Properties of Operations for Real Numbers

ID1050– Quantitative & Qualitative Reasoning

What Are Properties?

- These are rules that are always true…
  - … for the set we are working with (the set of real numbers)
  - … for the operations we are working with (addition and multiplication)
- They may seem ‘obvious’ because we are so familiar with them
  - The are not always true for other sets of things
- They form an important foundation for higher mathematics

Associative Property

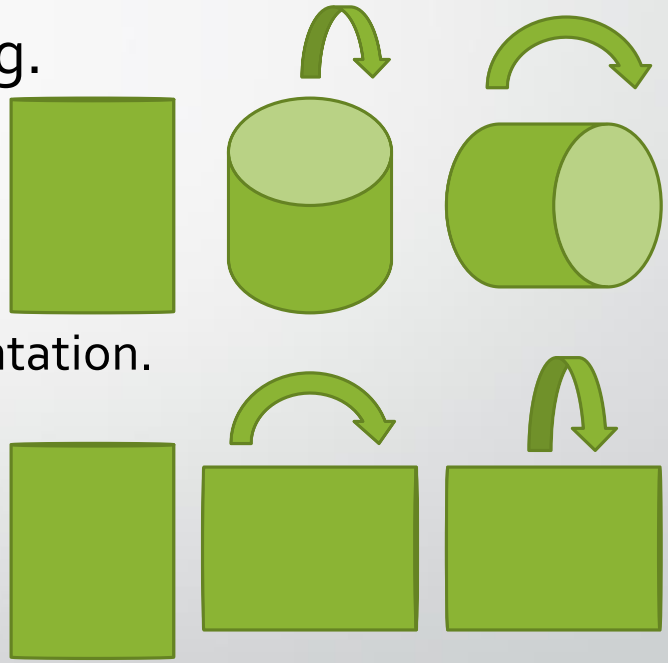
- Associative property of addition of real numbers (*associate*)
  - This answers the question “When we add numbers together, does *grouping* matter?”
  - No, it does not. In general, (A + B) + C = A + (B + C) and A, B, and C are any real number
    - Example: (⁻1 + 2) + 3 = ⁻1 + (2 + 3) = 4
  - Does this work for subtraction?
    - Counter-example: (⁻1 – 2) – 3 ≠ ⁻1 – (2 – 3)
    - But you can convert subtraction problems into addition problems, and it will work.
- Associative property of multiplication of real numbers
  - Since multiplication is a series of additions, multiplication is also associative.
  - In general, (A*B)*C = A*(B*C)
    - Example: (⁻1 * 2) * 3 = ⁻1 * (2 * 3) = ⁻6

Commutative Property

- Commutative property of addition of real numbers (*commute* or *travel*)
  - This answers the question "When we add numbers together, does order matter?"
  - No, it does not. In general, $A + B = B + A$ and A and B are any real number
    - Example: $^-2 + 3 = 3 + ^-2 = 1$
  - Does this work for subtraction?
    - Counter-example: $^-2 - 3 \neq 3 - ^-2$
    - But you can convert subtraction problems into addition problems, and it will work.
- Commutative property of multiplication of real numbers
  - Since multiplication is a series of additions, multiplication is also commutative.
  - In general, $A*B = B*A$
    - Example: $^-3 * 4 = 4 * ^-3 = ^-12$

Does the Order of Operations Ever Matter?

- We are so used to order not making a difference for addition of real numbers that we may wonder if it's true for addition of anything.
- Here's a counter-example: Rotations of an object
  - Hold an object (like a can) orientated like this:
  - First, rotate it *forward*, then to your *left*. Note the final orientation.
  - Now start over with your original orientation.
  - Now, rotate it to your *left* first, then *forward*.
    - The orientation is different than the first time
  - The order of adding rotations matters, so rotations *<u>don't</u>* commute.



Converting Subtraction & Division

- As we've seen, the properties for real numbers involve addition (and not subtraction) and multiplication (and not division).
- Subtraction problems can be converted to addition problems.
  - Example: $(^{-}1 - 2) - 3$ turns into $(^{-}1 + {}^{-}2) + {}^{-}3$
- Division problems can be converted into multiplication problems.
  - Example: $2 \div 3$ or $2/3$ or $\frac{2}{3}$ turns into $2 * \frac{1}{3}$
- So any problem involving the four basic operations can be manipulated using properties, like *associative* or *commutative*.
  - Example: $2 - 4/5$ turns into $2 + {}^{-}4 * \frac{1}{5} = {}^{-}\frac{1}{5} * 4 + 2$ (used commutivity)

Distributive Property

- This property combines both multiplication and addition
  - This changes the addition of two terms into the multiplication of two factors
  - We say '*multiplication distributes over addition*' for real numbers
  - In general, A*(B + C) = A*B + A*C and A, B, and C are any real number
    - Example: `2*(3 + 4) = 2*3 + 2*4 = 14`
  - Does multiplication distribute over subtraction?
    - Actually, yes, but it is because you can convert subtraction of a number into addition of its negative.
    - Example: `2*(3 – 4) = 2*(3 + ⁻4) = 2*3 + 2*⁻4 = ⁻2`
- Careful! Don't distribute multiplication over multiplication
  - This is wrong: `2*(3 * 4) ≠ 2*3 * 2*4`

Conclusion

- Operations on real numbers have some rules that are always true:
  - Associativity of addition (and multiplication): grouping doesn’t matter
  - Commutativity of addition (and multiplication): order doesn’t matter
  - Distributive property: Governs how addition and multiplication interact
- Knowing the properties of real numbers lets us manipulate equations to get them into a more useful form without ‘breaking’ the equality.