Math 3410 - Intro to Analysis - MathOnline
Department of Mathematics - University of Colorado at Colorado Springs
Instructor: **Dr. Radu C. Cascaval**
Phone: 719-255-3759, Fax: **1-888-951-4321**, Email: radu@uccs.edu

*To be read and completed by the Proctor: By signing below, I certify that the exam was administered in the time allowed (**75 minutes**) and observing the following rules: no open books/notes. Please make sure all 5 pages are returned.*

Proctor Name: _____ Proctor Signature (required): _____

(Please email or fax the completed exam **no later than Wed, 4/20, 5pm MT***, unless prior arrangements have been made.)*

April 19, 2016 Student Name: _____

Exam 2 - Math 3410 - Intro to Analysis

Dr. Radu C. Cascaval, Spring 2016

The exam has 5 problems on 5 pages. Students need to make sure ALL pages are returned. To receive credit, students need to show all work. No calculators allowed!

1. Determine whether the following statements are TRUE or FALSE. Justify your answers.

   (a) If $\limsup |a_n| = 0$ then $\lim a_n = 0$.

   (b) The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent for all $p \geq 1$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{10^n} = \frac{1}{9}$.

   (d) If the series $\sum a_n$ is convergent, then so is $\sum a_n^2$.

   (e) If the series $\sum |a_n|$ is convergent, then so is $\sum a_n^2$

   (f) If $\liminf_n a_n = 0$ then there exist a subsequence (a_{n_k}) of (a_n) with $\displaystyle\sum_k a_{n_k}$ convergent.

   (g) The series $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \ldots$ is absolutely convergent.

2. For each of the following series, determine whether it is absolutely convergent, conditionally convergent or divergent. Please indicate which test you are using and show all your work.

(a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

(b) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 + n} - n)$

(c) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2 + 1} - n)$

(d) $\sum_{n \geq 1} \frac{1}{1.01^n - 1000}$

(e) $\sum_{n \geq 1} \frac{1}{n^2 + \sqrt{n^2 + 1}}$

... continued on next page ...

(f) $\displaystyle\sum_{n=1}^{\infty} \log \frac{n+1}{n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n \log n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5n}{6n+1} \right)^n$

3. Calculate

$$\lim_{n \to \infty} \left(\frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \ldots + (-1)^n \frac{3}{2^n} \right)$$

4. (a) State the Cauchy criterium (property) for a series $\sum_n a_n$ and explain why it is equivalent with the convergence of the series.

(b) Using (a), prove that if $\sum_n |a_n|$ is convergent, then $\sum_n a_n$ is convergent. [Hint: Use Triangle inequality]

(c) Using (a), prove that if a series $\sum_n a_n$ is convergent, then $\lim_n a_n = 0$.

5. For each of the following series, determine the radius of convergence and the interval of convergence:

(a) $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{2n+1}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} x^n$

(c) $\sum_{n=1}^{\infty} n(x-3)^n$