Cohn path algebras have Invariant Basis Number

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Cohn path algebras

Let K be a field, and let $E = (E^0, E^1, s, r)$ be a directed graph.

The Cohn path K-algebra $C_K(E)$ of E with coefficients in K

is the K-algebra generated by a set $\{v \mid v \in E^0\}$, together with a set of variables $\{e, e^* \mid e \in E^1\}$, which satisfy the following relations:

(V) $vw = \delta_{v,w} v$ for all $v, w \in E^0$,

(E1) $s(e)e = er(e) = e$ for all $e \in E^1$,

(E2) $r(e)e^* = e^* s(e) = e^*$ for all $e \in E^1$, and

(CK1) $e^* e' = \delta_{e,e'} r(e)$ for all $e, e' \in E^1$.

Cohn path algebras

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Cohn path algebras

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Cohn path algebras

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is the K-algebra generated by the same set $\{v \mid v \in E^0\}$, together with the same set of variables $\{e, e^* \mid e \in E^1\}$, which satisfy the same set of relations (V), (E1), (E2), and (CK1), and also satisfy the additional relation

(CK2) $v = \sum_{\{e \in E^1 \mid s(e) = v\}} ee^*$ for every regular vertex $v \in E^0$.

Cohn path algebras

If E^0 is finite then both $C_K(E)$ and $L_K(E)$ are unital, each having identity $1 = \sum_{v \in E^0} v$.

These are clearly related:

$$L_K(E) \cong C_K(E)/N$$

where $N = \langle v \ - \ \sum_{\{e \in E^1 | s(e) = v\}} ee^* \mid v \text{ regular} \rangle \ \trianglelefteq C_K(E)$.

But they are ALSO related in a perhaps surprising way.

The graph $F(E)$

Let E be an arbitrary graph.

Let Y denote the set of regular vertices of E.

Let $Y' = \{v' \mid v \in Y\}$ be a disjoint copy of Y.

For $v \in Y$ and for each edge e in E such that $r_E(e) = v$, we consider a new symbol e'.

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We define the graph $F = F(E)$, as follows:

$$F^0 = E^0 \sqcup Y'; \quad F^1 = E^1 \sqcup \{e' \mid r_E(e) \in Y\};$$

and for each $e \in E^1$,

$$s_F(e) = s_E(e),\ s_F(e') = s_E(e),$$

$$r_F(e) = r_E(e),\text{ and } r_F(e') = r_E(e)'.$$

The graph $F(E)$

Less formally (focus on E finite)

$F = F(E)$ is built from E by:

1) adding a new vertex to E corresponding to each non-sink of E,

and

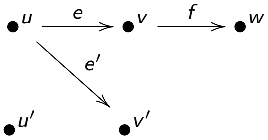
2) including new edges going into each of these new vertices as they were connected to the original vertices.

Cohn path algebras

Examples.

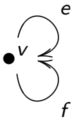
Let E be the graph

Then the graph $F = F(E)$ is:

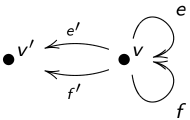


Cohn path algebras

Let R_2 be the graph



Then the graph $F = F(R_2)$ is:



Cohn path algebras

Note for later: The incidence matrix A_F of $F = F(E)$ is the $(n+t) \times (n+t)$ matrix in which, for $1 \leq i \leq t$, the i^{th} row is

$$(a_{i,1}, a_{i,2}, ..., a_{i,n}, a_{i,1}, a_{i,2}, ..., a_{i,t}),$$

and the remaining n rows are zeroes.

Cohn path algebras

Theorem. (P. Ara) Let E be any graph. Then there is an isomorphism of K-algebras

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Idea of Proof. Expressions of the form $v - \sum_{e \in s^{-1}(v)} ee^*$ are nonzero idempotents in $C_K(E)$.

"Replace" each regular v by $\sum_{e \in s^{-1}(v)} ee^*$ and think of v' as $v - \sum_{e \in s^{-1}(v)} ee^*$.

