Our Number System

ID1050– Quantitative & Qualitative Reasoning

Counting Numbers

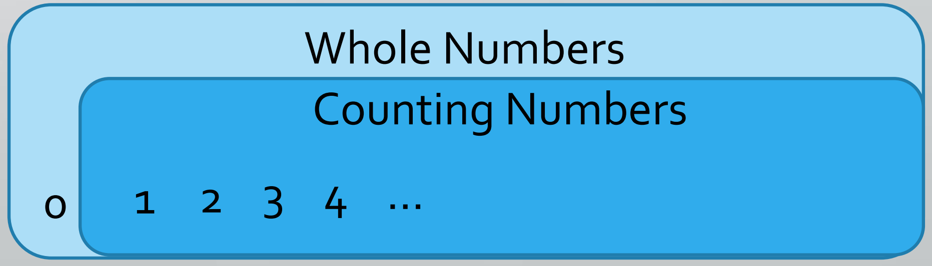
- Our system of expressing numbers has been purposefully constructed. We're going to explore the types of numbers used in that construction.
- The most basic need for a number system is to count. The first evidence of the use of notches for counting is forty thousand years old.
- Not all cultures keep track of all counting numbers. Australian aborigines have words only for 'one', 'two', and then 'many'.
- What we term the 'counting numbers' begins with 1 and sequentially adds one to the previous result, as many times as needed.
- The ***counting numbers***, then, are 1, 2, 3, 4, 5, … (the list is infinite, which means it continues as far as needed, without end).

Counting Numbers

1 2 3 4 …

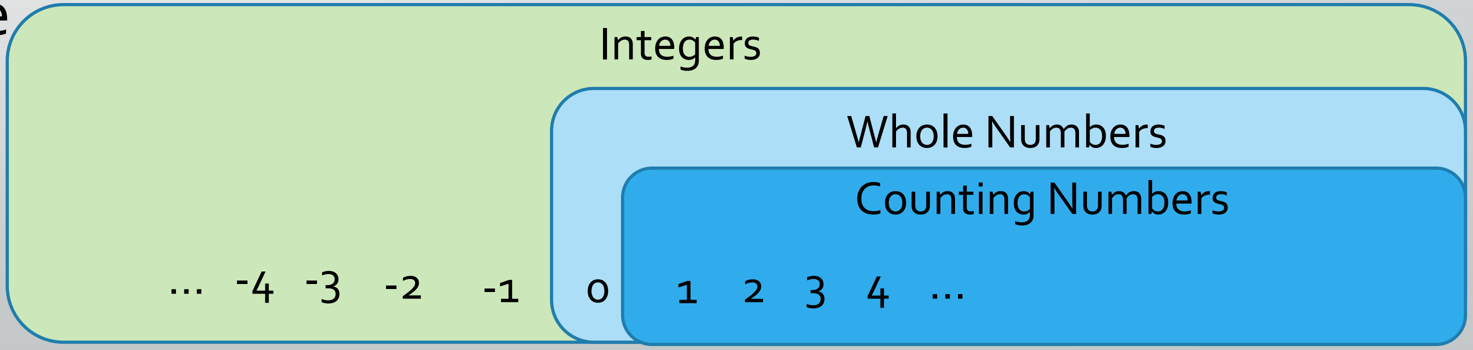
Whole Numbers

- Of course, we started counting from nothing, but we don’t use ‘zero’ to count anything.
- Never-the-less, the concept of ‘zero’ or ‘nothing’ is important in mathematics.
- Including ‘zero’ with the counting numbers forms the set of ***whole numbers.***
- Again, not all cultures have a mathematical need for ‘zero’. Our concept of ‘zero’ was invented in India, and independently by the Mayans and others.



Integers

- You can't count below zero, but you *can* have numbers that represent direction.
- Including a 'negative sign' in front of each counting number forms the set of ***integers***.
- Integers can be used to indicate height *above* or *below* sea-level, or money *earned* or *owed*, for instance.
- There are an infinite number of negative numbers now included with our infinite set of positive numbers.



Rational Numbers

- We sometimes need to express values that are not whole parts, but fractional parts.
- We define the set of ***rational numbers*** as any number that can be expressed as the ratio of an integer over another integer (other than zero).
- There are an infinite number of rational numbers just between zero and one.
  - Here are some of them: `1/2, 1/3, 1/4, 1/5,` …
  - Here are more: `2/3, 13/16, 1001/4812,` …
- There is an infinite number of rational numbers between *any* two integers.
- The set of integers is included in the set of rational numbers.
  - For instance, `4` can be expressed as the ratio `4/1`, so it is rational. So is zero.

Irrational Numbers

- It seems like all the space between zero and one is filled with rational numbers, but it isn't.
- There is *another* infinite set of numbers between zero and one. These are called the *irrational numbers.*
- ***Irrational numbers*** are numbers that <u>cannot</u> be expressed as the ratio of two integers.
- Numbers that are square roots, cube roots, etc. are likely to be irrational.
  - The 'radical' symbol, $\sqrt{}$, is often used to indicate a root
- Special numbers like π are also irrational.

