Using the z-Table: Given z, Find the Area

ID1050– Quantitative & Qualitative Reasoning

A Full Z-Table

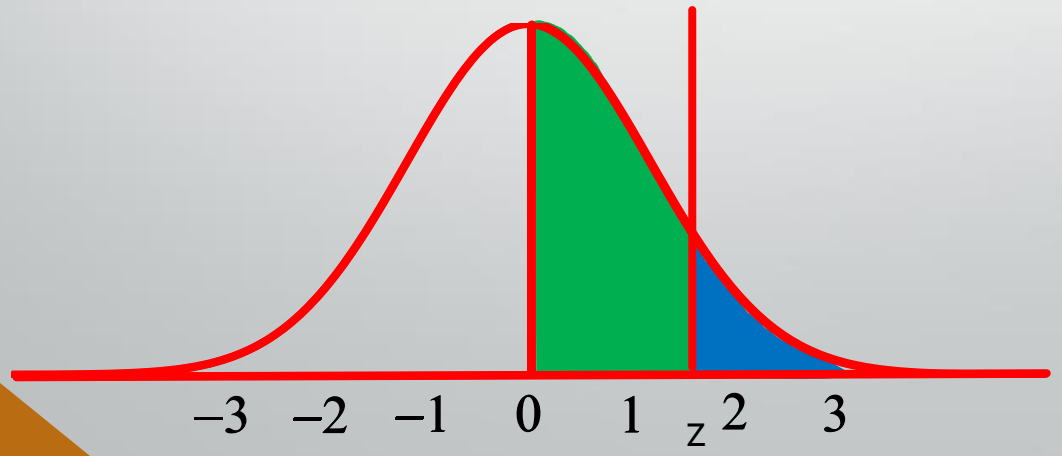
(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500
0.1	0.040	0.460
0.2	0.079	0.421
0.3	0.118	0.382
0.4	0.155	0.345
0.5	0.192	0.309
0.6	0.226	0.274
0.7	0.258	0.242
0.8	0.288	0.212
0.9	0.316	0.184
1.0	0.341	0.159
1.1	0.364	0.136
1.2	0.385	0.115

(A) z	(B) Area between mean and z	(C) Area beyond z
1.3	0.403	0.097
1.4	0.419	0.081
1.5	0.433	0.067
1.6	0.445	0.055
1.7	0.455	0.045
1.8	0.464	0.036
1.9	0.471	0.029
2.0	0.477	0.023
2.1	0.482	0.018
2.2	0.486	0.014
2.3	0.489	0.011
2.4	0.492	0.008
2.5	0.494	0.006

(A) z	(B) Area between mean and z	(C) Area beyond z
2.6	0.495	0.005
2.7	0.496	0.004
2.8	0.497	0.003
2.9	0.498	0.002
3.0	0.499	0.001

Recall What the Z-Table Represents

- We can now answer all of the types of questions we have seen before, like "what fraction of the population is above/below/between some value?" or "what score is needed to be at some percentile?"
- Our z-value can be negative, but because of symmetry, we ignore the sign on the z-value when we look it up in the z-table.
- The z-table has three columns:
  - Column A is the z-value. Due to symmetry, we only need positive values of z.
  - Column B is the area under the standard normal curve between z and zero (toward the hump.)
  - Column C is the area under the standard normal curve beyond z (toward the tail.)
- The area values in columns B and C represent some fraction of the entire population.



(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500
0.1	0.040	0.460
0.2	0.079	0.421
…	…	…

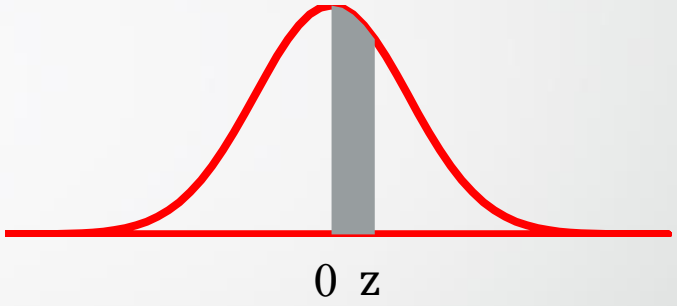
Given a Z-value, Find the Area

Here are some examples of how to answer this type of question:

- You should first sketch the normal curve, then draw a vertical line at the given z-value(s), and, finally, shade in the area that the question is asking about. This helps you figure out how to use the z-table value to find the answer to the question.
- These questions require you to look for the z-value(s) in Columns A, and then find the corresponding value in Column B or Column C.
- In some cases, the answer is directly in the table. In other cases, the value in the table is used to calculate the answer.

