

A Sandbox Analogy of the Number Line

ID1050– Quantitative & Qualitative Reasoning



How can we visualize negative numbers?

- Using counting numbers to visualize addition is no problem.

- Addition always gives another counting number.

$$\text{🍏} + \text{🍏} \text{🍏} = ?$$

- Using counting numbers to visualize subtraction is problematic.

- You can't 'take away' more than you start with.

$$\text{🍏} - \text{🍏} \text{🍏} = ?$$

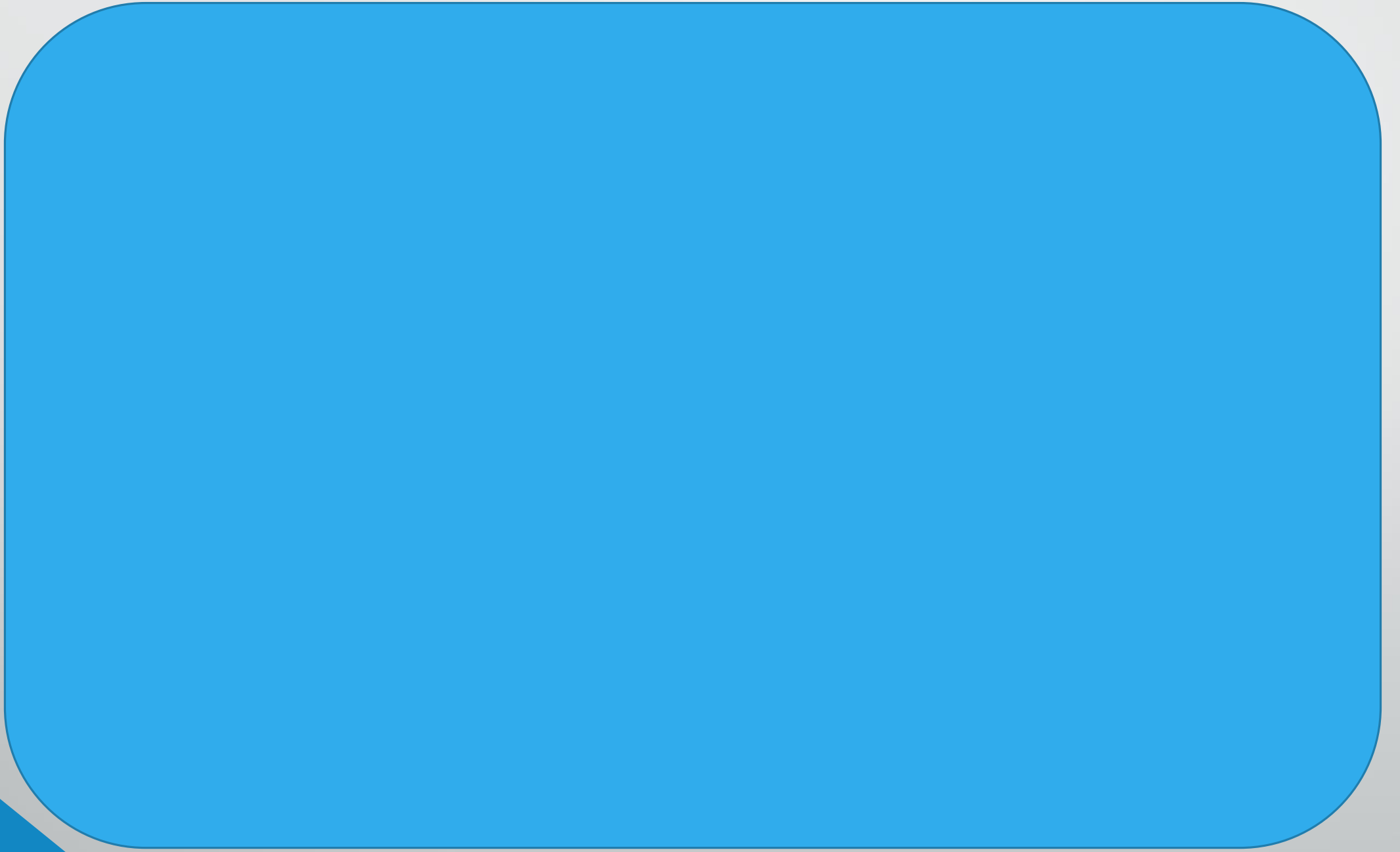
- The introduction of both positive and negative numbers (integers) allows us to solve any addition or subtraction problem and still get another integer.
- It is useful to look at an analogy to help us visualize both positive and negative integers.



Piles and Holes

- Representing integers as piles and holes in a sandbox is a useful analogy.
- We'll imagine a sandbox that is deep enough for any number of holes and can have sand piles arbitrarily high.
- We will imagine working only with 'complete' piles and holes, not fractional parts of piles and holes.
- We will envision the ability to make piles and holes of any size and not have them collapse.
- In this analogy, the untouched level of sand represents the integer 'zero'.
- Piles above this level represent positive integers.
- Holes below this level represent negative integers.

