[CHECKED EXERCISE 1]

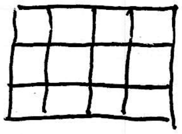
1-21-2016

MULTIPLICATION IS A SERIES OF ADDITIONS

$$3 + 3 + 3 + 3 = 4 \times 3 = 12$$

$$4 + 4 + 4 = 3 \times 4 = 12$$

$$(+3) \times (+4) = (+12)$$



FOR NEGATIVE NUMBERS,

$$(-3) + (-3) + (-3) + (-3) = 4 \times (-3) = -12$$

$$(-4) + (-4) + (-4) = 3 \times (-4) = -12$$

TAKING AWAY HOLES SAME AS ADDING SCOOPS

$$3 \times 4 = 12 \qquad (-4) \times (3) = -12$$

$$(-3) \times (-4) = 12 \qquad 4 \times (-3) = -12$$

SAME SIGN $\Rightarrow$ POSITIVE

OPPOSITE SIGNS $\Rightarrow$ NEGATIVE

MORE THAN TWO NUMBERS

EXAMPLE:

$$(-2) \times 3 \times 4 \times (-5) \times (-6) =$$

$$(-6) \times 4 \times (-5) \times (-6) =$$

$$(-24) \times (-5) \times (-6) =$$

$$(+120) \times (-6) =$$

$$(-720) = -720$$

ALTERNATIVELY, $\left.\begin{array}{l} 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720 \\ \text{ODD NUMBER OF } (-) \end{array}\right\} -720$

ODD # OF $(-)$ $\Rightarrow$ ANSWER IS NEGATIVE
EVEN # OF $(-)$ $\Rightarrow$ ANSWER IS POSITIVE

DIVISION IS A SERIES OF SUBTRACTIONS

$12 \div 4 = 3$

$$\left.\begin{array}{l} 12 - 4 = 8 \\ 8 - 4 = 4 \\ 4 - 4 = 0 \end{array}\right\}$$ 3 TIMES (3 OPERATIONS OF SUBT.)

$(-12) \div (-4) = +3$

$$\left.\begin{array}{l} (-12) - (-4) = -8 \\ (-8) - (-4) = -4 \\ (-4) - (-4) = 0 \end{array}\right\}$$ 3 TIMES

SAME SIGNS $\Rightarrow$ POSITIVE

$(-12) \div (+4) = -3$

$(+12) \div (-4) = -3$

OPPOSITE SIGNS $\Rightarrow$ NEGATIVE

TRUE FOR MULTIPLY AND DIVIDE

OTHER FORMATS FOR MULTIPLICATION:

3×4 $\quad$ $3 \cdot 4$ $\quad$ $(3)(4)$ $\leftarrow$ $\cancel{34}$

IF a REPRESENTS A NUMBER,

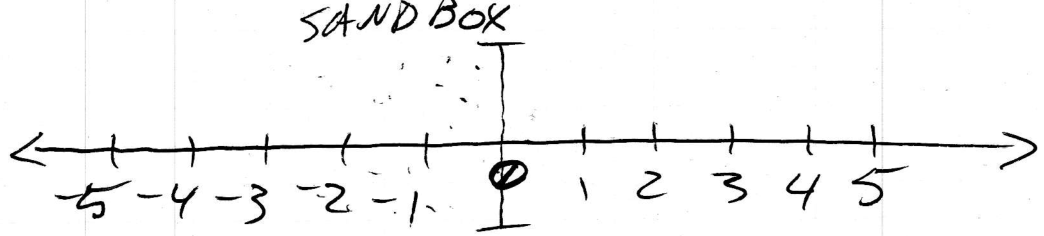
$3 \cdot a$ OR $a \cdot 3$ OR $3a$

OTHER FORMS OF DIVISION

$12 \div 4$ $\quad$ $4\overline{)12}$ $\quad$ $\frac{12}{4}$ $\quad$ $12/4$

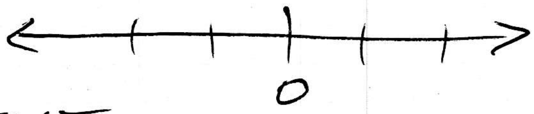
$\div$

REPRESENTING NUMBERS ON A NUMBER LINE
(GRAPHICAL FORM)



