Solving for the Unknown: Logarithms and Quadratics

ID1050– Quantitative & Qualitative Reasoning

Logarithms

Logarithm functions (common log, natural log, and their inverses) represent exponents, so they are the '*E*' in PEMDAS.

- Recall that the common log has a base of 10
  - Log(x) = y means 10^y=x (10^y is also known as the common anti-log)
- Recall that the natural log has a base of e
  - Ln(x) = y means e^y=x (e^y is also know as the natural anti-log)

(evaluating)→
PEMDAS
←(inverting)

Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

Function/Inverse Pairs
Log() and 10^x
LN() and e^x

Common Logarithm and Basic Operations

- 2-step inversion example: `2*log(x)=8`
  - Operations on x: **Common log** and **multiplication** by 2
  - **Divide** both sides by 2: `2*log(x)/2 = 8/2` or `log(x) = 4`
  - Take **common anti-log** of both sides: $10^{\log(x)} = 10^4$ or `x=10000`
- 3-step inversion example: `2 ·log(x)-5=-4`
  - Operations on x: **multiplication**, **common log**, and **subtraction**
  - **Add** 5 to both sides: `2 ·log(x)-5+5=-4+5` or `2 ·log(x)=1`
  - **Divide** both sides by 2: `2 ·log(x)/2=1/2` or `log(x)=0.5`
  - Take **common anti-log** of both sides: $10^{\log(x)} = 10^{0.5}$ or `x=3.16`

(evaluating)→
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Common Anti-Logarithm and Basic Operations

- 2-step inversion example: $5*10^x=6$
  - Operations on x: 10^x and **multiplication**
  - **Divide** both sides by 5: $5*10^x/5 = 6/5$ or $10^x=1.2$
  - Take **common log** of both sides: $\log(10^x)=\log(1.2)$ or $x=0.0792$
- 3-step inversion example: $10^x/2-5=0$
  - Operations on x: **division**, 10^x, and **subtraction**
  - **Add** 5 to both sides: $10^x/2-5+5=0+5$ or $10^x/2=5$
  - **Multiply** both sides by 2: $10^x/2*2=5*2$ or $10^x=10$
  - Take **common log** of both sides: $\log(10^x)=\log(10)$ or $x=1$

(evaluating)→
PEMDAS
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Natural Logarithm and Basic Operations

- 2-step inversion example: 2*LN(x)=4
  - Operations on x: **Natural log** and **multiplication** by 2
  - **Divide** both sides by 2: 2*LN(x)/2 = 4/2 or LN(x) = 2
  - Take **natural anti-log** of both sides: $e^{LN(x)} = e^2$ or x=7.39
- 3-step inversion example: 3LN(x)+5=8
  - Operations on x: **multiplication**, **natural log**, and **addition**
  - **Subtract** 5 from both sides: 3LN(x)-5+5=8-5 or 3LN(x)=3
  - **Divide** both sides by 3: 3LN(x)/3=3/3 or LN(x)=1
  - Take **natural anti-log** of both sides: $e^{LN(x)} = e^1$ or x=2.72

(evaluating)→
PEMDAS
←(inverting)

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LN() and e^x

Natural Anti-Logarithm and Basic Operations

- 2-step inversion example: $e^x/3=1$
  - Operations on x: e^x and **division**
  - **Multiply** both sides by 3: $e^x/3*3 = 1*3$ or $e^x=3$
  - Take **natural log** of both sides: $LN(e^x)=LN(3)$ or $x=1.10$
- 3-step inversion example: $2*e^x-1=2$
  - Operations on x: **multiplication**, e^x, and **subtraction**
  - **Add** 1 to both sides: $2*e^x-1+1=2+1$ or $2*e^x=3$
  - **Divide** both sides by 2: $2*e^x/2=3/2$ or $e^x=1.5$
  - Take **natural log** of both sides: $LN(e^x)=LN(1.5)$ or $x=0.405$

(evaluating)→
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Double Exponents

- An exponent can also have an exponent itself:
  - Example: $\mathbf{10}^{x^2}$
  - We invert ‘from the ground, up’
- Example: $\mathbf{10000} = \mathbf{10}^{x^2}$
  - Operations on x: **10ˣ** and **x²**
    - Invert **10ˣ** with **log()**: $log(\mathbf{10000}) = log(\mathbf{10}^{x^2})$ or **4=x²**
    - Invert **x²** with **square-root**: $\sqrt{x^2} = \sqrt{\mathbf{4}}$ or **x=2**

Quadratic Equations

- Quadratic equations have the form: **$ax^2+bx+c=0$**, where **a**, **b**, and **c** are just numbers, called 'coefficients'. The number in front of x^2 is **a**, the number in front of x is **b**, and the number by itself is **c**.
- Using the PEMDAS-and-invert process doesn't work well here.
- An inversion formula has been worked out to solve for the unknown *x*. It is called the Quadratic Formula
  - Here it is: $x = \frac{-b \pm \sqrt{b^2 - 4*a*c}}{2*a}$
  - The numbers for **a**, **b**, and **c** are placed into the formula
  - The 'plus-minus' symbol means that there are two answers for x; one if you use 'plus' in the formula, and the other if you use 'minus' in the formula.
  - Since there is an x^2 in the formula, there will be two answers
    - Simple example: $x^2=25$, answers are +5 and -5
  - In a cubic equation (x^3) there are 3 answers for x, in a quartic equation (x^4), there are 4 answers, etc.

Examples: Quadratic Formula

- Example: $2x^2+3x+1=0$
  - The numbers in front are $a=2$, $b=3$, $c=1$
  - Plugging into the quadratic formula: $x = \frac{-3 \pm \sqrt{3^2 - 4*2*1}}{2*2}$, so $x=-1$ and $x=-0.5$
- Example: $2x^2=-3x-1$
  - If one side of the equation doesn't equal zero, perform algebraic operations to get it that way.
  - In our example, you can add $3x$ and 1 to both sides to get $2x^2+3x+1=0$
  - In our case, the quadratic equation now looks the same as the first example.

Examples: Quadratic Formula

- Example: $x^2-2x+1=0$
  - If a term has no explicit number in front (e.g. x^2), then the number defaults to 1 (a=1)
  - If a term is subtracted (e.g. $-2x$), then the number in front is a negative number (b=-2)
  - Here, c=1
  - Plugging into the quadratic formula: $x = \frac{-(-2)\pm\sqrt{(-2)^2-4*1*1}}{2*1}$, so $x=1$ and $x=1$
  - This is an example of a 'double root'; both values of x are the same.
- Example: $x^2-9=0$
  - If any term doesn't appear, then the number in front is zero. Here, there is no x-term, so the coefficient, b, is zero (b=0)
  - Plugging into the quadratic formula: $x = \frac{-0\pm\sqrt{0^2-4*1*(-9)}}{2*1}$, so $x=3$ and $x=-3$

Conclusion

- Logarithms (common log and natural log) and their inverses appear in PEMDAS as 'E', exponents.
- A series of exponents (e.g. e^{x^3}) is inverted from the bottom, up.
- Quadratic equations are inverted using the Quadratic Formula.
  - Coefficients in front of powers of x are placed into the formula.
  - If a term has no coefficient, it defaults to 1. If a term doesn't appear, its coefficient is 0.