Using the z-Table: Working with Normal Data

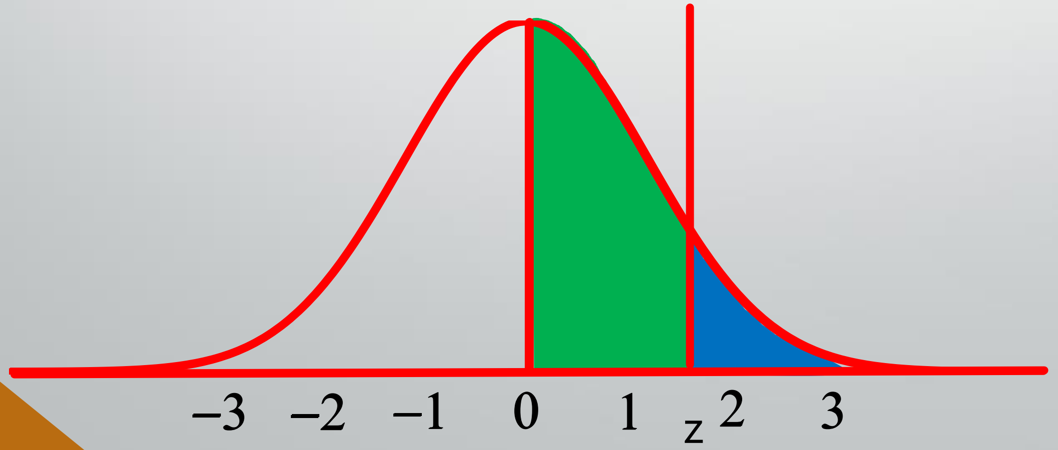
ID1050– Quantitative & Qualitative Reasoning

A Full Z-Table

(A) z	(B) Area between mean and z	(C) Area beyond z	(A) z	(B) Area between mean and z	(C) Area beyond z	(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500	1.3	0.403	0.097	2.6	0.495	0.005
0.1	0.040	0.460	1.4	0.419	0.081	2.7	0.496	0.004
0.2	0.079	0.421	1.5	0.433	0.067	2.8	0.497	0.003
0.3	0.118	0.382	1.6	0.445	0.055	2.9	0.498	0.002
0.4	0.155	0.345	1.7	0.455	0.045	3.0	0.499	0.001
0.5	0.192	0.309	1.8	0.464	0.036			
0.6	0.226	0.274	1.9	0.471	0.029			
0.7	0.258	0.242	2.0	0.477	0.023			
0.8	0.288	0.212	2.1	0.482	0.018			
0.9	0.316	0.184	2.2	0.486	0.014			
1.0	0.341	0.159	2.3	0.489	0.011			
1.1	0.364	0.136	2.4	0.492	0.008			
1.2	0.385	0.115	2.5	0.494	0.006			

Recall What the Z-Table Represents

- We can now answer all of the types of questions we have seen before, like "what fraction of the population is above/below/between some value?" or "what score is needed to be at some percentile?"
- Our z-value can be negative, but because of symmetry, we ignore the sign on the z-value when we look it up in the z-table.
- The z-table has three columns:
  - Column A is the z-value. Due to symmetry, we only need positive values of z.
  - Column B is the area under the standard normal curve between z and zero (toward the hump.)
  - Column C is the area under the standard normal curve beyond z (toward the tail.)
- The area values in columns B and C represent some fraction of the entire population.



(A) z	(B) Area between mean and z	(C) Area beyond z
0.0	0.000	0.500
0.1	0.040	0.460
0.2	0.079	0.421
…	…	…

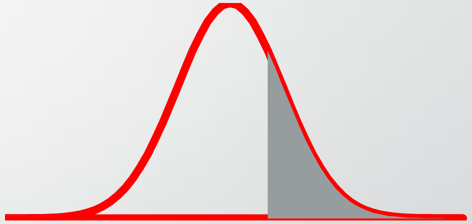
Working with Normal Data

- Our goal is to answer questions involving regular data, with an x-variable that has a normal distribution.
- We have learned how to use a z-table to answer questions about z-values and areas.
- To link these two concepts, we only need to convert our normal data's x-variables into z-values. The percentages in the z-table would be the same as if the table had been custom built for our x-values.
- The general procedure is to convert any given x-value to a z-value using the formula $z = \frac{(x-\mu)}{\sigma}$. Then proceed using the methods for answering questions like "given z, find the area" or "given the area, find z".

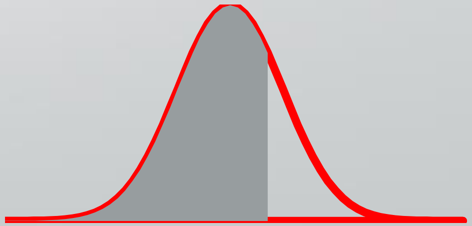
Example: Quiz Scores

A quiz is given, and the scores are found to be normally distributed, with a mean of 10 and a standard deviation of 2. Answer the following questions.

