Powers and Roots

ID1050– Quantitative & Qualitative Reasoning

Powers

- What does the notation B^p mean? (B is the 'base' and p is the 'power')
  - Positive integer powers – If p is a positive integer, then B^p is a short-hand for a series of multiplications of the base
    - $B*B*B*\ldots*B=B^p$
    - Numerical Example: $10^5 = 10*10*10*10*10$
  - Negative integer powers – If p is a negative integer, this means the reciprocal of $B^{|p|}$
    - $B^{-p} = \frac{1}{B^p} = \frac{1}{B*B*\ldots*B}$
    - Numerical Example: $10^{-4} = \frac{1}{10^4} = \frac{1}{10*10*10*10}$

Roots

- What does the notation $B^{1/r}$ mean? (B is the ‘base’ and r is the ‘root’)
  - The number ($B^{1/r}$) times itself r times is B
  - $(B^{1/r}) * (B^{1/r}) * \ldots * (B^{1/r}) = B$
- Numerical example
  - Cube root of 2 = 1.26…
  - So (1.26…) * (1.26…) * (1.26…) = 2
- Radical Notation
  - Cube root of 2 = $\sqrt[3]{2}$
  - Square root of five = $\sqrt[2]{5}$ = $\sqrt{5}$
- Fractional Notation
  - Cube root of 2 = $2^{1/3}$

Rules for Combining Powers

- $B^{p}*B^{q} = B^{p+q}$
  - $2^{3}*2^{4} = (2*2*2)*(2*2*2*2) = 2^{7} = 2^{3+4}$
- $\frac{B^{p}}{B^{q}} = B^{p-q}$
  - $\frac{5^{3}}{5^{2}} = \frac{5*5*5}{5*5} = 5^{1} = 5^{3-2}$
- $(B^{p})^{q} = B^{p*q}$
  - $(4^{3})^{2} = (4*4*4)*(4*4*4) = 4^{6} = 4^{3*2}$
- $(B^{p})^{1/q} = B^{p/q} = (B^{1/q})^{p}$
  - $(3^{2})^{1/3} = 3^{2/3} = (3^{1/3})^{2} = 2.08008...$

Powers and Roots Are Inverses of Each Other

- $(B^p)^{1/p} = B^{p/p} = B^1 = B$
- $(B^{p/r})^{r/p} = B^{pr/pr} = B^1 = B$
- Numerical examples
  - Radical notation example
    - $(\sqrt[3]{2})^3 = \sqrt[3]{(2^3)} = 2$
  - Fractional power notation examples
    - $(10^{3/2})^{2/3} = 10^1 = 10$

Examples: Evaluation

- The integer powers and roots are unary functions.
  - They take a single argument (number) and return the result.
- See the TI-30Xa calculator tutorial, or the manual for your calculator, to determine exactly how to enter these functions.
- Alternative notation:
  - **$10^{2/3}$ = 10^(2/3) = 4.641589…**

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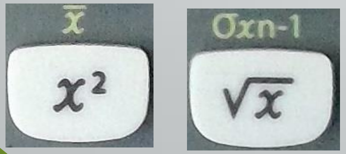
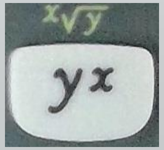
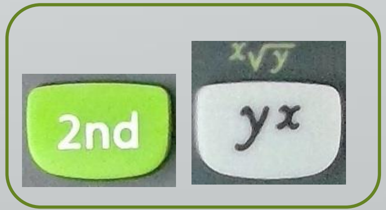
PEMDAS

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction







Solving Equations: Single Function

- To solve an equation for an unknown variable that is affected by **only one power or root operation**, you must apply the **inverse** of that operation to **both sides** of the equation.
- The operation and its inverse 'cancel each other', leaving just the unknown on one side, and its value on the other.
  - Example: $x^3 = 55$
    - The operation affecting 'x' is 'cube' or '3rd power'.
    - Its inverse is 'cube root' or '3rd root'. Taking the cube root of both sides yields:
    - $\sqrt[3]{(x^3)} = \sqrt[3]{(55)}$
    - $x = 3.80$
  - Example: $\sqrt{x} = 15$
    - The operation affecting 'x' is 'square root' or '2nd root'.
    - Its inverse is 'square' or '2nd power'. Taking the square of both sides yields:
    - $(\sqrt{x})^2 = (15)^2$
    - $x = 225$

Solving Equations: Single Function

- To solve an equation for an unknown variable that is affected by **only one fractional power operation**, you must apply the **inverse** of that operation to **both sides** of the equation.
- The inverse of a fraction is its **reciprocal** (e.g 3/2 is the reciprocal of 2/3)
- A fractional power is inverted by raising it to the **reciprocal** of that fractional power.
- The operation and its inverse 'cancel each other', leaving just the unknown on one side, and its value on the other.
  - Example: $x^{2/3} = 10$
    - The operation affecting 'x' is '2/3 power' or '2nd power, 3rd root'.
    - Its inverse is '2/3 root' or '2nd root, 3rd power'. Taking the 3/2 power of both sides yields:
    - $(x^{2/3})^{3/2} = (10)^{3/2}$
    - $x = 31.62$

Conclusion

- The functions we call ‘powers’ and ‘roots’ are inverse functions of each other
- We can express roots in either *radical* notation or *fractional power* notation
- To invert a function that is a fractional power, raise the function to the reciprocal of that fraction