They allow to incorporate the Leavitt algebras of any type (m, n) into the theory of graph algebras. A construction due to Exel and the presenter [2] allows to build a related algebra $L_K^{\mathrm{ab}}(E, C)$, which is more tractable, because the generating set of partial isometries is tame.

The lamplighter group is the wreath product $G = \mathbb{Z}_2 \wr \mathbb{Z} = (\bigoplus_{n\in\mathbb{Z}} \mathbb{Z}_2) \rtimes \mathbb{Z}$. This group provided the first counterexample to Atiyah's Conjecture on L_2-invariants, see [4] and [3]. We will formulate the general Atiyah's Problem, which consists in finding, for a given group Γ, what is the set of values taken by a certain canonical rank function rk on matrices over $\mathbb{C}[\Gamma]$. We will show how to obtain the group algebra $K[G]$ of the lamplighter group G as an algebra of the form $L_K^{\mathrm{ab}}(E, C)$, for a specific separated graph (E, C). We will also consider a certain subalgebra A of $K[G]$, which is relevant to the study of Atiyah's problem for the lamplighter group. This algebra A is also of the form $L_K^{\mathrm{ab}}(F, D)$ for a separated graph (F, D). We will describe the structure of the algebra A, and we will give a concrete presentation of its monoid of finitely generated projective modules.

References

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