Fall 2017 **STUDENT NAME:** ______________________
. **Student ID:** ____________________________

EXAM II – Math 4/5650 - Numerical Analysis
Take Home - Due Monday, Nov 20, 12:15pm

No team work of any kind! Answer each problem on a separate sheet of paper, with your name clearly visible.

1. Let $f(x) = e^x - 2x^2 - 1$.
(a) Use a Centered Three point formula with $h = .2$ to approximate $f'(2)$. Also, compute the error bound associated with this approximation (NOT the actual error!).
(b) Use a Forward Three point formula with $h = .2$ to approximate $f'(2)$. Also, compute the error bound associated with this approximation (NOT the actual error!).
(c) Use a Backward Three point formula with $h = .2$ to approximate $f'(2)$. Also, compute the error bound associated with this approximation (NOT the actual error!).

2. Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

$$M = N(h) + K_1h^2 + K_2h^4 + K_3h^6 + \dots,$$

for some constants $K_1, K_2, K_3, \dots$. Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $O(h^6)$ approximation to M.

3. Find the constants c_1, x_0, and x_1 so that the quadrature

$$\int_0^1 f(x)\,dx = \frac{1}{2}f(x_0) + c_1f(x_1)$$

has the highest possible degree of precision. What transformation must be made to implement the rule on $\int_a^b f(t)\,dt$? Use the rule to approximate $\int_0^2 t^3\,dt$ and compare to the actual solution.

4. The Corrected (composite) Trapezoidal Rule is given by

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)] - \frac{h^2}{12}[f'(b) - f'(a)]$$

where $h = (b-a)/n$ and $x_i = a + ih$ are n equidistant points in $[a, b]$.
Use this to approximate the integral

$$\int_0^\pi e^x \cos x\,dx$$

and compare its error with the error resulting from the composite Trapezoidal Rule and the composite Simpson's Rule.

5. Let $\displaystyle\int_1^2 (x^3 + \ln x)\,dx$.
(a) For what value of n is the Composite Simpson Rule approximation of the integral accurate to 10^{-4}?
(b) Compute the approximation using the calculated value of n found in the previous step.

6. Given the initial-value problem

$$y' = e^{t-y}, \qquad 0 \le t \le 1, \qquad y(0) = 1$$

(a) Use Euler's method with $h = 0.5$ to approximate the solution.
(b) Use Taylor's method of order two with $h = 0.5$ to approximate the solution.
(c) Use the answers generated in part(b) and quadratic interpolation to approximate $y(0.8)$.