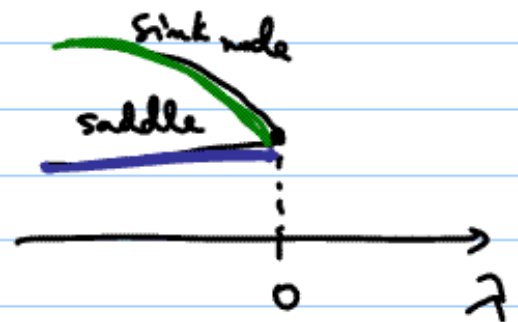


Examples of Elementary Bifurcations in the Plane (2D systems)

① SADDLE-NODE BIFURCATION

$$\begin{cases} x' = \lambda + x^2 \\ y' = -y \end{cases}$$



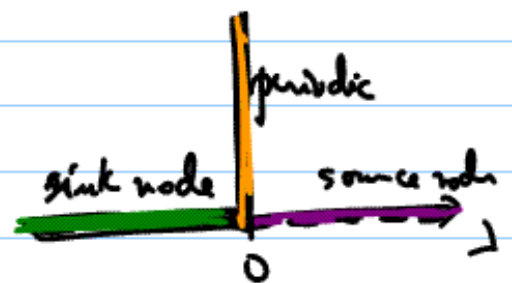
② PITCHFORK BIFURCATION

$$\begin{cases} x' = -\lambda x - x^3 \\ y' = -y \end{cases}$$



③ VERTICAL BIFURCATION

$$\begin{cases} x' = \lambda x + y \\ y' = -x + \lambda y \end{cases}$$



POINCARÉ - ANDRONOV - HOPF BIFURCATION

$$\begin{cases} x' = y + x(\lambda - x^2 - y^2) \\ y' = -x + y(\lambda - x^2 - y^2) \end{cases}$$


HOMOCLINIC OR SADDLE-LOOP BIFURCATION

$$\begin{cases} x' = y \\ y' = x + \lambda y - x^2 \end{cases}$$

For each system, find the equilibria, the linearization around equilibria as a function of λ and discuss the bifurcation diagram (by plotting a few representative phase portraits for different λ 's)