The graph monoid M_G

Suppose $|G^0| = n$. Construct the abelian monoid M_G:

For each regular vertex v_i $(1 \leq i \leq t)$ define

$$\vec{b_i} = (0, 0, ..., 1, 0, ...0) \in (\mathbb{Z}^+)^n.$$

Consider the equivalence relation $\sim_G$ in $(\mathbb{Z}^+)^n$, generated by setting

$$\vec{b_i} \sim_G (a_{i,1}, a_{i,2}, ..., a_{i,n})$$

for each regular vertex v_i. Define

$$M_G = (\mathbb{Z}^+)^n / \sim_G \ .$$

The operation in M_G is: $[\vec{a}] + [\vec{a'}] = [\vec{a} + \vec{a'}]$ for $\vec{a}, \vec{a'} \in (\mathbb{Z}^+)^n$.

The graph monoid M_G

Viewed another way:

$$\vec{b_i} \sim_G i^{th} \text{ row of } A_G$$

for each nonzero row of A_G.

i.e., for each v_i which is not a sink in G.

The graph monoid M_G

Example.

$$E = \bullet^u \xrightarrow{e} \bullet^v \xrightarrow{f} \bullet^w$$

Then M_E is the monoid $(\mathbb{Z}^+)^3$, modulo the relation $\sim_E$ generated by setting $(1,0,0) \sim_E (0,1,0)$ and $(0,1,0) \sim_E (0,0,1)$.

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We get $M_E \cong \mathbb{Z}^+$, via $[(a,b,c)] \mapsto a+b+c$.

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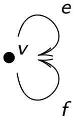
$R_2 =$ 

M_{R_2} is the monoid $(\mathbb{Z}^+)^1$, modulo the relation generated by setting $(1) \sim_{R_2} (2)$.

We get $M_{R_2} \cong \{0, x\}$, where $x + x = x$.

(N.b.: M_{R_2} is *not* the group $\mathbb{Z}_2$.)

Invariant Basis Number

Definition. R any unital ring. R has *Invariant Basis Number* (IBN) in case for each pair $m, m' \in \mathbb{N}$,

$${}_RR^m \cong {}_RR^{m'} \text{ as left } R\text{-modules } \Leftrightarrow \ m = m'.$$

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$$[P] + [Q] = [P \oplus Q].$$

Observation: R has IBN if and only if for every pair of distinct positive integers $m \neq m'$ we have $m[R] \neq m'[R]$ in $\mathcal{V}(R)$.

The graph monoid M_E

Theorem. (Ara, Moreno, Pardo, 2007) Let E be a finite graph with vertices $\{v_i \mid 1 \leq i \leq n\}$, and let K be any field. Then the assignment $[\vec{b_i}] \mapsto [L_K(E)v_i]$ yields an isomorphism of monoids

$$M_E \cong \mathcal{V}(L_K(E)).$$

In particular, under this isomorphism, if $\vec{\rho} = (1, 1, ..., 1) \in (\mathbb{Z}^+)^n$, we have $[\vec{\rho}] \mapsto [L_K(E)]$.

Corollary. Let F be any finite graph, and K any field. Let $\vec{\rho} = (1, 1, ..., 1) \in M_F$. Then $L_K(F)$ has IBN if and only if for any pair of positive integers $m \neq m'$, we have $m\vec{\rho} \not\sim_F m'\vec{\rho}$.

The graph monoid M_E

Example.

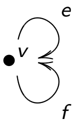
$$E = \ \bullet^u \xrightarrow{\ e\ } \bullet^v \xrightarrow{\ f\ } \bullet^w$$

Then $M_E \cong \mathbb{Z}^+$. In this identification,

$[\vec{\rho}] = [(1,1,1)] \mapsto 1 + 1 + 1 = 3$.

$m \neq m'$ obviously gives $m \cdot 3 \neq m' \cdot 3$ in $\mathbb{Z}^+$. So $L_K(E)$ has IBN.

