# Deductive Arguments: Basic Truth Tables

ID1050– Quantitative & Qualitative Reasoning

# Basic Truth Tables

- A compound statement is formed from simple statements combined using logical operators.
- The compound statement is either True or False, depending on the logical combination of its simple statements and whether they are True or False.
- This means we can go through all the possibilities of each simple statement being True or False and check whether the compound statement is True or False.
- Listing the rows of possible values of the simple statements and the resulting value of the compound statement is called 'forming a truth table.'
- We will begin this process with forming the truth tables for the basic logical operators AND, OR, and IF…THEN…
- Using these tables, we can form truth tables for more complicated statements.

# Setting Up the Truth Table

- If there are two variables, each of which can have two possible values (T or F), our table with have four rows.
  - The order of the rows isn't critical since each row is independent of the others, but the rows are usually presented in the standard order shown.
- It helps to have a concrete example to illustrate these abstract concepts. I'll use the idea of having two foods (pizza and quiche) available for lunch to explain the values of the basic truth tables.
- In the following slides, let the variables p and q represent these statements:
  - p="I'll have pizza for lunch."
  - q="I'll have quiche for lunch."

| p | q | [Binary logical statement] |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# Truth Table for AND

- Let's say that one day I make the statement "I'll eat pizza and quiche for lunch".
  - You follow me and see that I do have both pizza and quiche for lunch. In this case, the statement I made was True.
  - In row 1, in which p=T and q=T, the statement p ∩ q is True.
- On another day, I make the same statement: "I'll eat pizza and quiche for lunch".
  - You follow me and see that I eat pizza, but not quiche. In this case, the statement I made was False.
  - In row 2, in which p=T and q=F, the statement p ∩ q is False.
- On yet another day, I make the same statement.
  - You follow me and see that I eat quiche, but not pizza.
  - In row 3, in which p=F and q=T, then the statement p ∩ q is False.
- Finally, after making the same statement, I eat neither pizza nor quiche.
  - Row 4 (both p and q are F) for the statement p ∩ q is False.
- This completes the truth table for AND in all cases of p and q being T or F.

| p | q | p ∩ q |
| --- | --- | --- |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

# Truth Table for OR

- Let's say that one day I make the statement "I'll eat pizza or quiche for lunch".
  - I do have both pizza and quiche for lunch. The statement I made was True.
  - For row 1, p ∪ q is True
- On another day, I make the same statement: "I'll eat pizza or quiche for lunch".
  - I had pizza but not quiche, but the statement I made was still True. For row 2, p ∪ q is True
- On a third day, I make the same statement.
  - This time I have quiche and not pizza. The statement I made was still True.
  - For row 3, then, p ∪ q is True again.
- On a fourth day, I make the same statement.
  - I have neither pizza nor quiche for lunch. The statement I made is finally False.
  - For row 4, p ∪ q is False
- This completes the truth table for inclusive OR in all cases of p and q being T or F.

| p | q | p ∪ q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

# Truth Table for IF..THEN..

- Let's say that one day I make the statement "If I eat pizza for lunch, then I'll have quiche". This is saying that eating pizza will be followed by eating quiche.
  - I have pizza and quiche for lunch. Eating pizza was followed by eating quiche. For row 1, $p \rightarrow q$ is True.
- On another day, I make the same statement.
  - I have pizza but not quiche. Eating pizza was not followed by eating quiche. For row 2, $p \rightarrow q$ is False.
- On a third day, I make the same statement.
  - I don't have pizza, but I do have quiche. Now this one is tricky. Is the statement I made True or False? I didn't have the pizza, so you can't check to see if eating pizza is followed by eating quiche or not.
  - In the case where we can't check, we give the speaker the benefit of the doubt, and assume they told the truth.
  - So, for row 3, $p \rightarrow q$ is True.
- On a fourth day, I again don't have pizza, nor do I have quiche.
  - Again, you can't check to see if eating pizza is followed by eating quiche or not.
  - Giving the benefit of the doubt again, for row 4, $p \rightarrow q$ is True.
- This completes the truth table for IF..THEN.. in all cases of *p* and *q* being T or F.

