

# Using the z-Table: Given an Area, Find z

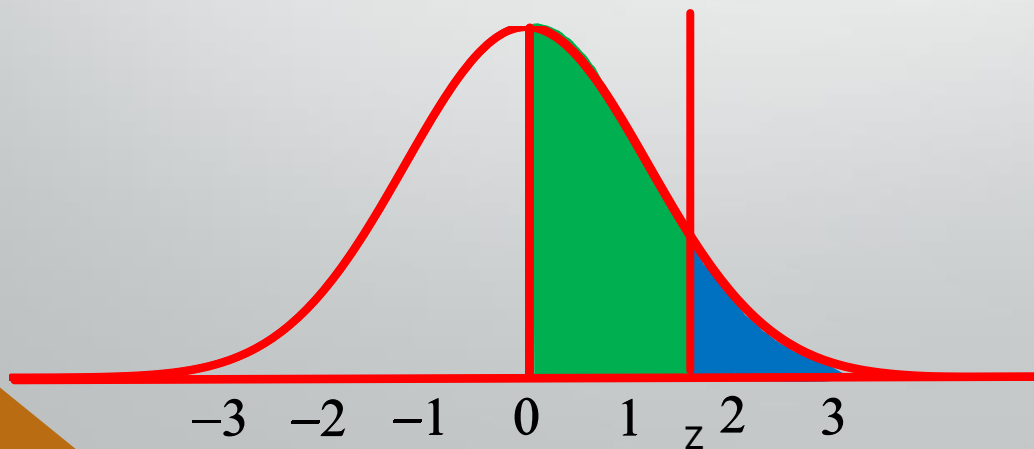
ID1050– Quantitative & Qualitative Reasoning

# A Full Z-Table

(A) z	(B) Area between mean and z	(C) Area beyond z	(A) z	(B) Area between mean and z	(C) Area beyond z	(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500	1.3	0.403	0.097	2.6	0.495	0.005
0.1	0.040	0.460	1.4	0.419	0.081	2.7	0.496	0.004
0.2	0.079	0.421	1.5	0.433	0.067	2.8	0.497	0.003
0.3	0.118	0.382	1.6	0.445	0.055	2.9	0.498	0.002
0.4	0.155	0.345	1.7	0.455	0.045	3.0	0.499	0.001
0.5	0.192	0.309	1.8	0.464	0.036			
0.6	0.226	0.274	1.9	0.471	0.029			
0.7	0.258	0.242	2.0	0.477	0.023			
0.8	0.288	0.212	2.1	0.482	0.018			
0.9	0.316	0.184	2.2	0.486	0.014			
1.0	0.341	0.159	2.3	0.489	0.011			
1.1	0.364	0.136	2.4	0.492	0.008			
1.2	0.385	0.115	2.5	0.494	0.006			

# Recall What the Z-Table Represents

- We can now answer all of the types of questions we have seen before, like “what fraction of the population is above/below/between some value?” or “what score is needed to be at some percentile?”
- Our z-value can be negative, but because of symmetry, we ignore the sign on the z-value when we look it up in the z-table.
- The z-table has three columns:
  - **Column A is the z-value.** Due to symmetry, we only need positive values of z.
  - **Column B is the area under the standard normal curve between z and zero** (toward the hump.)
  - **Column C is the area under the standard normal curve beyond z** (toward the tail.)
- The area values in columns B and C represent some fraction of the entire population.



(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500
0.1	0.040	0.460
0.2	0.079	0.421
...	...	...

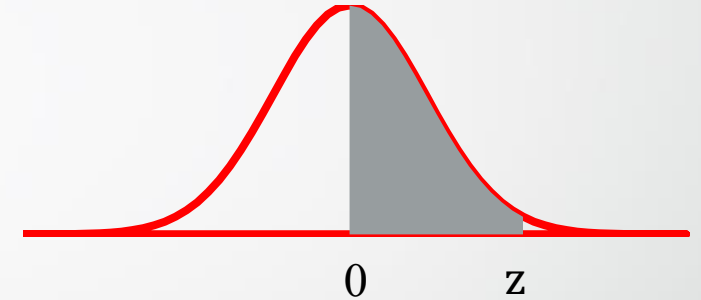
# Given an Area, Find the Z-value

Here are some examples of how to answer this type of question:

- In some cases, the answer is directly in the table. In other cases, the value in the table is used to calculate the answer.
- You should first sketch the normal curve, then draw a vertical line where you estimate the z-value should be, and, finally, shade in the area that the question is asking about. This helps you figure out how to use the z-table value to find the answer to the question.
- These questions require you to look for values in Columns B or C, and then find the corresponding z-value.
- In some cases, the value may be between two rows. In these cases, you may pick the row with the closest value. (Alternatively, interpolation could be used for greater accuracy.)

