MATH 4/5420 - Optimization - Spring 2017
Sample Final Exam

Problem 1

(a) Consider the problem of minimizing a general quadratic function subject to a linear constraint:

$$\begin{aligned} \min \frac{1}{2} x^T Q x - c^T x \\ \text{subject to } Ax = b \end{aligned}$$

where $Q = Q^T > 0, A \in \mathbb{R}^{m \times n}, m < n$, rank A =m.
Derive a closed-form solution to the problem.

(b) The shortest distance from the origin to the line in 3D is computed by minimizing

$$\begin{aligned} \min \frac{1}{2}|x|^2 = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2) \\ \text{subject to } x_1 + x_2 - x_3 = 3 \\ x_1 - x_2 + 2x_3 = 1 \end{aligned}$$

Solve this optimization problem, either directly or using part (a). State the necessary and sufficient condition of optimality (1st order and 2nd order).

Problem 2

For the linear programming problem

$$\begin{aligned} \min \quad x_1 + x_2 + 3x_3 \\ \text{subject to } x_1 - x_2 = 1 \\ x_1 + x_3 \geq 2 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

(a) Find the dual problem and solve it using the graphical method.

(b) Write down the KKT optimality conditions for the primal problem and solve it.

Problem 3

Solve the following linear programming problem using the simplex tableau method

$$\begin{aligned} \max \quad 3x_1 + 2x_2 \\ \text{subject to } x_1 \leq 4, \\ x_2 \leq 1 \\ 2x_1 + x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{aligned}$$