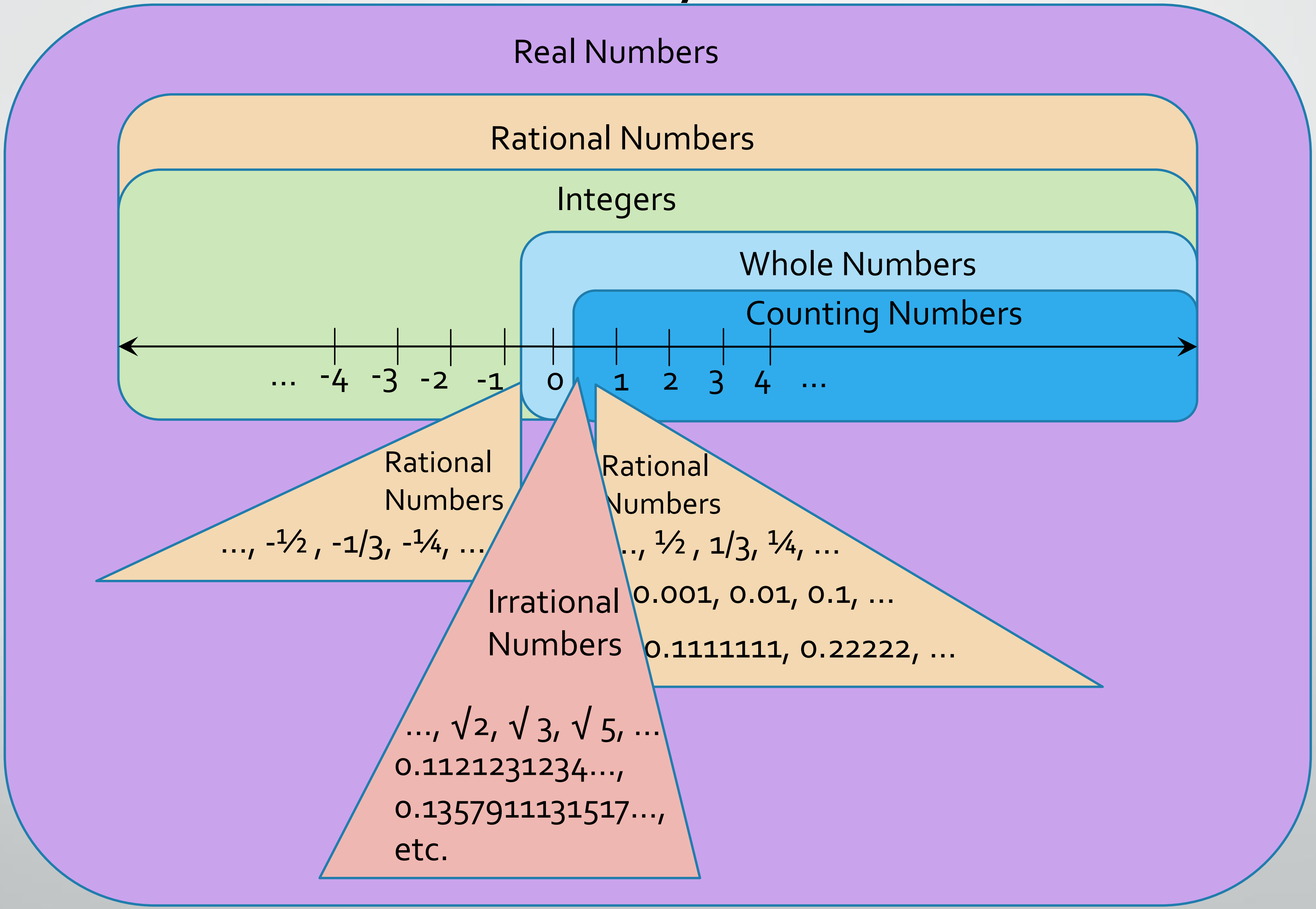
Rational or Irrational?

- Numbers are either rational or irrational. How do you tell which it is?
- A good way is to express the number in decimal form (using a calculator).
- If a number is *rational*, its decimal form will either…
  - …have a set of digits that *repeats* forever (0.333…, or 8.131313…, or −1.00…)
  - Or, its decimal form will *truncate* (0.5, or 4.125).
- If a number is *irrational*, its decimal form will…
  - …have *no set of digits that repeats* (0.1121231234…, -0.1257911131517…, 3.14159265…)
- Roots often indicate that a number is *irrational* ($\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ …)
  - Check on the calculator, though. $\sqrt{4} = 2$, which is rational because it truncates.

The Real Numbers

- Together, the set of rational and irrational numbers make up what we call the ***real numbers.***
- This course will deal exclusively with the set of real numbers.

Summary



Real Numbers
Rational Numbers
Integers
Whole Numbers
Counting Numbers
… -4 -3 -2 -1 0 1 2 3 4 …
Rational Numbers
…, -½ , -1/3, -¼, …
Rational Numbers
…, ½ , 1/3, ¼, …
0.001, 0.01, 0.1, …
0.1111111, 0.22222, …
Irrational Numbers
…, √2, √ 3, √ 5, …
0.1121231234…,
0.1357911131517…,
etc.