Practice Exam 2 – MATH 3400 - Spring 2016

Intro to Differential Equations - Dr. Radu C. Cascaval

READ EACH PROBLEM CAREFULLY! To get full credit, you must show all work! No calculators allowed!

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1. Consider the equation
$$y'' + y' - 6y = 0.$$
(a) Find the general solution.
(b) Find the solution satisfying the initial conditions $y(0) = 0, y'(0) = 3$.

2. Consider the differential equation
$$y'' + y' - 2y = 0$$
(a) Find two linearly independent solutions and write the general solution.
(b) Find the solution satisfying the initial conditions
$$y(0) = 1, y'(0) = -2$$

3. Use the method of reduction of order to find a second solution $y_2(x)$ for
$$xy'' + (x+1)y' + y = 0, \quad x > 0$$
given that $y_1(x) = e^{-x}$ is one solution. Then write the general solution.

4. Use the method of reduction of order to find a second solution $y_2(t)$ for
$$x^2y'' - x(x+2)y' + (x+2)y = 0$$
given that $y_1(x) = x$ is one solution. Write down the general solution of the differential equation.

5. Given the non-homogeneous equation
$$y'' - 2y' + y = 2e^x,$$
(a) verify that $y_p(x) = x^2e^x$ is a solution.
(b) find the solution satisfying the initial condition $y(0) = 2, y'(0) = 3$.

6. Given the non-homogeneous equation
$$y'' + y = 2\cos x$$
(a) Find the general solution $y(x)$.
(b) Solve the initial value problem with initial conditions
$$y(0) = -1, y'(0) = 0$$

7. Solve the differential equation
$$y'' + 4y = \sin^2 x.$$

8. Solve the differential equation
$$y'' - 4y = xe^x.$$

9. The motion of a forced spring-mass system is described by the equation $mx''+kx = F\cos\omega t$, where $m = 4\,g$, $k = 16\,g/s^2$ and the forcing has amplitude $F = 12\,g/cm\cdot s^2$. Initially, the system is at rest ($x(0) = 0\,cm$, $x'(0) = 0\,cm/s$).

   (a) Determine the value $\omega = \omega_0$ for which resonance is occurring and write down $x(t)$ in this case.
   (b) Assuming ω is different than the value found in (a), determine the motion $x(t)$.
   (c) Use the identity $\cos\alpha t - \cos\beta t = 2\sin\frac{\beta-\alpha}{2}t\sin\frac{\alpha+\beta}{2}t$ to express the motion in (a) using a product of two sine functions.
   (d) Show that if ω approaches ω_0, then the solution found in part (c) tends to the solution found in (a).