- What is the probability a student scored 11 or higher?
  - Here, x=11. Convert to a z-score: $z = \frac{(11-10)}{2} = 0.5$
  - Sketch what the question is asking:



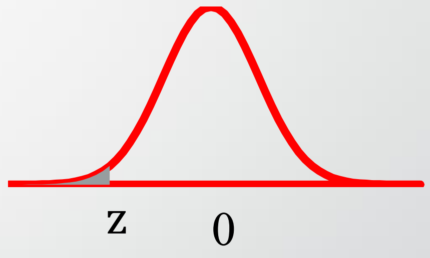
  - This is the area beyond z, found in column C in the row for z=0.5.
  - Answer: 0.309 (or about 31%)
- At what percentile is a student with this score?
  - Percentile is the percent the student beat. Since 31% is above a score of 11, 69% scored below 11. So the student's score is at the 69^{th} percentile.



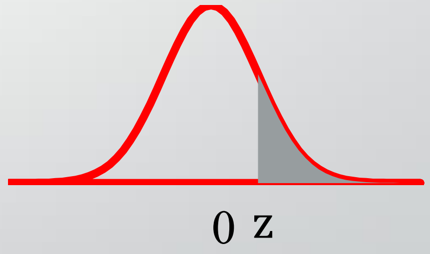
Example: Circuit Board Failures

The failure rate of circuit boards from a manufacturer is found to be normally distributed; an average of 8 boards fail end-of-assembly testing per day, with a standard deviation of 3.

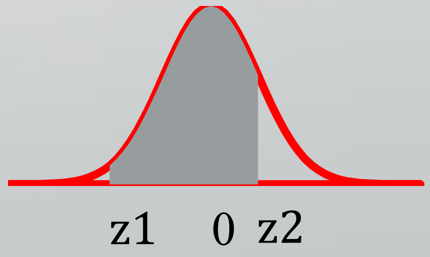
- What is the probability that 3 or fewer boards fail during a day?
  - $z = \frac{(3-8)}{3} = -1.7$. Desired area is left of z (column C). Answer: 0.045



- What is the probability that 10 or more boards fail during a day?
  - $z = \frac{(10-8)}{3} = 0.7$. Desired area is right of z (column C). Answer: 0.242



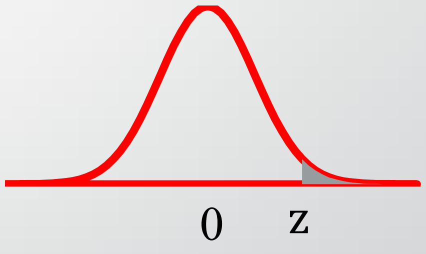
- What is the probability that between 3 and 10 boards fail?
  - Desired area is between $z1 = -1.7$ and $z2 = +0.7$ (opposite signs). Look up column B for +1.7 and +0.7, and add. Answer: 0.455+0.258=0.713 (or 1.000-0.045-0.242).

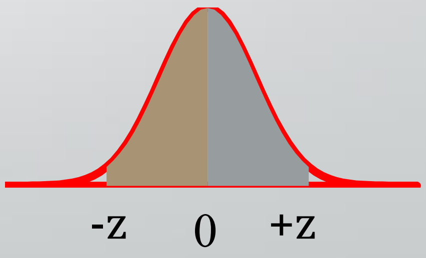


Example: Defective Cells

Regular blood samples have a certain number of malfunctioning cells which follows a normal distribution with μ=8 parts per million (ppm) and σ=3 ppm. Answer the following questions about some particular blood samples.

- What are the chances the blood sample has 13 ppm or more of malfunctioning cells?
  - $z = \frac{(13-8)}{3} = 1.7$. Desired area is right of z (column C). Answer: 0.045
- What range of malfunctioning cells do 95% of people have in their blood? (This one is tricky.)
  - We want the area between +/-z to be 95%, so the area between z and the mean is half of this, or 47.5%.
  - Look up 0.475 in column B. The z-value is about 2.0.
  - Finally, what x-values correspond to z=+/- 2.0? Solve $2.0 = \frac{(x-8)}{3}$ for x (x=14) and solve $-2.0 = \frac{(x-8)}{3}$ for x (x=2).
  - Answer: x is in the range 2 to 14





Conclusion

- Working with data that is normally distributed involves:
  - Converting x-values to z-values.
  - Using the z-table by either:
    - Looking up z (column A) and calculating the desired area from column B or C value.
    - Or calculating the column B or C value from the desired area, and looking up the corresponding z-value.