

Sample Final Exam - Math 135 - Spring 2008

(1) Evaluate the following limits

$$\lim_{x \rightarrow \pi/2} \frac{1 - \sin^2 x}{\cos^2 x} =$$

$$\lim_{x \rightarrow 2} \frac{x^2 - x + 1}{x + 2} =$$

$$\lim_{x \rightarrow 0^+} x \ln^2 x =$$

(2) Find all vertical and horizontal asymptotes (if any) of the curve

$$y = \frac{3x^2 + 1}{x^2 - x - 6}.$$

(3) Find the first and second derivative of the function $f(x) = x \cos x$.

(4) Find the equation of the tangent line to the parabola $y = -x^2 + 6x + 1$ at the point with coordinate $x = 3$. Sketch the parabola and the tangent line found above.

(5) Compute the derivative of each function

- $f(x) = \ln \sin x$
- $g(u) = e^{2u} + e^{-3u}$
- $h(t) = (t^2 - 3)^4$
- $l(x) = \frac{\sin x}{x}$

(6) Using logarithmic differentiation, compute $\frac{dy}{dx}$, where

$$y = (1 + x)^x.$$

(7) Find all critical points of the function

$$f(x) = x^{1/3}(x + 1)^{2/3}$$

(8) Let $f(x) = (x^2 - 3)^2$.

- (a) Find all critical points, relative maxima and minima for f .
- (b) Find the inflection points for f , if any.
- (c) Sketch the graph of f .

- (9) A farmer plans to fence three sides of a rectangular field next to a river, leaving the fourth side facing the river unfenced (see picture). If the desired area of the field is 1800 ft^2 , what is the minimum length of fence he will need to use?
- (10) Decide whether the following statements are TRUE or FALSE. EXPLAIN!

$$\int \frac{x}{x^2 + 1} dx = \frac{1}{2} \ln(x^2 + 1) + C$$

$$\int e^{-x^2} dx = \frac{1}{2} e^{-x^2} + C$$

- (11) Evaluate the following definite integrals

$$\int_0^{2\pi} \sin x \, dx =$$

$$\int_1^e \frac{1}{x} \, dx =$$

- (12) Using the Fundamental theorem of Calculus, find the derivative of the following function

$$f(x) = \int_1^{\sqrt{x}} \frac{1}{1+t^2} dt$$

Topics to be reviewed for the final (each group will represented approx 1/4 of the final)

- Limits and Derivatives(1.3-1.6, 3.7, 2.1, 2.2)
 - Methods of Computing Limits
 - Limits at Infinity, l'Hopital's rule
 - Definition of Derivative
 - Derivative as a Function
- Rules of differentiation (2.3-2.6, 3.1-3.3, 3.5)
 - Power Rule
 - Product and Quotient Rule
 - Chain Rule
 - Derivative of exponential and logarithmic functions, inverse trig functions
- Application of Derivatives (2.7,2.8, 3.4, 4.1-4.5)
 - Rates of Change in Science
 - Related Rates
 - Curve Sketching (Max, Min, Critical points, Concavity, Inflection points, etc)
 - Optimization problems
- Antiderivatives and Integrals (4.7,5.1-5.5)
 - Antiderivatives and Indefinite Integrals
 - Areas and the Definite Integral
 - Fundamental Theorem of Calculus
 - Substitution Rule