| p | q | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

# Unary NOT operator

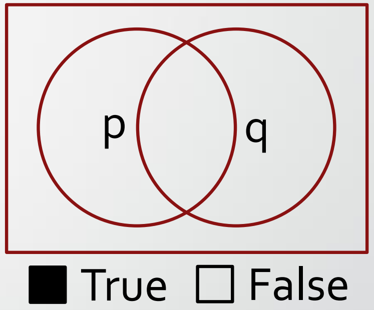
- The NOT operator acts on a single variable.
- NOT is like the negative sign in arithmetic.  It changes False to True and True to False.
- Since there is only a single variable, there are only two rows in our truth table, corresponding to the variable being True or False.
- Assume I make the statement "I won't have pizza for lunch."
  - If I do have pizza for lunch, I made a False statement.  Row 1 for ~p is False.
  - If I don't have pizza, I made a True statement.  Row 2 for ~p is True.

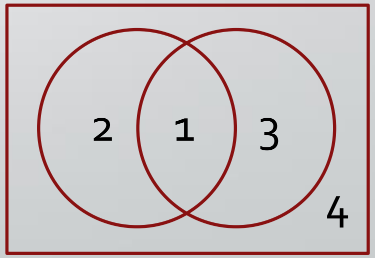
| p | ~p |
|---|---|
| T | F |
| F | T |

# Visualizing Truth Tables

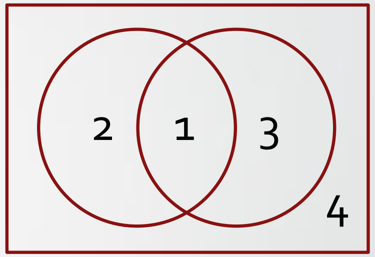
We can use a 2-circle Venn diagram to illustrate the information in our truth tables.

- The circle on the left represents the quality '*p*' and the circle on the right represents the quality '*q*'. The intersection represents having both qualities, and the exterior represents having neither quality.
- We'll use a filled region to represent True, and an unfilled region to represent False.
- There are four distinct regions, each representing one row in our two-variable truth table.
- With four regions, each of which can be filled or unfilled, there are 16 possible combinations, all of which represent a logical statement consisting of AND, OR, IF..THEN.., and NOT and their combinations. We will only look at a few of the interesting ones.



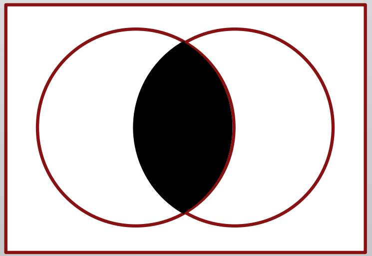



# Shading the Venn Diagram

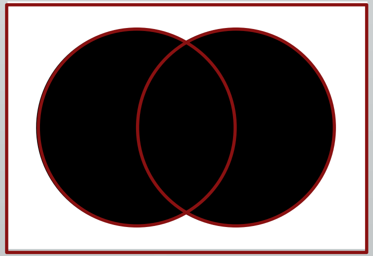



- Here is our truth table for AND, with row numbers added.
  - Only row 1 is True, so only region 1 is shaded in the diagram.
  - Visually, this represents the part of both circles that has both quality 'p' and quality 'q', which is what AND means. It is also sometimes called 'intersection'.
- Here is our truth table for OR.
  - The first three rows are True, so the first three regions are shaded.
  - Visually, this represents the fact of having either quality 'p', quality 'q', or both qualities.
  - This is also sometimes called 'union', which explains the symbol 'U'.

