Solving for the Unknown: Basic Operations & Trigonometry

ID1050– Quantitative & Qualitative Reasoning

What is Algebra?

- An expression is a combination of numbers and operations that leads to a numerical result
  - Example: If x=2, what is $3x^2+2$?
  - For these problems, we 'plug and chug'. We *substitute* the number for x, and we *evaluate* the expression.
- In algebra, we have an equation, and we don't know the value of x (or whatever variable name we are using.)
  - Example: $3x^2+2=14$
  - For these problems, we need to *solve for the unknown x*, so we used rules of algebra to *invert* all of the operations acting on x, leaving x by itself on one side of the equation
  - At every inversion step, we also apply the same inversion to the number on the other side, eventually leaving the result of what x is. For our example, x=2.

Order of Operations and Inversions

- When *evaluating an expression*, we adhere to a specified order when we apply operations like addition, squaring, etc. This is called the *order of operations*.
  - The order is listed to the right. Expressions in parentheses should be evaluated first. Exponent operations should be evaluated next, followed by multiplication, etc.
  - One mnemonic for PEMDAS is "please excuse my dear Aunt Sally".
- When *inverting an equation*, we must go *backwards through* PEMDAS.
  - We invert the operations on the unknown x in reverse order of how the operations would have been applied if we had known x from the start.

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Basic Operations

- We have seen examples of inverting single operations like addition or division. We don't need PEMDAS for this.
- When we have two or more operations, we need PEMDAS to determine the correct order. If the two operations are the same (addition and addition), the order doesn't matter.
  - 2-step inversion example: $4x-1=3$
  - We ask "what operations are acting on x?" They are multiplication by 4 and subtraction of 1.
  - PEMDAS says we invert Subtraction first (by addition of 1), then invert Multiplication next (by division by 4.)
  - Remember to apply the inversion operation to *both sides*.
    - Invert subtraction: $4x-1+1 = 3+1$ or $4x=4$
    - Invert multiplication: $4x/4 = 4/4$ or $x=1$

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Function/Inverse Pairs
Multiply and divide
Addition and subtract

Basic Operations

- 3-step inversion example: $2x/3+4=6$
  - What operations act on x?
    - Multiplication by 2
    - Division by 3
    - Addition of 4
  - PEMDAS says we should invert addition first, division next, and multiplication last.
    - Subtract 4 from both sides: $2x/3+4-4 = 6-4$ or $2x/3 = 2$
    - Multiply both sides by 3: $2x/3*3 = 2*3$ or $2x=6$
    - Divide both sides by 2 : $2x/2 = 6/2$ or $x=3$
- Any number of basic operations can be inverted in this way.

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
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Division
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Subtraction

Function/Inverse Pairs
Multiply and divide
Addition and subtract

Trigonometry Functions and Basic Operations

- Trigonometry functions don't appear explicitly in PEMDAS. If they did, they would be between Parentheses and Exponents.
- 2-step inversion example: `tan(x)+1=0`
  - Operations on x: tangent and addition of 1
  - Subtract 1 from both sides: `tan(x)+1-1=0-1` or `tan(x)= -1`
  - Inverse tangent of both sides: $\tan^{-1}$`[tan(x)]=` $\tan^{-1}$`[-1]` or `x=-45°`
- 3-step inversion example: `2*sin(x)-1=0`
  - Operations on x: multiplication, sine, and subtraction
  - Add 1 to both sides: `2*sin(x)-1+1 = 0+1` or `2*sin(x)=1`
  - Divide both sides by 2: `2*sin(x)/2 = 1/2` or `sin(x)=0.5`
  - Inverse sine of both sides: $\sin^{-1}$`[sin(x)] =` $\sin^{-1}$ `[0.5]` or `x=30°`

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Function/Inverse Pairs
Sin() and Sin^{-1}()
Cos() and Cos^{-1}()
Tan() and Tan^{-1}()

Trigonometric Inverses and Basic Operations

- Trigonometric inverses don't appear explicitly in PEMDAS. If they did, they would be between Parentheses and Exponents.
- 2-step inversion example: $\tan^{-1}(x)+60°=30°$
  - Operations on x: inverse tangent and addition
  - Subtract 60° from both sides: $\tan^{-1}(x)+60^0-60^0=30°-60^0$ or $\tan^{-1}(x)=-30°$
  - Apply tangent to both sides: $\tan[\tan^{-1}(x)]= \tan[-30°]$ or $x=-0.577$
- 3-step inversion example: $2*\cos^{-1}(x)/3=40°$
  - Operations on x: division, inverse cosine, and multiplication
  - Multiply both sides by 3: $2*\cos^{-1}(x)/3*3 = 40^0*3$ or $2*\cos^{-1}(x)=120^0$
  - Divide both sides by 2: $2*\cos^{-1}(x)/2 = 120^0/2$ or $\cos^{-1}(x)=60^0$
  - Apply cosine to both sides: $\cos[\cos^{-1}(x)] = \cos[60^0]$ or $x=0.5$

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Function/Inverse Pairs
$\text{Sin}()$ and $\text{Sin}^{-1}()$
$\text{Cos}()$ and $\text{Cos}^{-1}()$
$\text{Tan}()$ and $\text{Tan}^{-1}()$

Conclusion

- The order of operations (PEMDAS) provides a plan for applying operations when evaluating expressions or solving equations.
- When solving for the unknown x, we invert operations in reverse order through PEMDAS.
- Trigonometry function and inverses would be done between the *P* and the *E.*
- If two operations are the same (e.g. addition, as in 3+x+6), then they can be applied or inverted left-to-right.