Counting in Different Bases

ID1050– Quantitative & Qualitative Reasoning

Counting in Base-10

- To understand counting in other bases, we'll examine how our number system behaves when we count in base-10 (decimal).
- You start with zero. Then for each object you count, you add one to the previous number.
- You go through the list of digits 0 to 9. When you add one to 9, there is no symbol for ten, so you start over at zero and 'carry' the one to the 'tens' column.
- You keep cycling through the digits 0 to 9. When you go to a number after any '9', you have to cycle back to '0' and 'carry the one'.
- Look closely at how this happens when you move from '99' to '100'.

0
1
2
3
:
8
9
10
11
12
13
:
18
19
20
21
:
97
98
99
100
101
:

Counting in Base-8

- Let's apply this same procedure to counting in base-8 (octal).
- You start with zero. Then for each object you count, you add one to the previous number.
- You go through the list of digits 0 to 7. When you add one to 7, there is no symbol for eight, so you start over at zero and 'carry' the one to the 'eights' column.
- You keep cycling through the digits 0 to 7. When you go to a number after any '7', you have to cycle back to '0' and 'carry the one'.
- Look closely at how this happens when you move from '77' to '100'.

Decimal	Octal
zero	0
one	1
two	2
three	3
:	:
six	6
seven	7
eight	10
nine	11
ten	12
:	:
fourteen	16
fifteen	17
sixteen	20
seventeen	21
eighteen	22
:	:
sixty-two	76
sixty-three	77
sixty-four	100
sixty-five	101
:	:

Some Things to Note

- Numbers less than the base itself are the same as in base-10
  - Six = 6 (base 10) = 6 (base 8)
- When you see '100' in a different base, don't think to yourself 'one-hundred' because that word is only for decimal numbers. Think instead 'one', 'zero', 'zero'.
- For base systems with a small base…
  - …more columns are usually needed to express a number
  - …you reach the end of the available digits more quickly and have to cycle back to zero.

Counting in Base-3

- Let's apply the procedure to counting in base-3 (trinary).
- You start with zero. Then for each object you count, you add one to the previous number.
- You go through the list of digits 0 to 2. When you add one to 2, there is no symbol for three, so you start over at zero and 'carry' the one to the 'threes' column.
- You keep cycling through the digits 0 to 2. When you go to a number after any '2', you have to cycle back to '0' and 'carry the one'.
- Look closely at how this happens when you move from '22' to '100'.

Decimal	Trinary
zero	0
one	1
two	2
three	10
four	11
five	12
six	20
seven	21
eight	22
nine	100
ten	101
eleven	102
twelve	110
thirteen	111
fourteen	112
fifteen	120
sixteen	121
seventeen	122
eighteen	200
nineteen	201
twenty	202
twenty-one	210
:	:

Counting in Base-2

- Let's apply this same procedure to counting in base-2 (binary).
- You start with zero. Then for each object you count, you add one to the previous number.
- You go through the list of digits 0 to 1. When you add one to 1, there is no symbol for two, so you start over at zero and 'carry' the one to the 'twos' column.
- You keep cycling through the digits 0 to 1. When you go to a number after any '1', you have to cycle back to '0' and 'carry the one'.
- Look closely at how this happens when you move from '11' to '100'.

Decimal	Binary
zero	0
one	1
two	10
three	11
four	100
five	101
six	110
seven	111
eight	1000
nine	1001
ten	1010
eleven	1011
twelve	1100
thirteen	1101
fourteen	1110
fifteen	1111
sixteen	10000
seventeen	10001
eighteen	10010
nineteen	10011
twenty	10100
twenty-one	10101
:	:

Conclusion

- It is difficult to unlearn our base-10 counting system in order to learn to count in a different base, but keep practicing
- Look for the patterns