Example.

$R_2 =$ 

$M_{R_2} \cong \{0, x\}$, and $[\vec{\rho}] \mapsto x$.

Since $1x = 2x$, $L_K(R_2)$ does not have IBN.

The graph monoid M_E

Key question: In a monoid of the form M_E, how do you show that two elements $[\vec{a}]$ and $[\vec{a'}]$ are NOT equal?

One possible approach: Find an invariant for the generating relations of $\sim_E$.

Specifically: Find a function $\varphi : (\mathbb{Z}^+)^n \to \mathbb{Q}$ with the property that if $\vec{c} \sim_E \vec{c'}$ in $(\mathbb{Z}^+)^n$, then $\varphi(\vec{c}) = \varphi(\vec{c'})$.

So if for some pair $\vec{a}$ and $\vec{a'}$ we have $\varphi(\vec{a}) \neq \varphi(\vec{a'})$, then $[\vec{a}]$ and $[\vec{a'}]$ are not equal in M_E.

The key linear algebra result.

Proposition. Given integers a_{ij} $(1 \leq i \leq t,\ 1 \leq j \leq n)$, there exist $w_1, w_2, \ldots, w_{n+t}$ in $\mathbb{Q}$ which satisfy the following system of $t+1$ linear equations:

$$\begin{array}{ccccccccccccccc}
1 & = & w_1 & + & w_2 & + & \ldots & + & w_n & + & w_{n+1} & + & \ldots & + & w_{n+t} \\
w_1 & = & a_{11}w_1 & + & a_{12}w_2 & + & \ldots & + & a_{1n}w_n & + & a_{11}w_{n+1} & + & \ldots & + & a_{1t}w_{n+t} \\
w_2 & = & a_{21}w_1 & + & a_{22}w_2 & + & \ldots & + & a_{2n}w_n & + & a_{21}w_{n+1} & + & \ldots & + & a_{2t}w_{n+t} \\
\vdots & & & & & & & & & & & & & & \\
w_t & = & a_{t1}w_1 & + & a_{t2}w_2 & + & \ldots & + & a_{tn}w_n & + & a_{t1}w_{n+1} & + & \ldots & + & a_{tt}w_{n+t}
\end{array}$$

The key linear algebra result.

Proof. Consider this $(t+1) \times (n+t)$ matrix:

$$B = \begin{pmatrix} 1 & 1 & \ldots & 1 & \ldots & 1 & 1 & \ldots & 1 \\ a_{11}-1 & a_{12} & \ldots & a_{1t} & \ldots & a_{1n} & a_{11} & \ldots & a_{1t} \\ a_{21} & a_{22}-1 & \ldots & a_{2t} & \ldots & a_{2n} & a_{21} & \ldots & a_{2t} \\ \vdots & & & & & & & & \\ a_{t1} & a_{t2} & \ldots & a_{tt}-1 & \ldots & a_{tn} & a_{t1} & \ldots & a_{tt} \end{pmatrix}$$

Then the existence of desired rationals $w_1, w_2, \ldots, w_{n+t}$ is equivalent to the existence of a solution in $\mathbb{Q}^{n+t}$ to the system $B\vec{x} = (1, 0, 0, \ldots, 0)^t$.

The key linear algebra result.

$$B = \begin{pmatrix} 1 & 1 & \dots & 1 & \dots & 1 & 1 & \dots & 1 \\ a_{11}-1 & a_{12} & \dots & a_{1t} & \dots & a_{1n} & a_{11} & \dots & a_{1t} \\ a_{21} & a_{22}-1 & \dots & a_{2t} & \dots & a_{2n} & a_{21} & \dots & a_{2t} \\ \vdots & & & & & & & & \\ a_{t1} & a_{t2} & \dots & a_{tt}-1 & \dots & a_{tn} & a_{t1} & \dots & a_{tt} \end{pmatrix}$$

Claim: The $t+1$ rows of B are linearly independent in $\mathbb{Q}^{n+t}$.

