Factoring

ID1050– Quantitative & Qualitative Reasoning

Distributive Property and Factoring

- *Factoring* is a method of manipulating an equation.
- The *factor* is a common number or variable that appears in two terms that are being added together.
  - Example: *A* is a common factor in A*B + A*C
- When we *factor out* the common factor, we move that factor out in front of the two terms and group what's left of the terms between parentheses.
  - Example: (A * B + A * C)
- Factoring is really just an application of the distributive property.
- Following are some techniques to help perform this operation correctly:

Technique 1: Dividing by the factor

- If the quantity to factor out is given, you can find what terms should appear in the parentheses by dividing the given terms by this factor.
  - Example: Given the terms `ab + ac`, factor out `a`, or, in other words:
    - $ab + ac = a(? + ?)$
    - $\frac{ab}{a} + \frac{ac}{a} = \frac{a}{a}(? + ?)$ and the *a*'s *cancel out* or *reduce to one*, leaving: $b + c = (? + ?)$
    - So $ab + ac = a(b + c)$
  - Example: $10^2 + 10 = 10(? + ?)$ so $\frac{10^2}{10} + \frac{10}{10} = \frac{10}{10}(? + ?)$ therefore $10^2 + 10 = 10(10 + 1)$
  - Example: $a^2 + a = a^3(? + ?)$ so $\frac{a^2}{a^3} + \frac{a}{a^3} = \frac{a^3}{a^3}(? + ?)$ therefore $a^2 + a = a^3(\frac{1}{a} + \frac{1}{a^2})$
    - or $a^2 + a = a^3(a^{-1} + a^{-2})$

Technique 2: Comparing Exponents

- In this technique, we recall that when you multiply numbers with the same base, you add their exponents: $10^2 * 10^3 = 10^{(2+3)} = 10^5$
- In our problem: $10^2 + 10 = 10(? + ?)$ we know that the ?'s must be *10* to some exponent, so we need to find that exponent. Writing everything as *10* to some exponent:
  - $10^2 + 10^1 = 10^1(10^{▲} + 10^{■}) = 10^{1+▲} + 10^{1+■}$
- The first terms must be equal, so $10^2 = 10^{1+▲}$ so $2=1+▲$ therefore $▲=1$
- The second terms must be equal, so $10^1 = 10^{1+■}$ so $1=1+■$ therefore $■=0$
  - So $10^2 + 10 = 10(10^1 + 10^0) = 10(10 + 1)$

Technique 3: Eliminating Common Factors

- With this technique, we look for the common factor in all the terms and simply eliminate them. Whatever is left in the terms must be what goes in the parentheses.
  - Example: $a^2b^3 + ab^2 = ab(? + ?)$
  - Write out the powers: $a * a * b * b * b + a * b * b = a * b(? + ?)$
  - Cross out an a and a b in every term: $\cancel{a} * a * \cancel{b} * b * b + \cancel{a} * \cancel{b} * b = \cancel{a} * \cancel{b}(? + ?)$
  - This leaves the answer: $a * b * b + b = \ (? + ?) = (ab^2 + b)$
  - So our problem can be rewritten as: $a^2b^3 + ab^2 = ab(ab^2 + b)$
- If we eliminate everything in a term, it will be replaced by 1.
  - Example: $a^2b + ab = ab(? + ?)$
  - $a * \cancel{a} * \cancel{b} + \cancel{a} * \cancel{b} = \cancel{a} * a * \cancel{b} + \cancel{a} * \cancel{b} * 1 = (a + 1)$ and so $a^2b + ab = ab(a + 1)$

Conclusion

- Factoring involves factoring out a common value from some given terms and finding what is left.
- If the quantity to factor out is given, you can use one of several techniques to find what is left after factoring it out.
- Some techniques will work better for different problems; if one doesn't work out, try a different one.