

Quiz 0 - Jan 23, 2012
[15 minutes]

SOLUTIONS

Student Name _____

MATH 3400 - Intro to Differential Equations - Spring 2012

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Evaluate the following indefinite integrals: [Check your answer]

$$1. \int \frac{1}{x+1} dx = \boxed{\ln |x+1| + C}$$

$$2. \int \frac{1}{x^2+1} dx = \boxed{\arctan(x) + C}$$

$$3. \int \frac{1}{x^2-x} dx = \int \frac{1}{x(x-1)} dx = \int \frac{1}{x-1} - \frac{1}{x} dx \quad \left(\begin{array}{l} \text{partial fraction decomp.} \\ \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} \\ A = -1, B = 1 \end{array} \right)$$
$$= \ln|x-1| - \ln|x| + C$$
$$= \boxed{\ln \left| \frac{x-1}{x} \right| + C}$$

$$4. \int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \boxed{\frac{1}{2} \ln(x^2+1) + C}$$

Subs $u = x^2+1$
 $du = 2x dx$

$$5. \int \frac{1}{(3-x)^2} dx = - \int \frac{1}{u^2} du = - \int u^{-2} du = -(-u^{-1}) + C$$

Subs $u = 3-x$
 $du = -dx$

$$= \frac{1}{u} + C = \boxed{\frac{1}{3-x} + C}$$

$$6. \int \frac{1}{\sqrt{x-1}} dx = \int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = 2u^{1/2} + C$$

Subs $u = x-1$
 $du = dx$

$$= \boxed{2\sqrt{x-1} + C}$$

$$7. \int \frac{\ln x}{x} dx = \int u du = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C}$$

Subs $u = \ln x$
 $du = \frac{1}{x} dx$

$$8. \int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx = \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx$$

Int by parts
 $u = \ln x \quad dv = x dx$
 $du = \frac{1}{x} dx \quad v = \frac{1}{2} x^2$

$$= \boxed{\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C}$$

$$9. \int \sin 2x dx = \frac{1}{2} \int \sin u du = \frac{1}{2} (-\cos u) + C = \boxed{-\frac{1}{2} \cos 2x + C}$$

Subs $u = 2x$
 $du = 2 dx$
 $\Downarrow$
 $dx = \frac{1}{2} du$

$$10. \int \sin^2 x dx = \int \frac{1}{2} (1 - \cos 2x) dx = \frac{1}{2} \int 1 - \cos 2x dx$$

Trig identity
 $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$

$$= \frac{1}{2} x - \frac{1}{2} \int \cos 2x dx$$

$$= \boxed{\frac{1}{2} x - \frac{1}{4} \sin 2x + C}$$