Idea: Induction on the number of rows. The first two rows are linearly independent (Row 2 has two unequal entries).

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In a similar way, Row $j+1$ can't be a linear combination of the first j rows, since in Row $j+1$, the j^{th} entry $a_{jj}-1$ is different than the $(j+n)^{th}$ entry a_{jj}; but in each Row i $(1 \leq i \leq j)$, the i^{th} and $(i+n)^{th}$ entries are equal. □

The Main Result.

Theorem. Let E be any finite graph, and K any field. Then the Cohn path algebra $C_K(E)$ has the Invariant Basis Number property.

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Theorem. Let E be any finite graph, and K any field. Then the Cohn path algebra $C_K(E)$ has the Invariant Basis Number property.

Proof. By Ara's theorem we have $C_K(E) \cong L_K(F)$, where $F = F(E)$ as above. We show that $L_K(F)$ has IBN.

Recall that if $E^0 = \{v_1, v_2, ..., v_n\}$, and we label the regular vertices of E as $v_1, ..., v_t$, then:

1) $F^0 = \{v_1, v_2, ..., v_n, v_1', v_2', ..., v_t'\}$ (so that $|F^0| = n + t$), and

2) the only regular vertices of F are $\{v_1, v_2, ..., v_t\}$.

The Main Result.

So if $A = A_E = (a_{i,j})$ is the incidence matrix of E, then the monoid M_F is the monoid $(\mathbb{Z}^+)^{n+t}$, modulo the equivalence relation generated by setting

$$\vec{b_i} \sim_F (a_{i,1}, a_{i,2}, \ldots, a_{i,n}, a_{i,1}, a_{i,2}, \ldots, a_{i,t}) \quad \text{for each } 1 \leq i \leq t.$$

The Main Result.

Let $\vec{\rho}$ denote the element $(1, 1, ..., 1)$ of $(\mathbb{Z}^+)^{n+t}$. We establish, for any pair of positive integers $m \neq m'$, that $m\vec{\rho} \not\sim_F m'\vec{\rho}$.

Let $w_1, w_2, ..., w_{n+t} \in \mathbb{Q}$ be rationals which satisfy the linear system of the Linear Algebra result.

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Let $w_1, w_2, ..., w_{n+t} \in \mathbb{Q}$ be rationals which satisfy the linear system of the Linear Algebra result. Define $\Gamma : (\mathbb{Z}^+)^{n+t} \to \mathbb{Q}$ by

$$\Gamma((z_1, z_2, ..., z_{n+t})) = \sum_{\ell=1}^{n+t} z_\ell w_\ell.$$

Then Γ is clearly linear. Also, by the choice of $w_1, w_2, ..., w_{n+t}$, for any of the t generating relations for M_F we get that

$$\Gamma(\vec{b_i}) = \Gamma((a_{i,1}, a_{i,2}, ..., a_{i,n}, a_{i,1}, a_{i,2}, ..., a_{i,t})).$$

The Main Result.

So:

for any $\vec{a}, \vec{a'} \in (\mathbb{Z}^+)^{n+t}$ with $\vec{a} \sim_F \vec{a'}$, we have $\Gamma(\vec{a}) = \Gamma(\vec{a'})$. $(*)$

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But the w_ℓ have been chosen so that $\sum_{\ell=1}^{n+t} w_\ell = 1$. So in particular for any positive integer m we get

$$\Gamma(m\vec{\rho}) = \Gamma((m, m, ..., m)) = \sum_{\ell=1}^{n+t} m w_\ell = m \sum_{\ell=1}^{n+t} w_\ell = m \cdot 1 = m.$$

So for $m \neq m'$ we have $\Gamma(m\vec{\rho}) = m \neq m' = \Gamma(m'\vec{\rho})$.

So by (*) we conclude that if $m \neq m'$, then $m\vec{\rho} \not\sim_F m'\vec{\rho}$. $\square$

Some concluding remarks.