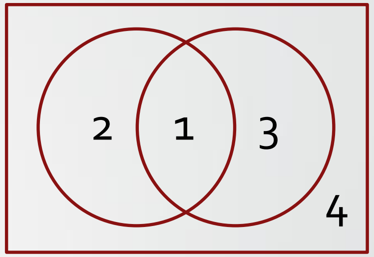
| Row | p | q | p ∩ q |
|---|---|---|---|
| 1 | T | T | T |
| 2 | T | F | F |
| 3 | F | T | F |
| 4 | F | F | F |



| p | q | p ∪ q |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

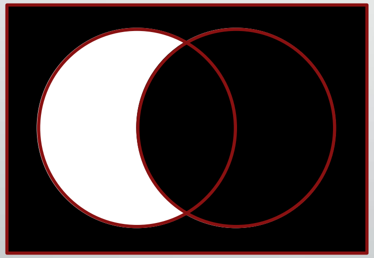


# Truth Tables and Their Diagrams

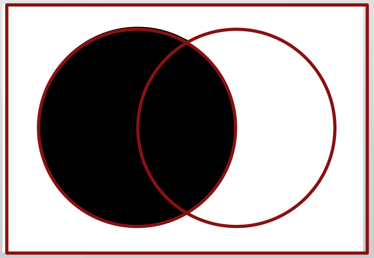



- The IF..THEN.. truth table is here, along with its Venn diagram.
  - It is not obvious what the diagram should look like before building the table and shading in the proper rows.
  - Note the lack of symmetry, which is unlike the diagrams for AND and OR.  Recall that order is important for the IF..THEN.. statement, but is not for AND and OR.
- The table and diagram for the statement ‘p’.
- The table and diagram for the statement ‘~p’.

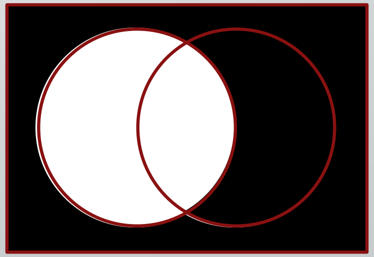
| p | q | p → q |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |



| p | q | p |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | F |
| F | F | F |



| p | q | ~p |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | T |
| F | F | T |

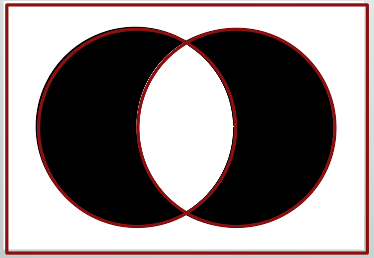


# More Truth Tables, More Diagrams

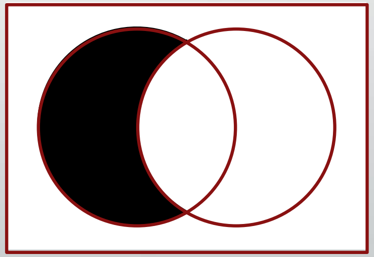
Here are some other interesting diagrams and tables we haven't seen yet.

- The *exclusive or* (XOR) table and diagram. (The symbol is ↮.)
- The '*p but not q*' table and diagram.
- The '*neither p nor q*' table and diagram.

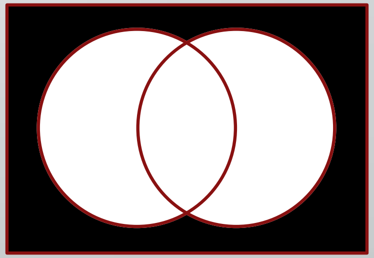
| p | q | p ↮ q |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |



| p | q | p ∩ ~q |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | F |
| F | F | F |



| p | q | ~(p ∪ q) |
|---|---|---|
| T | T | F |
| T | F | F |
| F | T | F |
| F | F | T |



# Conclusion

- Here is a combined truth table for the three binary logical operators AND, OR, and IF..THEN…
  - AND is only True if both of its simple statements are True.
  - OR is only False if both of its simple statements are False.
  - IF..THEN.. is only False for row p=T, q=F.
- Here is the truth table for unary logical operator NOT.
  - The NOT operator switches True to False and False to True.
- Venn diagrams can be used to visualize this information.

| p | q | p ∩ q | p ∪ q | p → q |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | F | T | T |
| F | F | F | F | T |

| p | ~p |
|---|---|
| T | F |
| F | T |