

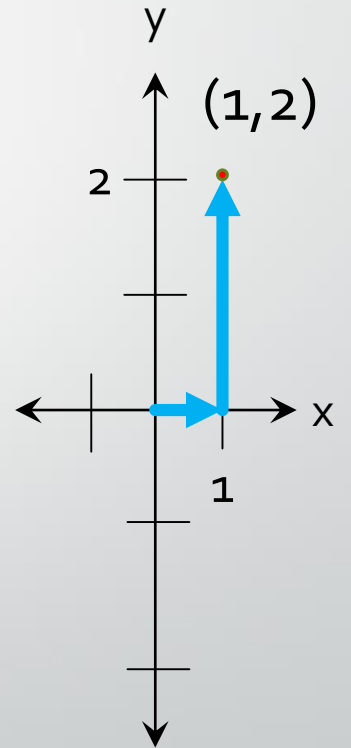


Functions and Graphing

ID1050– Quantitative & Qualitative Reasoning

Points on a Graph and Coordinates

- We can describe all the points in a plane by laying out two number lines that cross each other. We call these number lines *axes*.
 - The intersection of the two is called the *origin*.
 - The horizontal one is usually called the *x-axis*, and the vertical is the *y-axis*.
 - The scale and spacing can be the same or different between the axes.
- We notice that every point in the plane can be located by its value along the x-axis and its value along the y-axis.
 - The x and y values put together in that order are called the coordinates of the point they represent.
 - There is a one-to-one mapping of every point to two unique coordinates. Coordinates are always in order (x,y) .

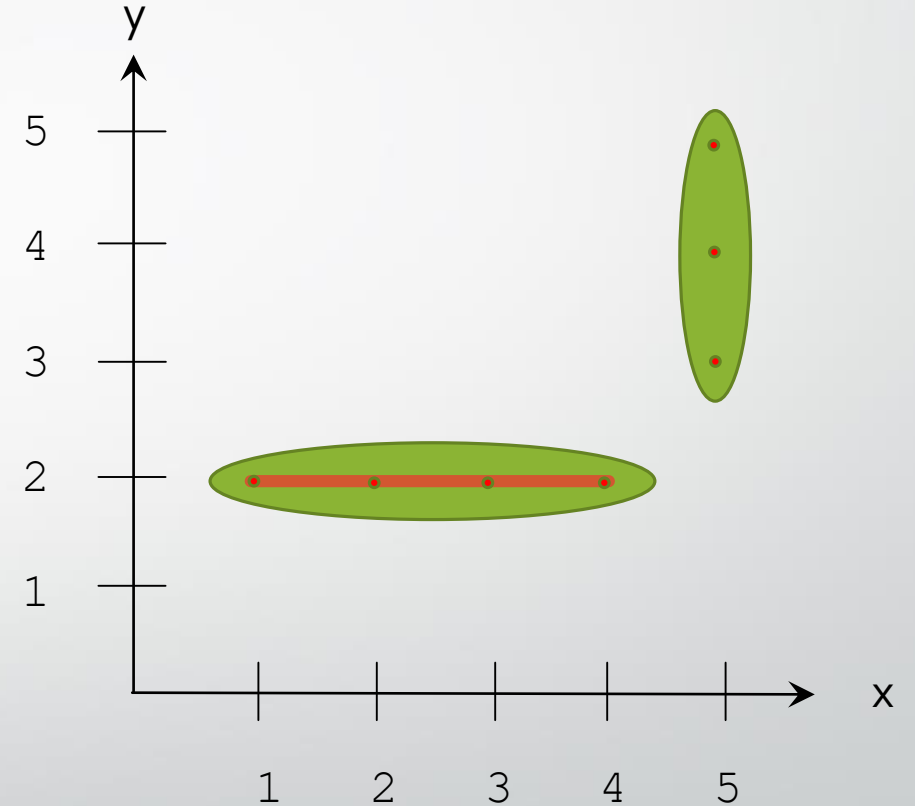


Relations and Functions

- A *Relation* is a collection of points (or their coordinates) that are related in some way.
- A *Function* is a relation that follows this rule:
 - No two unique coordinates have the same first number (x-value)
- A function can consist of discrete points or a continuous set of points.
- A function can be a finite or an infinite number of points.

