Deductive Arguments: Checking for Validity

ID1050– Quantitative & Qualitative Reasoning

Basic Truth Tables

- Here is a combined truth table for the three binary logical operators AND, OR, and IF..THEN…
  - AND is only True if both of its simple statements are True.
  - OR is only False if both of its simple statements are False.
  - IF..THEN.. is only False for row p=T, q=F.
- Here is the truth table for unary logical operator NOT.
  - The NOT operator switches True to False and False to True.

p	~p
T	F
F	T

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Steps for Checking Validity

An argument is VALID if, in all cases where the premises are True, the conclusion is also True.

- If there is *even one case* where all the premises are True but the conclusion is False, the entire argument is INVALID.
- So testing for validity is really searching for a row that invalidates the argument. If you can't find such a row, then the argument must be VALID.
- Rows that don't have all true premises can be ignored. They do not contribute to the validity of the argument.

- When presented with a deductive argument, you must first build a truth table for all of the statements and for all permutations of the basic statements being True or False.
  - There will be a row for each True/False permutation of the basic statement logical variables.
  - For two variables, like *p* and *q*, there are $2^2=4$ rows. In general there are 2^n rows, where n is the number of variables. Three variable arguments have 8 rows, etc.
  - There will be a column for each premise and for the conclusion. The column will have a value for each row depending on whether it is True or False under those conditions of the variables.
- Once the table is built, look for any row that has all premises = True but the conclusion = False. (i.e. 'TT....TTF')

Examples to follow

- Following are seven examples of executing this process and determining validity of a deductive argument.
- There are four 2-variable, 3-statement arguments.
  - Three of these are classic argument types, and have fairly simple statements.
  - The fourth is a slightly more complex argument.
- There is one example of a 2-variable, 4-statement argument.
- There are two examples of 3-variable, 3-statement arguments.
  - Some of these are arguments we discussed before and guessed were either valid or invalid, but the proof is now furnished.

Example 1: two variables, 3 statements

- Here is an argument we have seen before:
  - "If I eat apples, then I'll eat bananas." $a \rightarrow b$
  - "I ate apples." a
  - "Therefore, I ate bananas." b
- Fill in the truth table.
- Check for 'TTF'.
  - There is no 'TTF', so this argument is VALID.

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

a	b		P1: a → b	P2: a	C b
T	T		T	T	T
T	F		F	T	F
F	T		T	F	T
F	F		T	F	F

Example 2: two variables, 3 statements

- This argument seems similar:
  - "If I eat apples, then I'll eat bananas." $a \rightarrow b$
  - "I ate bananas." b
  - "Therefore, I ate apples." a
- Fill in the truth table.
- Check for 'TTF'.
  - Row 3 has a 'TTF', so this argument is INVALID.
  - (That is a case with all True premises, but a conclusion that is False, which invalidates the argument.)

p	q	$p \cap q$	$p \cup q$	$p \rightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

a	b	P1: $a \rightarrow b$	P2: b	C a
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Example 3: two variables, 3 statements

- One more variation:
  - "If I eat apples, then I'll eat bananas." $a \rightarrow b$
  - "I didn't eat bananas." $\sim b$
  - "Therefore, I didn't eat apples." $\sim a$
- Fill in the truth table.
- Check for 'TTF'.
  - There is no 'TTF', so this argument is VALID.

p	q	$p \cap q$	$p \cup q$	$p \rightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

a	b		P1: $a \rightarrow b$	P2: $\sim b$	C $\sim a$
T	T		T	F	F
T	F		F	T	F
F	T		T	F	T
F	F		T	T	T

Example 4: A more complex argument

- Here is an argument:
  - P1: c ∪ ~d
  - P2: c ⟶ d
  - C: c ∩ d
- Fill in the truth table.
  - A helper column for ~d is needed here.
- Check for 'TTF'.
  - Row 4 has a 'TTF', so this argument is INVALID.
  - (The 'TTT' in row 1 makes no difference.)

p	q	p ∩ q	p ∪ q	p ⟶ q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

c	d	~d	P1: c ∪ ~d	P2: c ⟶ d	C c ∩ d
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	T	T	F

Example 5: two variables, 4 statements

- A simple four-statement argument.
  - P1: ~g
  - P2: g ∪ h
  - P3: h ∩ ~g
  - C: ~h
- Fill in the truth table.
- Check for 'TTTF'.
  - Row 3 has a 'TTTF', so this argument is INVALID.

p	q	p ∩ q	p ∪ q	p → q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

g	h		P1: ~g	P2: g ∪ h	P3: h ∩ ~g	C ~h
T	T		F	T	F	F
T	F		F	T	F	T
F	T		T	T	T	F
F	F		T	F	F	T

Example 6: Dogs, Animals, and Things That Eat

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

- Here is an argument we have seen before:
  - "All dogs are animals." $d \rightarrow a$
  - "All animals must eat." $a \rightarrow e$
  - "Thus, all dogs must eat." $d \rightarrow e$
- There are 3 variables, so we will have 8 rows.
- Fill in the truth table.
- Check for 'TTF'.
  - There is no case where all premises are true and yet the conclusion is false. So this argument is VALID.

d	a	e	P1: d → a	P2: a → e	C d → e
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Example 7: Dogs, Animals, and 4-Legged Things

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

- Here is another argument we have seen before:
  - "All dogs are animals." $d \rightarrow a$
  - "All dogs have 4 legs." $d \rightarrow f$
  - "Thus, all animals have 4 legs." $a \rightarrow f$
- Fill in the truth table.
- Check for 'TTF'.
  - There is at least one case of 'TTF', so this argument is INVALID.

d	a	f	P1: $d \rightarrow a$	P2: $d \rightarrow f$	C $a \rightarrow f$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	T	T
F	F	F	T	T	T

Conclusion

- An argument is valid if there are no cases with all True premises but a False conclusion.
- To check for validity, we build the truth table for the argument.
  - There will be enough rows to accommodate all permutations of the number of variables. (2^n, where n=# of variables)
  - There will be one column for each statement (premise or conclusion). You should leave room for helper columns.
  - Complete the truth table using the basic truth tables for logical operators.
- Check for any instance of 'TT….TTF'.
  - If one or more exist, the argument is INVALID.
  - Otherwise, the argument is VALID.