# Given an Area, Find the Z-value

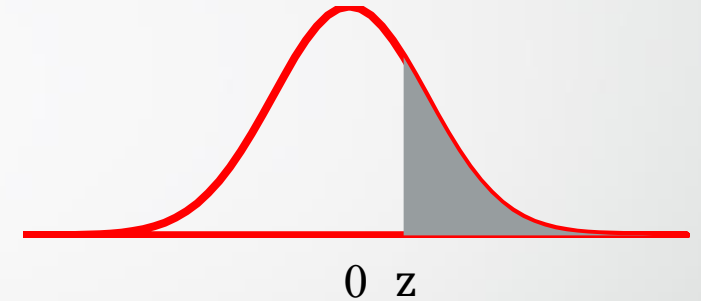
- Question: “The area between the mean and an unknown positive  $z$  is 0.445. Find this  $z$ -value”.
  - Sketch the curve and shade the area of interest. Since  $z$  is positive, it is to the right of the center. (The entire right half of the normal curve has an area of 0.5, so our  $z$  is probably pretty far to the right, but accuracy is not critical.)
  - The area we want is between  $z$  and the mean, which can be found in column B.
  - Search through column B until we find a number near 0.445. In this case, we find exactly this value.
  - The corresponding  $z$ -value is 1.6, and we are told that it is positive.
  - Answer:  $z=+1.6$



(A) $z$	(B) Area between mean and $z$	(C) Area beyond $z$
:	:	:
1.6	0.445	0.055
:	:	:

# Given an Area, Find the Z-value

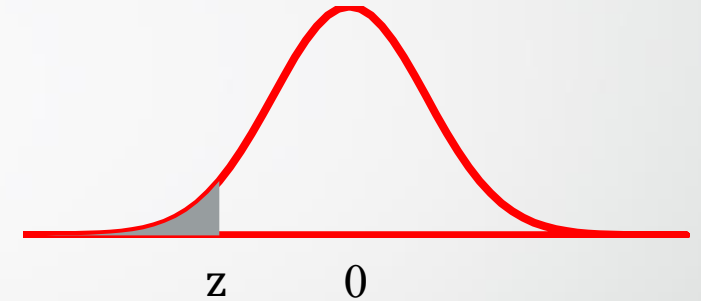
- Question: “The area to the right of an unknown  $z$  is 0.300. Find this  $z$ -value”.
  - Sketch the curve and shade the area of interest. The given area is less than half, so our  $z$ -value must be to the right of zero.
  - The area we want is beyond  $z$ , which can be found in column C.
  - Search through column C until we find a number near 0.300. In this case, the number is between two rows.
  - Let’s choose the row with the closest value to 0.300. This would be the row with  $z=0.5$ .
  - Answer:  $z=+0.5$



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
0.5	0.192	0.309
0.6	0.226	0.274
:	:	:

# Given an Area, Find the Z-value

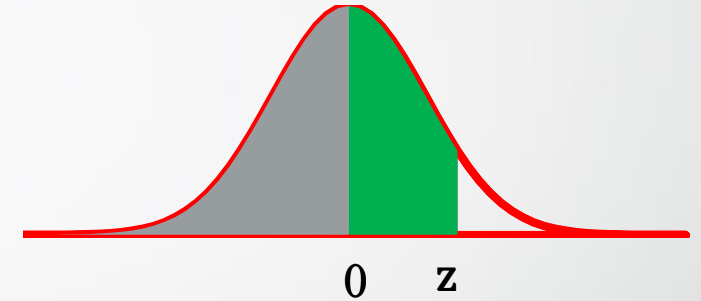
- Question: “The area to the left of an unknown  $z$  is 0.115. Find this  $z$ -value”.
  - Sketch the curve and shade the area of interest. The given area is less than half, so our  $z$ -value must be to the left of zero.
  - The area we want is beyond  $z$ , which can be found in column C.
  - Search through column C until we find a number near 0.115. In this case, exactly this value is found on the row where  $z=1.2$ .
  - We know that  $z$  is left of zero, so it must be negative.
  - Answer:  $z=-1.2$



(A) $z$	(B) Area between mean and $z$	(C) Area beyond $z$
:	:	:
1.2	0.385	0.115
:	:	:

# Given an Area, Find the Z-value

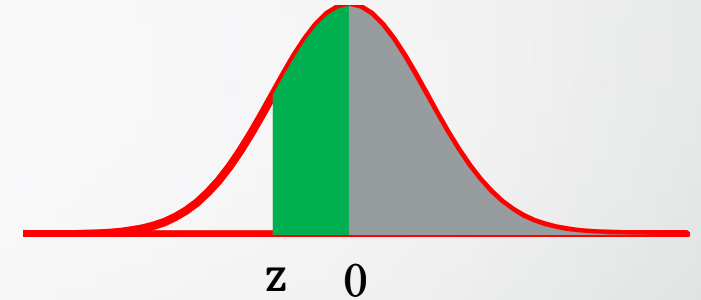
- Question: “The area to the left of an unknown  $z$  is 0.841. Find this  $z$ -value”.
  - Sketch the curve and shade the area of interest. The given area is more than half, so our  $z$ -value must be to the right of zero, since it must include the left half plus some more area.
  - The left half of the curve has an area 0.500. To this we need to add an additional 0.341 to get to the given 0.841. The additional area is between the mean and  $z$ , which can be found in column B.
  - Search through column B until we find a number near 0.341. In this case, we find exactly this number in the row with  $z=1.0$ .
  - Answer:  $z=+1.0$