Battleship Example

- In the game of Battleship, ships are placed on a grid, and guessing the points underneath them is the goal of the game.
- Ship 1's points $(5,3)$, $(5,4)$, and $(5,5)$ are a relation, but not a function:
 - The three unique points share the same first number.
 - Essentially, functions can't have points on the same vertical line.
- Ship 2's points $(1,2)$, $(2,2)$, $(3,2)$, and $(4,2)$ are a relation and also a function.
 - If every point between $(1,2)$ and $(4,2)$ are also included, then:
 - x is continuous, and y is discrete.
 - Points are specified by: $1 \leq x \leq 4$ and $y=2$.



Function Gun Analogy

- Functions take a number, perform an operation on it, and return another number.
- One way to understand how a function operates is visualize it as a function gun.
 - The function gun can take any type of bullet (e.g. BB, .22 caliber, 9 mm, etc.)
 - For each type of bullet, the function gun always hits the same point on the target.
 - The bullet is the x-value, and the hole in the target is the y-value
- The rules of functions make them reversible.
 - Observing the hole in the target, one can deduce the bullet that made it.



Ways to Express Functions

Now we have several ways of expressing a function:

- We can simply list the points that the function relates: (x,y)
 - If there are an infinite number of them, we can list a representative sample.
- We can instead list the points in table format.
 - One column holds the x 's, the other column holds the y 's.
- We can draw the points on a graph
 - If there are an infinite number of them, we can draw a few and connect the dots
- We can write the relationship between x and y as an equation.
 - y is a function of x , written $y = f(x)$
 - The function f takes an x value and turns it into a y value.

Graphing an Equation

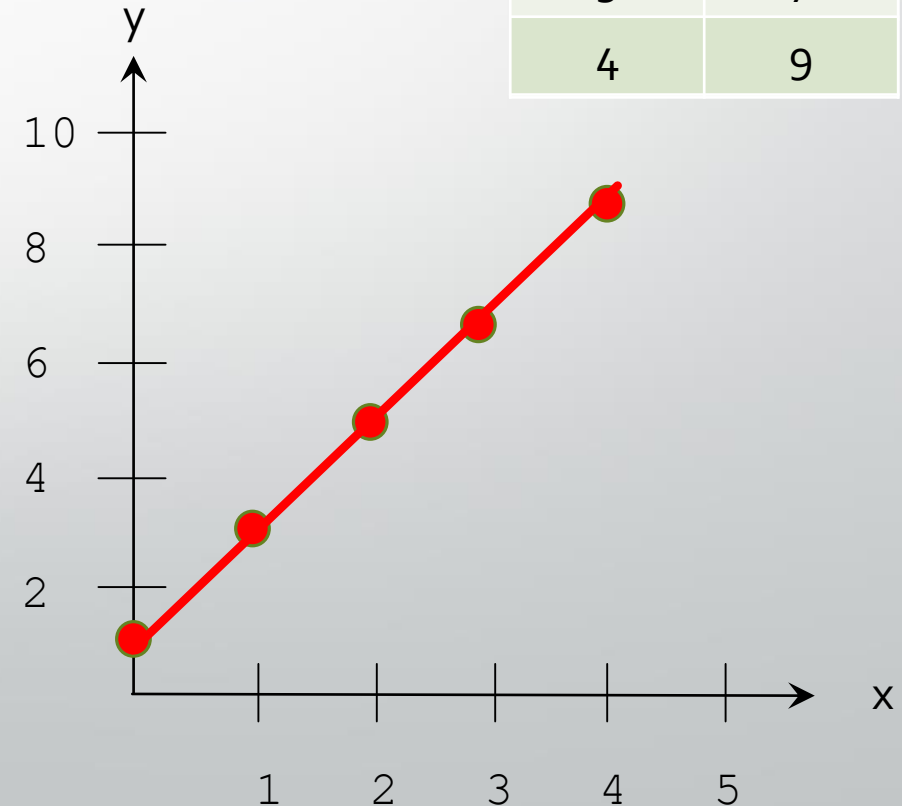
- If a function in equation form is to be put into graphical form, the possible x-values must be provided.
 - The possible x values are called the Domain
 - The resulting possible y values are called the Range
- You can form a table of the x and y values .
 - List the x-values (or a representative set of them)
 - Use the equation to find the corresponding y values
- You can now sketch the axes.
 - You need to wait until the table is formed to know where to start and end the axes.
- Now draw the points for each (x,y) pair, and then connect the dots.
 - 5-20 points is usually enough to get an idea of the complete graph.

