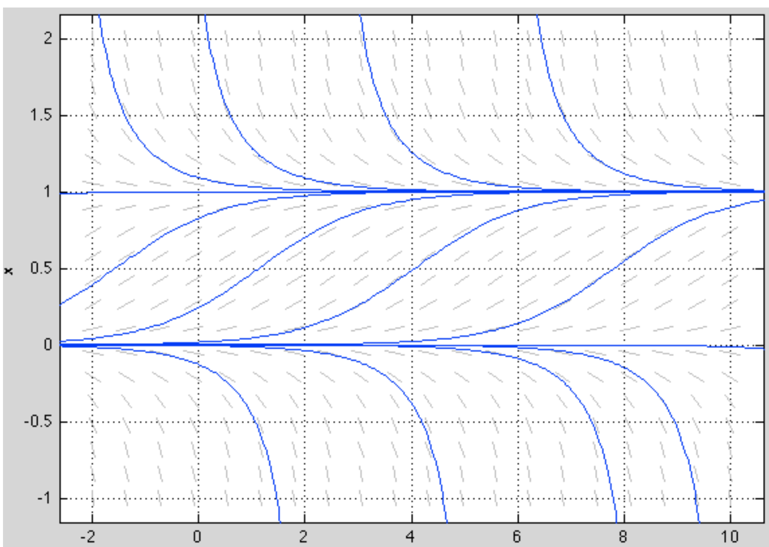
Practice for Final Exam
Math 3400 - Intro to Differential Equations
Spring 2016 - Dr. Radu Cascaval

The first 15 (multiple-choice) problems are 6 points each, the remaining 5 (essay) problems are 14 points each (Show all work to get full credit on the essay problems).

1. (Circle all that apply!) The differential equation $x\dfrac{dy}{dx} = x - y$ is
   (a) separable, (b) linear, (c) homogeneous, (d) exact

2. The general solution of the equation $x' + 5x = 3$ is
   (a) $x(t) = \frac{3}{5} + e^{-5t}$
   (b) $x(t) = 3 + C\sin 5t$
   (c) $x(t) = \frac{3}{5} + Ce^{-5t}$
   (d) $x(t) = C\cos 3t$

3. The following represents the slope field and several solution curves of the equation



   (a) $x' = t(1-x)$ (b) $x' = x(x-1)$ (c) $x' = t^2 - t$ (d) $x' = x(1-x)$

4. A solution of the initial value problem $x' + 8x = 1 + e^{-6t}, x(0) = 0$ is
   (a) $x(t) = \frac{1}{8} + \frac{1}{2}e^{6t} - \frac{5}{8}e^{8t}$
   (b) $x(t) = \frac{1}{8} + \frac{1}{2}e^{-6t} - \frac{5}{8}e^{-8t}$
   (c) $x(t) = 4 - e^{2t} + 3e^{8t}$
   (d) $x(t) = 4 - e^{-2t} + 3e^{-8t}$

5. Two linearly independent solutions of the equation $y'' + y' - 6y = 0$ are
   (a) e^{-3x} and e^{2x}, (b) e^{-2x} and e^{3x}, (c) e^{-x} and e^{6x}, (d) e^{-6x} and e^{x}

6. A particular solution for the differential equation $y'' + 2y' + y = 3 - 2\sin x$ is
   (a) $A + B\sin x$
   (b) $A + Bx^2 + C\cos x + D\sin x$
   (c) $A + Bx\cos x + Cx\sin x$
   (d) $A + B\cos x + C\sin x$

7. The solution of the initial value problem $x^2y'' - xy' - 3y = 0, y(1) = 1, y'(1) = -2$ is
(a) $\frac{5}{4}x^{-1} - \frac{1}{4}x^3$, (b) $\frac{1}{4}x + \frac{3}{4}x^{-3}$, (c) $\frac{5}{4}e^{-x} - \frac{1}{4}e^{3x}$, (d) $\frac{1}{4}e^x + \frac{3}{4}e^{-3x}$

8. The Laplace Transform of $f(t) = te^t \sin(2t)$ is
(a) $\frac{4(s+1)}{(s-1)^2(s^2+4)}$, (b) $\frac{4(s-1)}{((s-1)^2+4)^2}$, (c) $\frac{2}{(s-1)^2+4}$, (d) $\frac{2(s-1)}{(s-1)^2+4}$

9. The inverse Laplace Transform of the function $F(s) = \frac{4s+2}{s^2+2s+5}$ is
(a) $e^{-t}\cos 2t + 2e^{-t}\sin 2t$, (b) $e^{-t}\cos 2t - 4e^{-t}\sin 2t$, (c) $4e^{-t}\cos 2t + e^{-t}\sin 2t$,
(d) $4e^{-t}\cos 2t - e^{-t}\sin 2t$