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
1.0	0.341	0.159
:	:	:

# Given an Area, Find the Z-value

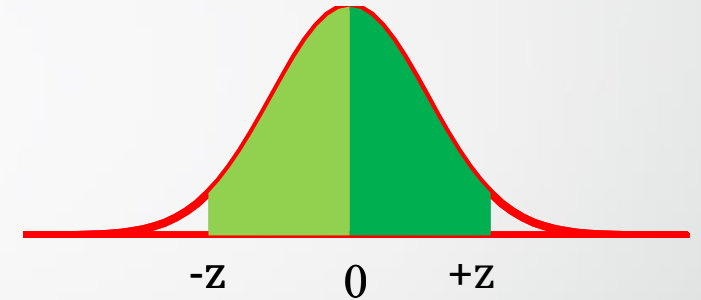
- Question: "The area to the right of an unknown  $z$  is 0.750. Find this  $z$ -value".
  - Sketch the curve and shade the area of interest. The given area is more than half, so our  $z$ -value must be to the left of zero, since it must include the right half plus some more area.
  - The right half of the curve has an area 0.500. To this we need to add an additional 0.250 to get to the given 0.750. The additional area is between the mean and  $z$ , which can be found in column B.
  - Search through column B until we find a number near 0.250. In this case, we must choose between two rows. For row  $z=0.7$ , the column B value (0.258) is closer to 0.250.
  - Since  $z$  must be left of zero, it is negative.
  - Answer:  $z=-0.7$



(A) $z$	(B) Area between mean and $z$	(C) Area beyond $z$
:	:	:
0.7	0.258	0.242
0.8	0.288	0.212
:	:	:

# Given an Area, Find the Z-value

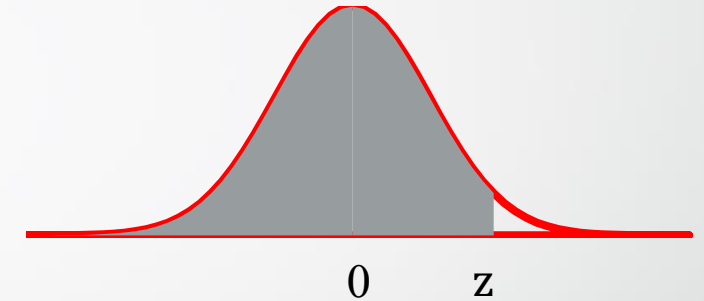
- Question: "The area between +/- z is 0.800. Find this z-value".
  - Sketch the curve and shade the area of interest. The area is between +z and -z, so draw vertical lines equidistant on either side of zero.
  - The area between +z and the mean is the same as the area between -z and the mean due to symmetry. Each must be half of 0.800, or 0.400.
  - Search through column B until we find a number near 0.400. In this case, the row for z=1.3 has a column B value (0.403) quite close to 0.400.
  - Answer: **z=1.3** ( or -z=-1.3, +z=+1.3)



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
1.2	0.385	0.115
1.3	0.403	0.097
:	:	:

# Percentile

- When the percentile is given and required z-value for this percentile is unknown, this is equivalent to the area being given and the z-value is desired.
- Question: "What z-value yields the 90<sup>th</sup> percentile?".
- This is equivalent to the question: "The area to the left of an unknown z is 0.900. Find this z-value".
  - Sketch the curve and shade the area of interest. The given area is more than half, so our z-value must be to the right of zero, since it must include the left half plus some more area.
  - The left half of the curve has an area 0.500. To this we need to add an additional 0.400 to get to the given 0.900 (90<sup>th</sup> percentile). The additional area is between the mean and z, which can be found in column B.
  - Search through column B until we find a number near 0.400. In this case, we find almost exactly this number in the row with z=1.3.
  - Answer: **z=+1.3**



(A) z	(B) Area between mean and z	(C) Area beyond z
:	:	:
1.3	0.403	0.097
:	:	:

# Conclusion

- The method for answering questions where the area (or fraction or percentage of the population) is given and the z-value is desired.
- Answering the question involves looking up values in Column B or Column C, (depending on what the question is asking), and finding the corresponding z-value in Column A.
- In some cases, the area is between two rows, so the nearest row can be chosen.
- If the given area is less than half, then it will appear directly in Column B or Column C.
- If the given is more than half, first subtract 0.500 from it, and look for the remainder in Column B or Column C.
- Be careful about the sign for z in your final answer. Consult the question and the sketch you made to decide what the sign should be.
- Percentile questions where percentile is given can be rephrased into the form 'given an area, find the z-value'. Percentile always refers to the area left of a z-value.