Graphing Example

Let's start with a simple example: $y=2x+1$ with $0 \leq x \leq 4$

- We need 5-20 points, so if we step the x-value from 0 to 4 with steps of 1, we'll get 5 points.
- We form a table and fill in the x-values:
- We put each x-value into our equation to get the corresponding y-value
- We note that x and y are always positive, so we sketch the following axes:
- We plot each point.
- We connect the dots. Our graph looks like a line (and it is). We can even predict how it will look outside of our plot range.

x	y
0	1
1	3
2	5
3	7
4	9

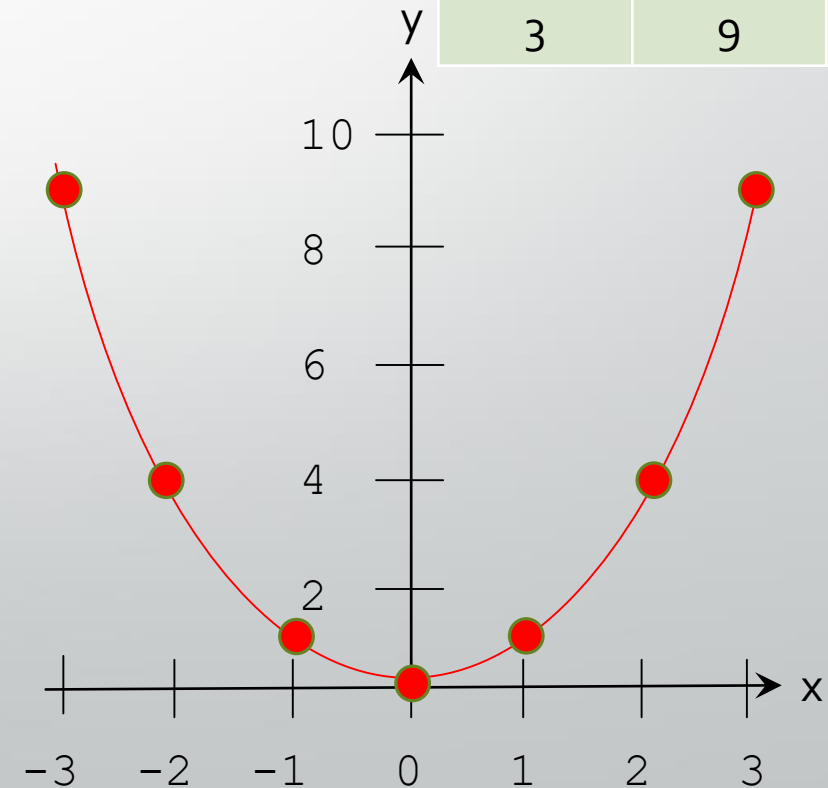


Another Graphing Example

Let's start with a simple example: $y=x^2$
with $-3 \leq x \leq 3$

- If we step the x-value from -3 to 3 with steps of 1 , we'll get 7 points.
- We form a table, fill in the x-values, and compute the y-values.
- We note that y is always positive, so we sketch the following axes:
- We plot each point and connect the dots.
- This graph looks quite different from the first.

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Conclusion

- Functions operate on a number and give a result
- Functions are reversible
- Functions can be expressed as a series of points, as a table, as an equation, and as a graph
- To graph a function:
 - Figure out a good number of x-values to use (based on the domain)
 - Compute the corresponding y-values (using a calculator)
 - Sketch a set of axes and plot the points
 - Connect the dots