Given a Z-value, Find the Area

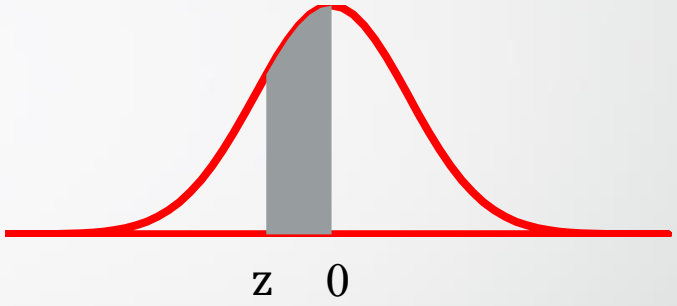
- Question: "Using the z-table, find the area between z=0.4 and the mean (z=0)".
  - Sketch the curve and shade the area of interest.
  - The area we want is between z and the mean, which is found directly in column B.
  - Answer: 0.155 (or 15.5%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.4	0.155	0.345
:	:	:

Given a Z-value, Find the Area

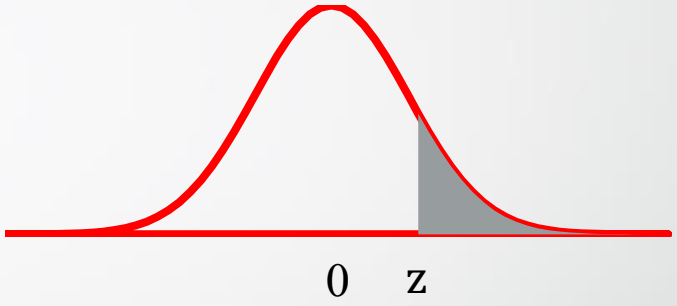
- Question: "Using the z-table, find the area between z=-0.6 and the mean (z=0)".
  - Sketch the curve and shade the area of interest.
  - The area we want is between z and the mean, which is found directly in column B.
  - We ignore the negative sign on z and look up 0.6 in the table.
  - Answer: 0.226 (or 22.6%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.6	0.226	0.274
:	:	:

Given a Z-value, Find the Area

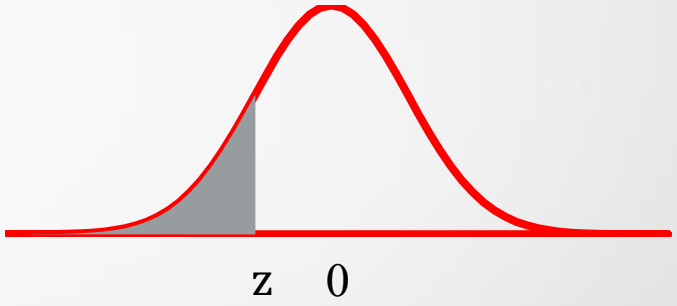
- Question: "Find the area to the right of $z=0.8$".
  - Sketch the curve and shade the area of interest.
  - The area we want is beyond z to the right, which is found directly in column C.
  - Answer: 0.212 (or 21.2%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.8	0.288	0.212
:	:	:

Given a Z-value, Find the Area

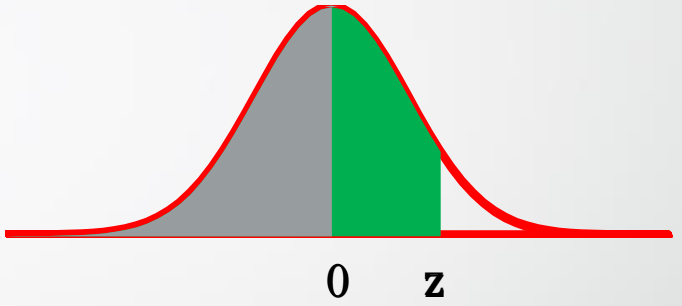
- Question: "Find the area to the left of z=-0.7".
  - Sketch the curve and shade the area of interest.
  - The area we want is beyond z to the left, which is found directly in column C.
  - We ignore the negative sign on z and look up 0.7 in the table.
  - Answer: 0.242 (or 24.2%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.7	0.258	0.242
:	:	:

Given a Z-value, Find the Area

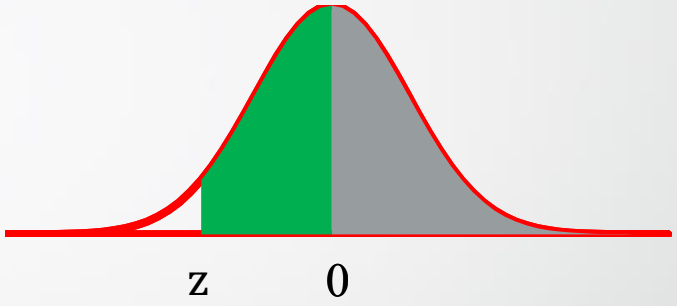
- Question: "Find the area to the left of z=1.0".
  - Sketch the curve and shade the area of interest.
  - The area we want includes both the area between z and the mean, and the area from the mean all the way to the left.
  - Recall that the area under the left half of the normal curve is 0.5.
  - We look up z=1.0 in column B (0.341), and add 0.5 to this value.
  - Answer: 0.841 (or 84.1%)
    - (Another way to answer this question is to take away the area we don't want from the total area under the curve. Here we take the part under the tail (0.159) from the whole area (1.000), to get 0.841.)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
1.0	0.341	0.159
:	:	:

