Logarithms

ID1050– Quantitative & Qualitative Reasoning

History and Uses

- We noticed that when we multiply two numbers that are the same base raised to different exponents, that the result is the base raised to the sum of the exponents
  - Example: $10^2 * 10^3 = 10^{2+3} = 10^5$
- This is the basis of the idea of logarithms: We can do complicated multiplication operations by working with only the exponents, which are added (an easier task).
- There are two bases typically used: **10**, and the natural number **e=2.71828…**
  - If the base is 10, this is called the 'common' log
  - If the base is e, this is called the 'natural' log (abbreviated *ln*)

History and Uses

- The trick is to find the exponent to which the base must be raised to get the desired number.
  - Example: What is the logarithm of 10? We know $10=10^1$, so the answer is **1**
  - To what power do we raise 10 to get 100? $100=10^2$, so the answer is **2**
  - Now one that is not obvious: $45 = 10^?$ The answer must be between 1 and 2, but what is it?
  - This used to be done using a table, but is now done using a calculator. The answer to our example is $45 = 10^{1.653212514}$ or $\log(45)=1.653212514$

Using Common Logarithms to Perform Calculations

- Let's start with our simple example: **$10^2 * 10^3 = ?$**
- We reduce **10^2** to just its exponent by looking up 100 in a log table or using a calculator. We find that **log(100)=2**.
- We reduce **10^3** in the same way: **log(1000)=3**.
- Now we add the exponents: **2+3=5**
- Finally, we convert from an exponent back to a normal number by using a reverse look-up in our log table, or using the inverse log function on the calculator: **10^x**
- We get the answer: **10^5**

Using Common Logarithms to Perform Calculations

- Let's see how this works for a more complicated example:
  45 * 16 = ?
  - We know a procedure for multiplication of multi-digit numbers, but this method can be error-prone, especially for numbers with many digits.
- Using the logarithm method:
  - Convert 45 to an exponent using logarithms: $\log(45)=1.653212514$
  - Convert 16 to an exponent using logarithms: $\log(16)=1.204119983$
  - Add the exponents: $1.653212514 + 1.204119983 = 2.857332497$
  - Convert back from an exponent: $10^{2.857332497} = 720$ (this is our answer)

Using Common Logarithms to Perform Calculations

- For small numbers, doing the multiplication may be easier than all this, but imagine multiplying two 10 digit numbers:
  - Multiplication involves each digit of one number times the other number (100 multiplications), and then adding all the columns to get the result.
  - The logarithm method involves three table-look-ups and adding two 10-digit numbers; a much simpler operation!

Using Natural Logarithms to Perform Calculations

- The method is exactly the same if you are using natural logarithms, except you would use the natural logarithm table or the natural logarithm function, ln(x)
  - We reduce one number to just its exponent by looking it up in a natural log table or using a calculator.
  - We reduce the other number in the same way.
  - Now we add the exponents.
  - Finally, we convert from an exponent back to a normal number by using a reverse look-up in our natural log table, or using the inverse natural log function on the calculator: e^x

Using Natural Logarithms to Perform Calculations

- Let's see how this works for our example: **45 * 16 = ?**
  - Convert 45 to an exponent using natural logarithms: **ln(45)=3.80666249**
  - Convert 16 to an exponent using natural logarithms: **ln(16)=2.772588722**
  - Add the exponents: **3.80666249 + 2.772588722 = 6.579251212**
  - Convert back from an exponent: $e^{6.579251212} = 720$ (this is our answer)

Examples: Evaluation

- The common and natural logarithms are unary functions, as are their inverses.
  - They take a single argument (number) and return the result.
- See the TI-30Xa calculator tutorial, or the manual for your calculator, to determine exactly how to enter these functions.

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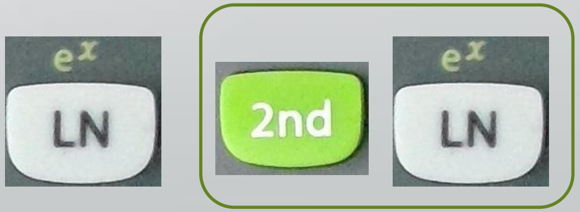
Parentheses
Exponents
Multiplication
Division
Addition
Subtraction



Solving Equations: Single Function

- To solve an equation for an unknown variable that is affected by **only one logarithmic or exponential operation**, you must apply the **inverse** of that operation to **both sides** of the equation.
- The operation and its inverse 'cancel each other', leaving just the unknown on one side, and its value on the other.
  - Example: $10^{x} = 45$
    - The operation affecting 'x' is '10^x' or common anti-logarithm or exponential.
    - Its inverse is 'common logarithm' or log(). Taking the logarithm of both sides yields:
    - $\log(10^{x}) = \log(45)$
    - $x = 1.65$
  - Example: $\ln(x) = -4$
    - The operation affecting 'x' is 'natural logarithm' or LN().
    - Its inverse is 'e^x' or natural anti-logarithm or exponential. Taking the anti-logarithm of both sides yields:
    - $e^{\ln(x)} = e^{-4}$
    - $x = 0.0183$

Conclusion

- Logarithm functions and their inverses can be used to simplify complicated arithmetic operations:
  - Multiplication problems become addition problems.
  - Exponent problems become multiplication problems.
- The base of the logarithm is not important; the method works the same for any base as long as an appropriate table (or calculator function) is available.
  - Usually base-10 (common log) or base-e (natural log) are used.
- Logarithms show up in many other areas in mathematics and science.