
Thursday, March 3, 2016

STUDENT NAME: _____

EXAM I – MATH 3400, SPRING 2016

READ EACH PROBLEM CAREFULLY! To get full credit, you must show all work! **The exam has 6 problems on 6 pages. The 7th page contains the review sheet of methods for solving DEs which can be used during the exam. Make sure you turn in ALL pages. No graphing calculator is allowed!**

Exercise 1 *[16 points]*

(a) Solve the initial value problem

$$y' + 2y = 2, \quad y(0) = 2$$

(b) Find $\lim_{t \rightarrow \infty} y(t)$.

Exercise 2 *[16 points]*

Solve the differential equations

$$(a) \quad y' - 3y = e^{3x}$$

$$(b) \quad xy' = y^2 + 1$$

Exercise 3 *[17 points]*

Determine whether the following equations are exact or not. Then **solve only those** that are exact.

$$(a) \quad (2y - x)dx + (y + 2x)dy = 0$$

$$(b) \quad (y^2 + 1)dx + (x^2 + 1)dy = 0$$

Exercise 4 [*7 points*]

(a) Find the general solution for the differential equation below

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

(b) Find the solution satisfying the initial condition $y(1) = -2$.

Exercise 5 [18 points] A tank initially contains 300 gal of pure water. A salt water mixture with a concentration of 5 pounds per gallon enters the tank at the rate of 20 gallon per minute. The well mixed solution leaves the tank at the same rate. Let $x(t)$ be the quantity of salt (pounds) in the tank at time t . [Hint: The differential equation $\frac{dx}{dt} = r_{in}c_{in} - r_{out}c_{out}$ describes the rate of change of the amount of salt in the tank]

(a) Set up the initial value problem for $x(t)$.

(b) Find the particular solution of the above IVP.

(c) How much salt is in the tank after 30 minutes?

Exercise 6 [16 points] A population of a certain species satisfies the following differential equation:

$$\frac{dP}{dt} = 20 - \frac{1}{5}P^2$$

- (a) Find the equilibria, and determine their stability.
- (b) Assuming the initial population is $P(0) = 5$, solve the IVP.
- (c) Is there a limiting population? If so, what is $\lim_{t \rightarrow +\infty} P(t)$?