Deductive Arguments: More Truth Tables

ID1050– Quantitative & Qualitative Reasoning

Basic Truth Tables

- Here is a combined truth table for the three binary logical operators AND, OR, and IF..THEN…
  - AND is only True if both of its simple statements are True.
  - OR is only False if both of its simple statements are False.
  - IF..THEN.. is only False for row p=T, q=F.
- Here is the truth table for unary logical operator NOT.
  - The NOT operator switches True to False and False to True.

p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

More Truth Tables

p	~p
T	F
F	T

- Using the basic truth tables, we can now build truth tables for more complicated statements.
- The letters *p* and *q* are placeholders. We will be using different letters for our examples.
- We'll start with statements based on two simple statements that we'll call *a* and *b*.
  - That means we'll have four rows, representing the four permutations of our simple statements being T or F.
- We will include some space for 'helper columns' if we need them.

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Helper columns

a	b			
T	T			
T	F			
F	T			
F	F			

Example 1

- Let's try the compound statement: "I'll eat apples and I won't eat bananas", or $a \cap (\sim b)$
- Our final column will hold the truth table for our target statement.
- To answer the $(\ ) \cap (\ )$ part of the question, we need a truth table column for the statement before the $\cap$ and a column for the statement after the $\cap$.
  - We have a column for the statement before, which is a.
  - We don't have a column with $\sim b$ in it, so let's put this in one of our helper columns.
  - Now we fill in the $\sim b$ column. Each entry is just the opposite of the value in the b column.

p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Helper columns

a	b	~b		a ∩ ~b
T	T	F		
T	F	T		
F	T	F		
F	F	T		

Example 1 (continued)

- [Our compound statement is a ∩ (~b).]
- To answer the ( ) ∩ ( ) part of the question, we take the values of what is before and after the ∩ for each row and check our basic truth tables for the answer in the ∩ column.
  - Row 1: *a* is T and ~*b* is F. Our table says T ∩ F is F.
  - Row 2: *a* is T and ~*b* is T. Our table says T ∩ T is T.
  - Row 3: *a* is F and ~*b* is F. Our table says F ∩ F is F.
  - Row 4: *a* is F and ~*b* is T. Our table says F ∩ T is F.
- Our truth table for a ∩ (~b) is now completed.

p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

a	b	~b		a ∩ ~b
T	T	F		F
T	F	T		T
F	T	F		F
F	F	T		F

Example 2

- The compound statement: "If I don't eat apples then I'll eat bananas", or $(\sim a) \rightarrow b$
- To answer the $(\ ) \rightarrow (\ )$ part of the question, we need a helper column for $\sim a$.
- Since order is important for the IF…THEN.., a helper column with b in it would be useful.
- Now fill in the target column:
  - Row 1: $\sim a$ is F and b is T. Our table says F → T is T.
  - Row 2: $\sim a$ is F and b is F. Our table says F → F is T.
  - Row 3: $\sim a$ is T and b is T. Our table says T → T is T.
  - Row 4: $\sim a$ is T and b is F. Our table says T → F is F.

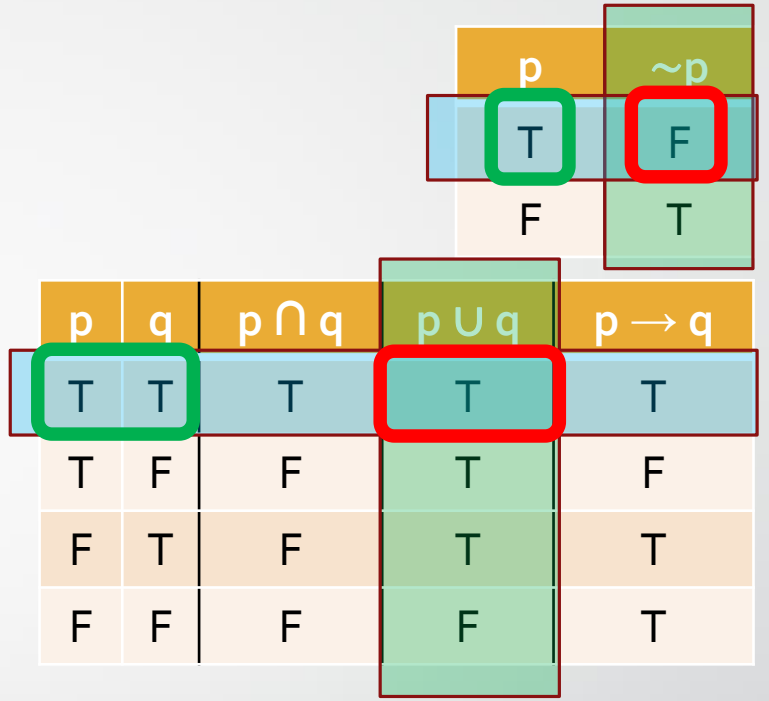
p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

a	b	~a	b	~a → b
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

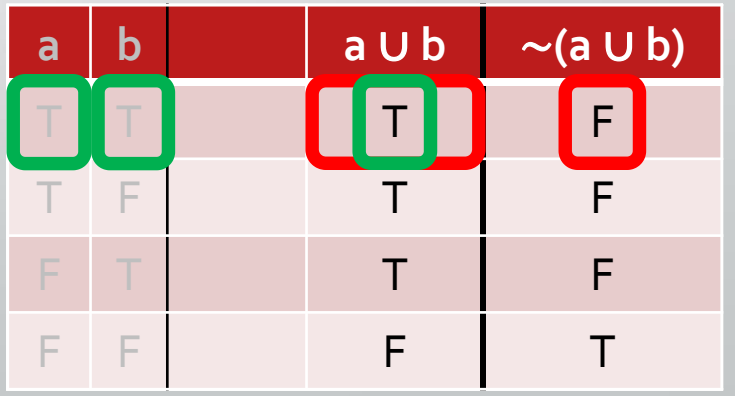
Example 3

- The compound statement: "It is not the case that I will eat apples or bananas", or ~ (a ∪ b)
- To answer the ~ ( ) part of the question, we need a helper column for a ∪ b.
- Now fill in the helper column a ∪ b :
  - Row 1: *a* is T and *b* is T. Our table says T ∪ T is T.
  - Row 2: *a* is T and *b* is F. Our table says T ∪ F is T.
  - Row 3: *a* is F and *b* is T. Our table says F ∪ T is T.
  - Row 4: *a* is F and *b* is F. Our table says F ∪ F is F.
- Finally, fill in the target column ~ (a ∪ b) :
  - Row 1: a ∪ b is T. Our table says ~T is F.
  - Row 2: a ∪ b is T. Our table says ~T is F.
  - Row 3: a ∪ b is T. Our table says ~T is F.
  - Row 4: a ∪ b is F. Our table says ~F is T.



p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T



a	b		a ∪ b	~(a ∪ b)
T	T		T	F
T	F		T	F
F	T		T	F
F	F		F	T

Example 4 – Try it on your own

p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Predict the answers below:

- The compound statement is:
  - "If I don't eat apples, then I won't eat bananas"
    - What is this symbolically?
    - Answer: (~a) → (~b)
- What helper columns will you need?
  - Answer: ~a and ~b
- Fill in the helper column for ~a.
- Fill in the helper column for ~b.
- Fill in the target column (~a) → (~b).

a	b	~a	~b	(~a) → (~b)
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Conclusion

- We can form the truth table for a complicated logical statement by using the basic truth tables.
- Helper columns can be useful for intermediate results.
- The resulting truth table shows the truth or falseness of the logical statement for each combination of its constituent simple statements being True or False.