Exponential Numbers

ID1050– Quantitative & Qualitative Reasoning

In what ways can you have $2000?

- Just like fractions, you can have a number in some denomination

Number	Denomination	Mantissa	Power of 10
20,000	Dimes	20,000.	10^{-1}
2000	Singles	2,000.	10^{0}
200	Tens	200.0	10^{1}
20	Hundreds	20.00	10^{2}
2	Thousands	2.000	10^{3}
0.2	Ten Thousands	0.2000	10^{4}

  - A number in this form has a mantissa (the number) and an exponent of 10 (the denomination)
  - Notice the pattern: as the decimal moves left in the mantissa (decreasing its value), the exponent of 10 moves up (increasing its value), and vice versa.

Numbers in Exponential Notation

- Any number can be expressed in this type of exponential format.
- It is especially useful for really big and really small numbers.
  - One trillion = 1,000,000,000,000 = 1.00×10^{12}
  - One billionth = 0.000000001 = 1.00×10^{-9}
- Scientific calculators allow you to enter numbers in this format.
  - (see the calculator tutorial)

Numbers in Floating Point Notation

- This is our common way of expressing numbers.
- The number is written with the decimal point in whatever place is appropriate.
- There is no multiplication by a power of ten.
  - Example: `456.78` is in floating point notation.
- A number in floating point notation can be easily converted to exponential notation:
  - The floating point number becomes the mantissa
  - Multiply by ten to the zero power (which is, after all, equal to one)
  - Example: `456.78` in exponential notation becomes 456.78×10^0

Numbers in Scientific Notation

- A particular form of exponential notation is called scientific notation
- In this form, the mantissa must be between 1 and 10.
- This results in a single, non-zero digit, followed by the decimal point, and then perhaps more digits.
  - Examples: 1.2345×10^{3} and 5.0×10^{-4} (but *not* 0.65×10^{1})
- Most calculators use scientific notation as their default way to express exponential numbers.
  - A number in exponential format can have the decimal anywhere in the mantissa, but the calculator will convert this into scientific notation.

Precision and Rounding

- The number of digits in the mantissa is a measure of the number's precision. We call this the number of *significant figures*.
- We could require the mantissa to have only 3 digits of precision, for example.
- We would need to truncate (drop) any digits after the third one (the second digit after the decimal point)
- If the mantissa has fewer than three digits, fill in with zeros on the right.
- Before we drop the 4th digit and beyond, we need to check to see if we should *round* the 3rd digit first:
  - If the 4th digit is between 0 and 4, don't change the 3rd digit
  - If the 4th digit is between 5 and 9, increase the 3rd digit by one
  - Example: 1.234×10^{3} becomes 1.23×10^{3}
  - Example: 4.56789×10^{-8} becomes 4.57×10^{-8}
  - Example: 5.0×10^{1} becomes 5.00×10^{1}

Addition/Subtraction in Exponential Format

There is a simple method for adding numbers in exponential format:

- Get both number's exponents to be the same by adjusting the decimal point of one of them. Use the rules 'exponent up, decimal left' or 'exponent down, decimal right'
- Keeping this common exponent for the power of ten, add the mantissas.
- Adjust the decimal and exponent to put the answer into proper scientific notation, and round to 3 significant figures
- Example:

$$\begin{array}{r} 1.23\times10^{3} \\ +\ 4.56\times10^{2} \end{array} \qquad \begin{array}{r} 1.23\ \times10^{3} \\ +\ 0.456\times10^{3} \end{array} \qquad \begin{array}{r} 1.23\ \times10^{3} \\ +\ 0.456\times10^{3} \\ \hline 1.686\times10^{3} \end{array} \qquad \boxed{1.69\ \times10^{3}}$$

- Subtraction is done in exactly the same way, except you *subtract* the mantissas
- You can also just use a scientific calculator.

Multiplication/Division in Exponential Format

Multiplication of numbers in exponential format is even simpler:

- Multiply the mantissas.
- The power of 10 in the answer is the sum of the powers of 10 of the two numbers.
- Adjust the decimal and exponent to put the answer into proper scientific notation, and round to 3 significant figures
- Example:

$$\begin{array}{rr} & 3.7 \times 10^{5} \\ * & \underline{5.6 \times 10^{1}} \\ & 20.72 \times 10^{6} \end{array} \qquad \begin{array}{rr} & 3.7 \times 10^{5} \\ + & \underline{5.6 \times 10^{1}} \\ & \end{array}$$

2.07×10^{7}

- Division works exactly the same, except you *divide* mantissas and *subtract* the powers
- Example:

$$\begin{array}{rr} & 3.7 \times 10^{5} \\ \div & \underline{5.6 \times 10^{1}} \\ & 0.66071 \times 10^{4} \end{array} \qquad \begin{array}{rr} & 3.7 \times 10^{5} \\ \div & \underline{5.6 \times 10^{1}} \\ & \end{array}$$

6.61×10^{3}

Conclusion

- Exponential notation is a form of expressing numbers, especially big and small numbers.
- Scientific notation is a particular type of exponential notation
- We can specify a precision in our answer and round to that precision
- The operations of addition, subtraction, multiplication, and division can be performed using a certain method, or using a calculator.