USING THE NUMBER LINE TO DO OPERATIONS

(1) • ADDING A POSITIVE NUMBER ⇒ MOVING RIGHT BY THAT NUMBER

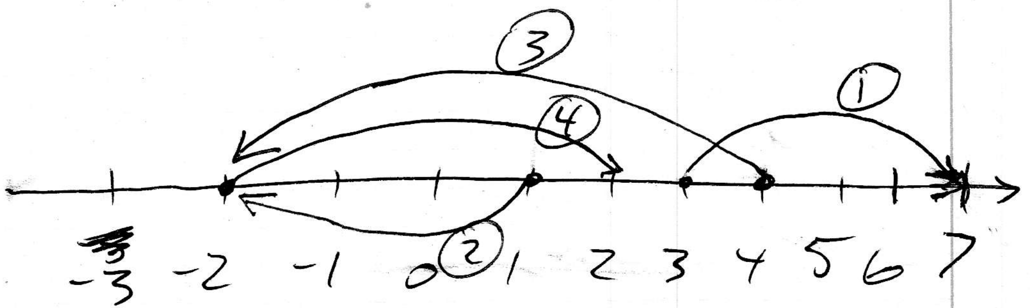


(2) • ADDING A NEGATIVE # ⇒ MOVING LEFT BY THAT #

(3) • SUBTRACTING A POSITIVE # ⇒ MOVING LEFT

(4) • SUBTRACTING A NEGATIVE # ⇒ MOVING RIGHT

EXAMPLES



(1) • $3+4=7$

(3) • $4-6=-2$

(2) • $1+(-3)=-2$

(4) • $-2-(-4)=2$

ORDERING TWO NUMBERS

'LARGER' = FARTHER RIGHT ON THE NUMBER LINE

'SMALLER' = " LEFT " " "

$5 < 10$ $\qquad$ $-2 < +2$

$10 > 5$ $\qquad$ $-5 > -10$

RATIONAL NUMBERS

A RATIONAL NUMBER CAN BE EXPRESSED AS THE RATIO OF INTEGERS

$\ldots -2, -1, 0, 1, 2 \ldots$

SOME EXAMPLES: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$

$\frac{2}{3}$, $\frac{4}{5}$, $\frac{17}{64}$, ...

$-\frac{1}{2}$, $-\frac{1}{3}$, etc.

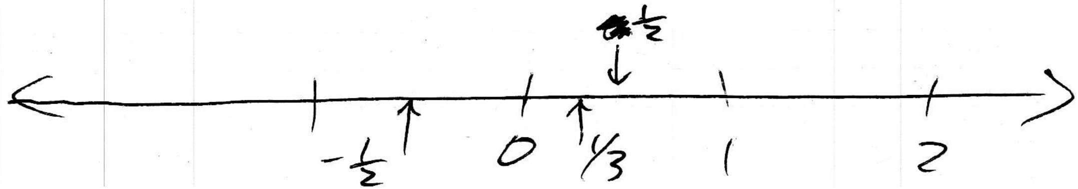
1? $\quad 1 = \frac{1}{1}$ RATIONAL

2? $\quad 2 = \frac{2}{1}$ RATIONAL

-3? $\quad -3 = \frac{-3}{1}$ RATIONAL

ALL INTEGERS ARE RATIONAL

0? $\quad 0 = \frac{0}{1}$ RATIONAL



WHAT ABOUT NUMBERS IN DECIMAL FORM?

$$\frac{1}{2} = 0.5 \quad \text{RATIONAL}$$

$$\frac{1}{3} = 0.3333333\ldots \quad \text{RATIONAL}$$
$$= 0.3333\overline{3}$$
$$= 0.\overline{3}$$

IF # TRUNCATES (0.5)

OR HAS A REPEATING SET OF DIGITS $(0.\overline{3})$

THEN THE # IS RATIONAL

$0.1212\overline{12}$? RATIONAL

SIMPLE PROOF

$$N = 0.1212\overline{12}$$

2 DIGITS $\Rightarrow$ 100

$$\begin{aligned} 100 \cdot N &= 12.1212\overline{12} \\ - \quad N &= 0.1212\overline{12} \\ \hline 99N &= 12 \end{aligned}$$

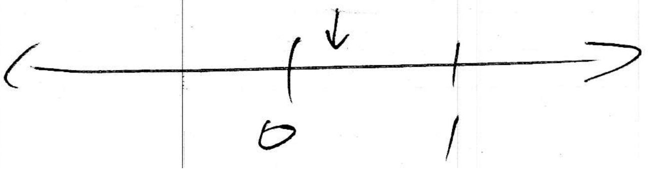
$N = \frac{12}{99}$ RATIONAL

IRRATIONAL NUMBERS ARE NOT RATIONAL

THERE IS NO SET OF REPEATING DIGITS IN DECIMAL FORM

EXAMPLE: $0.112123123412345123456\ldots$

IRRATIONAL



RADICAL $\rightarrow \sqrt{2} = 1.414213562\ldots$ IRRATIONAL

$\sqrt{3} = 1.732050808\ldots$ IRRATIONAL

$\sqrt{4} = 2$ RATIONAL

$\sqrt[3]{2} = 1.259923105$ IRRATIONAL

SOME SPECIAL IRRATIONAL #S

PI $\pi = 3.1415926536\ldots$ IRRATIONAL

EULER'S NUMBER $e = 2.7182818284 5\ldots$ IRRATIONAL

(ALSO THESE #'S ARE TERMED TRANCENDENTAL)