Summary of Convergence/Divergence Tests for Series

Math 3410 - Spring 2015 - Dr. Radu Cascaval

Geometric Series Test: The series $\sum_{n=0}^{\infty} ar^n$ is convergent only if $|r| < 1$ and is divergent if $|r| \geq 1$.

p-Series Test: The series $\sum \frac{1}{n^p}$ converges only if $p > 1$ and diverges if $p \leq 1$.

Divergence Test: If a sequence (a_n) does not converge to 0, then the series $\sum a_n$ diverges.

Absolute Convergence Test: If the series $\sum |a_n|$ is convergent, then the series $\sum a_n$ is convergent.

Comparison Test: Given a series $\sum a_n$ and another (comparison) series $\sum b_n$ with $b_n \geq 0$,

- If $|a_n| \leq b_n$ and $\sum b_n$ is convergent, then $\sum a_n$ is (abs.) convergent.
- If $|a_n| \geq b_n$ and $\sum b_n$ is divergent then $\sum a_n$ is divergent.

Limit Comparison Test:* Given a series $\sum a_n$ and another (comparison) series $\sum b_n$ with $b_n \geq 0$,

- If $\limsup \left|\frac{a_n}{b_n}\right| < +\infty$ and $\sum b_n$ is convergent, then $\sum a_n$ is (abs.) convergent.
- If $\liminf \left|\frac{a_n}{b_n}\right| > 0$ and $\sum b_n$ is divergent then $\sum a_n$ is divergent.

Ratio Test: Given a series $\sum a_n$ with nonzero terms,

- If $\limsup \left|\frac{a_{n+1}}{a_n}\right| < 1$, then $\sum a_n$ is (abs.) convergent.
- If $\liminf \left|\frac{a_{n+1}}{a_n}\right| > 1$, then $\sum a_n$ is divergent.

Root Test: Given a series $\sum a_n$,

- If $\limsup \left|a_n\right|^{1/n} < 1$, then $\sum a_n$ is (abs.) convergent.
- If $\limsup \left|a_n\right|^{1/n} > 1$, then $\sum a_n$ is divergent.

Integral Test: Given a series $\sum a_n$ with $a_n = f(n)$ for a function f which is continuous, positive and nonincreasing on an interval $[c, \infty)$, then

- If $\int_c^{\infty} f(x)\, dx$ is convergent, then $\sum a_n$ is convergent.
- If $\int_c^{\infty} f(x)\, dx$ is divergent, then $\sum a_n$ is divergent.

Alternating Series Test: A series $\sum_{n=0}^{\infty} (-1)^n a_n$ for which the sequence (a_n) is nonincreasing and convergent to zero, is convergent.

Dirichlet's Test:* If (a_n) is a nonincreasing sequence of convergent to 0, ($0 \leq a_{n+1} \leq a_n$ and $a_n \to 0$) and (b_n) is a sequence with bounded partial sums ($|\sum_{k=1}^{n} b_k| \leq M$ for all n), then $\sum a_n b_n$ converges.

Abel's Test:* If (a_n) is a monotone bounded sequence and $\sum b_n$ converges, then $\sum a_n b_n$ is convergent.

Cauchy's Test:* If (a_n) is nonincreasing sequence, then $\sum a_n$ is convergent if and only if $\sum 2^k a_{2^k}$ is convergent.

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* - not presented in the K. Ross textbook, but covered in class, so can be used during the Exam.