Name:

Sample Exam 2 Questions
Math 4650/5650: *Numerical Analysis*

Fall 2017

1. Let $f(x) = (x-1)e^x$ and take $h = 0.01$.

(a) Calculate approximation to $f'(2.3)$ using the two-point forward-difference formula. Also, compute the actual error and an error bound for you approximation.

(b) Solve part *(a)* using the two-point backward-difference formula.

2. Use the most accurate formula to determine approximations that will complete the following table.

x	f(x)	$f'(x)$
8.1	16.94410	
8.3	17.56492	
8.5	18.19056	
8.7	18.82091	

3. Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

$$M = N(h) + K_1 h + K_2 h^2 + K_3 h^3 + \ldots,$$

for some constants $K_1, K_2, K_3, \ldots$. Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $O(h^3)$ approximation to M.

4. Find the constants c_0, c_1, and x_1 so that the quadrature

$$\int_0^1 f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision. What transformation must be made to implement the rule on $\int_a^b f(t)\,dt$? Use the rule to approximate $\int_0^2 e^{2t}\,dt$ and compare to the actual solution.

5. Consider approximating the integral

$$\int_0^2 \frac{2}{x^2+4}\,dx.$$

(a) Use the Composite Trapezoidal rule with $n = 4$.

(b) Use the Composite Simpson's rule with $n = 4$.

(c) Use the Composite Midpoint rule with $n = 4$.

6. How large would n have to be to obtain a Composite Simpson Rule (or Composite Trapeziodal Rule) approximation of $\displaystyle\int_{-1}^{2} (x^3 + e^{-x})\, dx$ accurate to five decimal places?

7. Construct a quadrature rule of the form

$$\int_{-1}^{1} f(x)\, dx \approx a_1 f(-0.5) + a_2 f(0) + a_3 f(0.5)$$

which has degree of accuracy 2. What transformation must be made to implement the rule on $\int_a^b f(t)\, dt$? Use the rule to approximate $\int_0^2 e^{2t}\, dt$ and compare to the actual solution.

8. Given the initial-value problem

$$y' = te^{3t} - 2y, \qquad 0 \le t \le 1, \qquad y(0) = 0$$

(a) Use Euler's method with $h = 0.5$ to approximate the solution.

(b) Use Taylor's method of order two with $h = 0.5$ to approximate the solution.

(c) Use the Modified Euler method with $h = 0.5$ to approximate the solution.

(d) Use the answers from part b) and quadratic interpolation to approximate $y(0.4)$.