1) Is the Main Result just an artifact of some more general result about the Leavitt path algebras of graphs with sinks?

Every Cohn path algebra has IBN.

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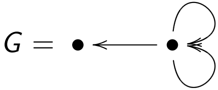
No. There are many graphs with sinks for which the corresponding Leavitt path algebra does not have IBN. For instance:



Then in M_G we have

$$1 \cdot \vec{\rho} = (1,1) = (1,0) + (0,1) \sim_G (2,1) + (0,1) = (2,2) = 2 \cdot \vec{\rho},$$

so that $L_K(G)$ does not have IBN.

Every Cohn path algebra has IBN.

2) Is the Main Result just an artifact of some more general result about the elements of $\mathcal{V}(C_K(E))$?

Every Cohn path algebra has IBN.

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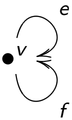
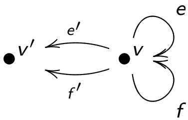
No. There can be elements $\vec{a}$ in $\mathcal{V}(C_K(E)) \cong \mathcal{V}(L_K(F(E))$ for which $m\vec{a} = m'\vec{a}$ with $m \neq m'$. For instance:

$E = R_2 =$ 

$F = F(R_2) =$ 

$C_K(E) \cong L_K(F)$ has IBN. But consider $\zeta = (1,2) \in M_F$. Then

$$\zeta = (1,0) + (0,2) \sim_F (2,2) + (0,2) = (2,4) = 2\zeta.$$

Every Cohn path algebra has IBN.

3) Is the Main Result just an artifact of a more general result which says that if you don't put the (CK2) condition at SOME vertex then the resulting algebra must have IBN?

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No. "Relative Cohn path algebras".

For any m, n with $m \geq 1$, we can construct a graph $E_{n,m}$ having n vertices and having a subset of m vertices, so that if we impose the (CK2) condition at those m vertices, the resulting relative Cohn path algebra does not have IBN.

Every Cohn path algebra has IBN.

4) There is a somewhat stronger conclusion:

Every Cohn path algebra has Invariant Matrix Number.

$$\mathrm{M}_d(C_K(E)) \cong \mathrm{M}_{d'}(C_K(E)) \ \Leftrightarrow d = d'.$$

Every Cohn path algebra has IBN.

5) There is another proof (given by P. Ara) of the Main Result, using an alternate description of $\mathcal{V}(C_K(E))$ (one which does NOT use properties of Leavitt path algebras).

Every Cohn path algebra has IBN.

6) Where are the tensor products?

Original motivating question:

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If E and E' each have a cycle, then we know the answer. (No)

If just one of E, E' has a cycle, and the other has at least one edge, then we DON'T know the answer.

Every Cohn path algebra has IBN.

Can we gin up an example of a nontrivial acyclic graph E, and a graph with a cycle E', for which $C_K(E) \otimes C_K(E') \cong C_K(G)$ for some graph G?

If $E = \bullet \longrightarrow \bullet$, then $C_K(E) \cong K \oplus \mathrm{M}_2(K)$. (easy)

So $C_K(E) \otimes C_K(E') \cong C_K(E') \oplus \mathrm{M}_2(C_K(E'))$ for any E'.

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So $C_K(E) \otimes C_K(E') \cong C_K(E') \oplus \mathrm{M}_2(C_K(E'))$ for any E'.

So **IF** we could find E' for which $\mathrm{M}_2(C_K(E')) \cong C_K(E')$, then $G = E' \sqcup E'$ would work.

So we looked for a graph E' for which $\mathrm{M}_2(C_K(E')) \cong C_K(E')$. Where to start the search? Find a graph E' for which $C_K(E') \cong C_K(E') \oplus C_K(E')$ as left $C_K(E')$-modules.

Every Cohn path algebra has IBN.

And, of course, we couldn't find one.

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We're still working on the tensor product question.

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Questions?