10. The Laplace Transform of the function $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2-t, & t \ge 1 \end{cases}$ is
(a) $\frac{1-2e^{-s}}{s^2}$, (b) $\frac{2}{s}$, (c) $\frac{1}{s^2} - \frac{2e^{-s}}{s}$, (d) $\frac{1}{s^2} + \left(\frac{2}{s} - \frac{1}{s^2}\right)e^{-s}$

11. For the initial value problem $x'' + 4x' + 13x = t, \quad x(0) = -1, x'(0) = 1$, the Laplace transform $X(s)$ of the solution $x(t)$ is
(a) $\frac{1}{s^2+4s}\left(-s - 3 - \frac{1}{s} + \frac{1}{s^2}\right)$
(b) $\frac{1}{s^2+4s}\left(-s - \frac{1}{s} + \frac{1}{s^2}\right)$
(c) $-\frac{s+3}{s^2+4s+13} + \frac{1}{s^2(s^2+4s+13)}$
(d) $-\frac{s}{s^2+4s+13} + \frac{1}{s^2(s^2+4s+13)}$

12. The partial fraction decomposition of $\frac{s+4}{(s-1)^2(s^2+4)}$ is
(a) $\frac{A}{(s-1)^2} + \frac{Bs+C}{s^2+4}$
(b) $\frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{Cs+D}{s^2+4}$
(c) $\frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s^2+4}$
(d) $\frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{Cs}{s^2+4}$

13. Which of the following equations can be rearranged into separable equations?
(a) $(x+y)y' = x - y$
(b) $y' - e^y = e^{x+y}$
(c) $y' = \ln(xy)$
(d) None of the above

14. A body with mass $m = \frac{1}{2}\,kg$ is attached to the end of a spring that is stretched $2\,m$ by a force of $100\,N$. It is set in motion with initial position $x_0 = 1$ m and initial velocity $v_0 = -5\,m/s$. The position function of the body is given by
(a) $x(t) = \cos 10t - \frac{1}{2} sin 10t$
(b) $x(t) = \cos 5t - \sin 5t$
(c) $x(t) = e^{-t}(\cos 9t - 3\sin 9t)$
(d) $x(t) = e^{-10t}(\frac{1}{10}\cos t - \sin t)$

15. The appropriate form of a particular solution y_p of the equation $y'' + 2y' + y = e^{-t}$ is
(a) Ae^{-t}
(b) Ate^{-t}
(c) At^2e^{-t}
(d) $(A + Bt)e^{-t}$

The following problems will be graded based on the complete work!! You must show all work to get full credit.

Problem 1.
(i) Solve the differential equations $y' = -2xy$
(ii) Verify that the equation $(x^3 + \frac{y}{x})\,dx + (y^2 + \ln x)\,dy = 0$ is exact and solve it!

Problem 2. Use the method of reduction of order to find a second solution $y_2(x)$ for

$$xy'' + (x-2)y' + y = 0, \quad x > 0$$

given that $y_1(x) = x^3 e^{-x}$ is one solution.

Problem 3. An instrument at an initial temperature of 40 degrees C is placed in a room whose temperature is 20 degrees C. For the next 5 h the room temperature $Q_0(t)$ gradually rises and is given by $Q_0(t) = 20 + 10t$, where t is measured in hours.
(a) Use Newton's Law of Cooling $\frac{dT}{dt} = -k(T - Q_0)$, with $k = 1$, to determine the temperature of the instrument $T(t)$ at any time $t > 0$.
(b) Is there any time when the instrument equals the room temperature? Sketch the graph of T and Q_0 on the same plot.

Problem 4. A 400-gal tank initially contains 100 gal of brine containing 50 lb of salt. Brine containing 1 lb of salt per gallon enters the tank at a rate of 5 gals/s, and the well mixed brine in the tank flows out at a rate of 3 gal/s. How much salt will be the tank contain when it is full of brine?

Problem 5. A spring-mass system ($m = 2$, $k = 10$) with damping ($c = 8$) is initially stretched by $x(0) = 2$ and has initial velocity $x'(0) = 0$. The governing differential equation is

$$mx'' + cx' + kx = f(t)$$

(a) Determine its subsequent motion $x = x(t)$ if forced by an external force $f(t) = 2\cos t$.
(b) Determine the amplitude C and phase angle α of the persistent motion found in (a). That is, write the persistent motion in the form $x_p(t) = C\cos(\omega t - \alpha)$.