Given a Z-value, Find the Area

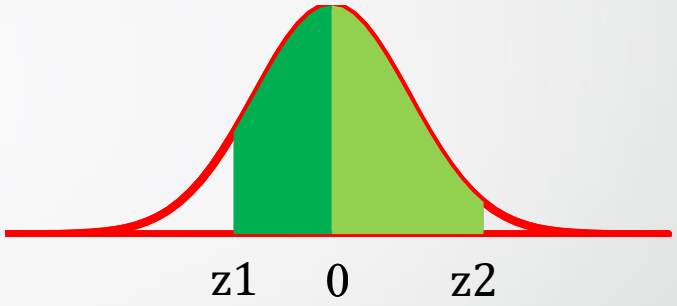
- Question: "Find the area to the right of z=-1.2".
  - Sketch the curve and shade the area of interest.
  - The area we want includes both the area between z and the mean, and the area from the mean all the way to the right.
  - Recall that the area under the right half of the normal curve is 0.5.
  - We ignore the sign on z and look up z=1.2 in column B (0.385), and add 0.5 to this value.
  - Answer: 0.885 (or 88.5%)
    - (Or 1.000 – 0.115 = 0.885.)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
1.2	0.385	0.115
:	:	:

Given a Z-value, Find the Area

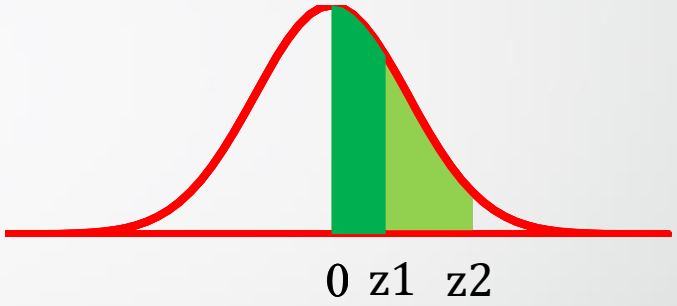
- Question: "Find the area between z1=-0.9 and z2=+1.4".
  - Sketch the curve and shade the area of interest.
  - Here we have two z-values of opposite sign.
  - The area we want is the sum of (the area between z1 and the mean) and (the area between z2 and the mean). These are both from column B.
  - We ignore the sign on z1 and look up z=0.9 in column B (0.316). For z2, we look up z=1.4 in column B (0.419). Then we add these two values together.
  - Answer: 0.735 (or 73.5%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.9	0.316	0.184
:	:	:
1.4	0.419	0.081
:	:	:

Given a Z-value, Find the Area

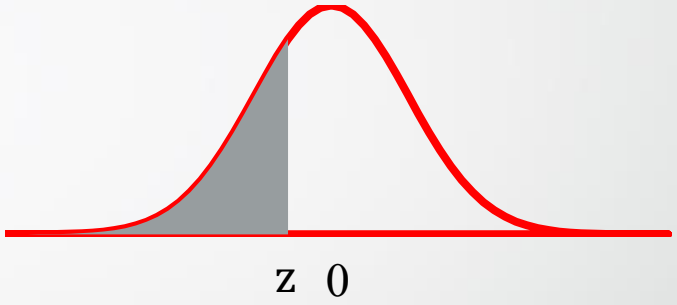
- Question: "Find the area between z1=0.5 and z2=1.3".
  - Sketch the curve and shade the area of interest.
  - Here we have two z-values of the same sign.
  - The area we want is the difference between (the area between z1 and the mean) and (the area between z2 and the mean). These are both from column B.
  - For z1, we look up z=0.5 in column B (0.192). For z2, we look up z=1.3 in column B (0.403). Then we subtract the smaller from the larger (areas are always positive).
  - Answer: 0.211 (or 21.1%)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.5	0.192	0.309
:	:	:
1.3	0.403	0.097
:	:	:

Percentile

- When the z-value is given and the percentile is unknown, this is equivalent to the z-value being given and the area is desired.
- Question: "What percentile does a z-value of -0.4 yield?".
- This is equivalent to the question: "Find the area to the left of z=-0.4".
  - Sketch the curve and shade the area of interest.
  - The area is to the left of z (beyond z). Ignore the sign on z and look up z=0.4. The desired area is found in column C (0.345, or 34.5%).
  - Answer: z=-0.4 is at the 34.5^{th} percentile



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.4	0.155	0.345
:	:	:

Conclusion

- The method for answering questions where the z-value(s) is given and area (or fraction or percentage of the population) is desired.
- In some cases, the answer is directly in the table. In other cases, the table value(s) are used to calculate the answer.
- If two z-values are given and the area between them is desired:
  - Look up column B for both z-values (ignoring the negative value on z, if present).
    - (Column B for "Between")
  - If the z-values have opposite signs, add the column B values.
  - If the z-values have the same signs, subtract the column B values.
    - If the subtraction gives a negative number, drop the sign. Areas are always positive.
- Percentile questions where the z-value is given can be rephrased into the form 'given a z-value, find the area'. Percentile always refers to the area left of a z-value.