Wed, Mar 4, 2015

Student Name: ____________________

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Exam 1 - Math 3410

Intro to Analysis

Dr. Radu C. Cascaval, Spring 2015

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1. Determine whether the following statements are TRUE or FALSE. Justify your answers.

   (a) $\sqrt{2}+\sqrt{3}$ is irrational.

   (b) Every open interval (a, b) contains infinitely many *irrationals*.

   (c) If $\liminf s_n = -\infty$ then $\lim s_n = -\infty$.

   (d) If (s_n) and (t_n) converge to s and t respectively, and $s_n < t_n$ for all n, then $s < t$.

   (e) Every open interval (a, b) contains infinitely many irrationals.

   (f) If S is a bounded set of real numbers, then $\sup S \in S$.

   (g) If $\lim s_n = +\infty$ then s_n is monotone.

2. Show by mathematical induction that $n! > 2^n$, for all $n > 3$.

3. Find the following limit:

$$\lim_{n}(\sqrt{n^2+2n} - \sqrt{n^2-3n}) =$$

4. Let (s_n) be defined by $s_1 = \sqrt{2}$ and $s_{n+1} = \sqrt{2 + s_n}$, for $n \in \mathbb{N}$.

(a) Show that each term s_n is an irrational number.

(b) Prove that (s_n) is monotone and bounded.

(c) Find $\lim_n s_n$.

5. Given two sets of real numbers S and T, we define the new set

$$S + T = \{s + t \mid s \in S, t \in T\}.$$

(a) Show that, if S and T are bounded

$$\inf(S + T) = \inf(S) + \inf(T)$$

and the same for the sup.

(b) Describe the set $S + T$, where $S = [0, 1] \bigcap \mathbb{Q}$ and $T = [0, 1] \setminus \mathbb{Q}$.

6. (a) State the Bolzano-Weierstrass theorem.

(b) Suppose (s_n) is a bounded sequence such that all *convergent* subsequences converge to the same number s. Prove that (s_n) must converge to s. [Hint: Assume that $s_n \nrightarrow s$, then apply Bolzano-Weierstrass to a conveniently chosen subsequence].

(c) Determine whether the sequence

$$s_n = \frac{n}{n+1} \cos \frac{n\pi}{2}$$

is bounded or not. Does it have convergent subsequences? Is (s_n) convergent itself? Explain!