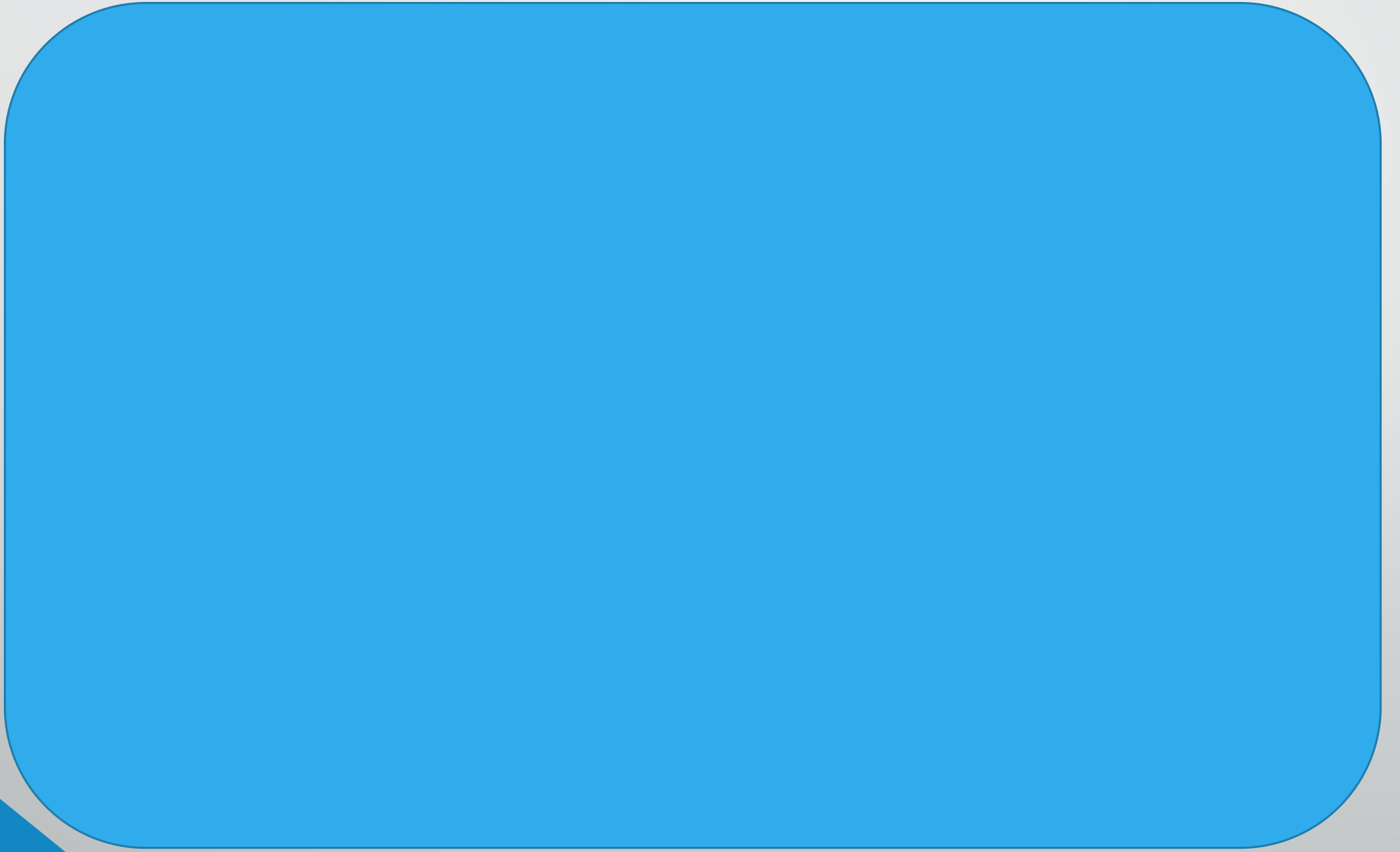




Doing Math with the Sandbox Analogy

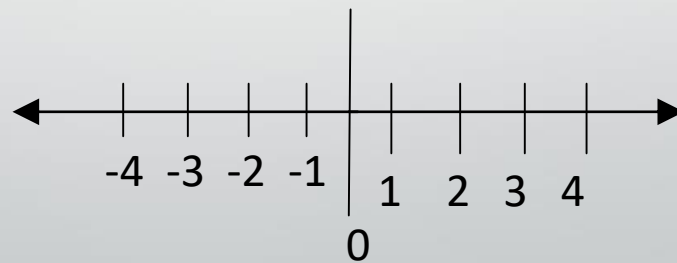
- We can use the sandbox analogy to represent the performance of mathematical operations like addition and subtraction.
- We can make the following equivalences:
 - 'Adding a scoop of sand' is equivalent to adding a positive integer. $+$ \Rightarrow $+$ $^+1$
 - 'Taking away a scoop of sand' is the same as subtracting a positive integer. $-$ \Rightarrow $-$ $^+1$
 - 'Adding a hole' is a way of representing adding a negative integer. $+$ \Rightarrow $+$ $^-1$
 - 'Taking away a hole' is subtracting a negative integer, but this is just another way of saying 'adding a scoop of sand', which itself is equivalent to adding a positive integer. $-$ \Rightarrow $-$ $^-1$ \Rightarrow $+$
- The resulting pile or hole represents the positive or negative integer result of the mathematical operation.





Turning the sandbox into a number line

- We can now imagine turning the sandbox on its side (and the sand can't fall!).
- We turn it clockwise, and label the 'piles' side with positive integers and the 'holes' side with negative integers.
- We place hash marks at the integers, but realize that there is the possibility of having 'partial piles' and 'partial holes' which represent fractional numbers.



Conclusion

- The sandbox analogy is a way to visualize both positive and negative numbers and operations using them.
- Similarly, the number line is a graphical way to express positive and negative numbers.
- Even though we only label the locations of integers, there is a continuous set of numbers all along the number line.

