

Practice Exam 1 – MATH 3400 - Spring 2016

Intro to Differential Equations

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READ EACH PROBLEM CAREFULLY! To get full credit, you must show all work! No graphing calculators allowed!

1. Consider the first order ODE:

$$ty' + (1 + t)y = 2e^{-2t}.$$

- (a) Find the general solution.
- (b) Find the solution to the IVP with $y(0) = 2$.

2. Consider the first order ODE:

$$y' - y = 3te^{-2t}.$$

- (a) Find the general solution.
- (b) Find the solution to the IVP with $y(0) = 2$.
- (c) Discuss the behavior of the solution in part (b) as $t \rightarrow +\infty$.
- (d) Find the value of y_0 for which the solution of the ODE with initial value $y(0) = y_0$ remains finite as $t \rightarrow \infty$.

3. Find the solution of the IVP. If possible, write the solution in the form $y = y(x)$.

$$\frac{dy}{dx} = \frac{x}{4y + x^2y}, \quad y(0) = 2.$$

4. Find the general solution of the following ODE. If possible, write the solution in the form $y = y(x)$.

$$3y + x^3y^4 + 3x\frac{dy}{dx} = 0$$

5. Find the general solution of the following ODE. If possible, write the solution in the form $y = y(x)$.

$$2x \sin y \cos y \frac{dy}{dx} + 4x^2 + \sin^2 y = 0.$$

6. Find the general solution of the following ODE. If possible, write the solution in the form $y = y(x)$.

$$(2y - 5x)\frac{dy}{dx} = x + 5y.$$

7. Find the solution of the following ODE. If possible, write the solution in the form $y = y(x)$.

$$xy' - 3y = 3x\sqrt[3]{y^4}$$

8. Find the general solution of the ODE. If possible, write the solution in the form $y = y(x)$.

$$2y^3y'' = 1$$

9. Suppose the number $x(t)$ (with t in months) of mice in a colony satisfies the ODE

$$\frac{dx}{dt} = 0.0001x^2 - 0.01x$$

- (a) Discuss the equilibrium solutions of this population model and their stability.
- (b) If initially there are 25 mice in the colony, solve the ODE to determine what happens to the mice population in the long run.
- (c) Repeat part (b), except with 150 mice initially. What happens to the population in this case?

10. The differential equation:

$$\frac{dP}{dt} = P(10 - P) - h$$

models a logistic population with harvesting at a rate h . Suppose $h = 9$, i.e. 9 individuals (fish, or whatever) are harvested per unit time. Find the equilibrium solutions for this value of h , and discuss if these equilibrium solutions are stable or not.

11. A projectile having mass $m = 1 \text{ kg}$ is launched from the ground straight up in the air with initial velocity $v_0 = 5 \text{ m/s}$ and obeys the Newton's Law of Motion

$$ma = F_R - mg$$

(a) Assuming that air resistance is proportional with the velocity $F_R = -kv$, determine the velocity $v(t)$ of the projectile at all time t in the upward motion. [Use the gravitational acceleration $g = 10 \text{ m/s}^2$ and $k = 5 \text{ kg/s}$.]

(b) When does the projectile reach maximum height? And what is this height?

(c) After it has reached its maximum height, the projectile falls back to the ground. Write down the differential equation that describes the velocity in the downward motion and solve it.

12. A tank contains 1000 gallons of a solution consisting of 500 lb of salt dissolved in water. Pure water is pumped into the tank at a rate of 5 gal/s, and the mixture - kept uniformly by stirring is pumped out at the same rate. How long will it take until the concentration of the salt in water is half of what it was at the initial time?

[Hint: The differential equation $\frac{dx}{dt} = r_{in}c_{in} - r_{out}c_{out}$ describes the rate of change of the amount of salt in the tank]