POWER AND ROOTS

INTEGER POWERS

B = BASE
P = POWER

$$B^{P} = \underbrace{B \cdot B \cdot B \cdot \ldots \cdot B}_{\text{'p' INSTANCES}}$$

$$B^{-P} = \frac{1}{B^{P}} = \frac{1}{B \cdot B \cdot B \cdot \ldots \cdot B}$$

$$5^{10} = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$$

$$3^{-1} = \frac{1}{3^{1}} = \frac{1}{3}$$

INTEGER ROOTS

$$\sqrt[r]{B}$$

B = BASE
r = ROOT

$$\underbrace{\left(\sqrt[r]{B}\right)\left(\sqrt[r]{B}\right) \cdot \ldots \cdot \left(\sqrt[r]{B}\right)}_{\text{'r' INSTANCES}} = B$$

$$\sqrt{2} = \sqrt[2]{2} = 1.4142\ldots$$

$$(1.4142)(1.4142) = 2$$

$$\left(\sqrt[3]{4}\right)\left(\sqrt[3]{4}\right)\left(\sqrt[3]{4}\right) = 4$$

ANOTHER FORM FOR ROOTS: FRACTIONAL EXPONENT

$$\sqrt{2} = \sqrt[2]{2} = 2^{1/2} = 1.4142\ldots$$

POWERS AND ROOTS ARE INVERSE FUNCTIONS

$\sqrt[3]{4}$ ON CALCULATOR:

$$\left.\begin{array}{ll} & \boxed{4}\ \boxed{2^{nd}}\ \overset{\sqrt[x]{y}}{\boxed{y^x}}\ \boxed{3}\ \boxed{=} \\ \text{OR} & \boxed{4}\ \boxed{y^x}\ \boxed{1}\ \boxed{a\,b/c}\ \boxed{3}\ \boxed{=} \end{array}\right\} 1,5874\ldots$$

$$\left(\sqrt[3]{4}\right)^3 = 4 \qquad \left(4^{1/3}\right)^3 = 4^{1/3}\cdot 4^{1/3}\cdot 4^{1/3}$$

$$= 4^1 = 4$$

$$\sqrt[3]{(4^3)} = 4 \qquad (4)^{\frac{1}{3}\cdot 3} = 4^1 = 4$$

$$\left(4^3\right)^{1/3} = 4^{3\cdot\frac{1}{3}} = 4^1 = 4$$

FRACTIONAL EXPONENT

$$B^{p/r} = \sqrt[r]{B^p} = \left(\sqrt[r]{B}\right)^p$$

$$\left(B^{p/r}\right)^{r/p} = B^{\frac{\cancel{p}}{\cancel{r}}\cdot\frac{\cancel{r}}{\cancel{p}}} = B^1 = B$$

EXAMPLES:

$$x^{1/3} = 2 \qquad \left(x^{1/3}\right)^3 = (2)^3 \quad \leftarrow \text{ SHOW}$$

OR

$$\text{OR} \quad \sqrt[3]{x} = 2 \qquad \left(\sqrt[3]{x}\right)^3 = (2)^3 \quad \leftarrow \text{ SHOW}$$

$$\boxed{x = 8}$$

$$x^{3/2} = 8$$

$$\left(x^{3/2}\right)^{2/3} = (8)^{2/3} \leftarrow \quad \boxed{8}\ \boxed{y^x}\ \boxed{2}\ \boxed{a\,b/c}\ \boxed{3}\ \boxed{=}$$

$$\boxed{x = 4}$$

LOGARITHMS

RECALL:

$$\begin{array}{r} 1 \times 10^3 \\ * \ 1 \times 10^4 \\ \hline 1 \times 10^7 \end{array}$$

$3+4=7$

"LOGARITHM OF x IS y" $\log x = y$

COMMON LOG OR BASE 10 LOG $\log_{10} x = y$

OR "WHAT $\underbrace{\text{EXPONENT}}_{y}$ CAN I RAISE A $\underbrace{\text{BASE}}_{10}$ TO TO GET x?"

$10^y = x$

SAME STATEMENT

FOR OUR EXAMPLE:

$$\begin{array}{r} 1 \times 10^3 \\ * \ 1 \times 10^4 \\ \hline 1 \times 10^7 \end{array} \quad \xrightarrow{\text{"LOG LAND"}} \quad \begin{array}{rr} \log(1 \times 10^3) & = 3 \\ + \log(1 \times 10^4) & = + 4 \\ \hline & 7 \end{array}$$

$$\longleftarrow$$

DONE BY
TABLE LOOK-UP

A DIFFERENT EXAMPLE

$$\begin{array}{r} {}^{6}_{1}{}^{4}_{1} \\ 476 \\ *\quad 82 \\ \hline 9^{1}52 \\ 3^{1}8080 \\ \hline 39032 \end{array} \xrightarrow[\text{LOGS}]{\text{BASE 10}} \begin{array}{r} 2.677606953 \\ +\ 1.913813852 \\ \hline 4.591420805 \end{array}$$

$$\begin{array}{r} 952 \\ 38080 \end{array} \leftarrow \quad 4.591420805$$

$$39032 \longleftarrow 10^{4.591420805}$$

NATURAL BASE

BASE IS $e = 2.718281828459O...$

'NATURAL BASE'

$\ln(x) = y$ OR $LN(x) = y$

$$e^{y} = x$$

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SOLVING FOR THE UNKNOWN:

$$10^{x} = 100$$

$$\log(10^{x}) = \log(100) \leftarrow$$

$$\boxed{X = 2}$$

$$10^{x} = 54$$

$$\log(10^{x}) = \log(54) \leftarrow$$

$$\boxed{X = 1.732...}$$

- $\log x = 7$

$10^{\log x} = 10^{7} \leftarrow$

$\boxed{x = 10{,}000{,}000}$

- $\underset{\nwarrow \text{1st}}{10}^{x^{3 \leftarrow \text{2nd}}} = 2000$ START FROM THE GROUND, UP

$\log(10^{x^3}) = \log(2000) \leftarrow$

$x^3 = 3.301\ldots$

$\sqrt[3]{x^3} = \sqrt[3]{3.301\ldots} \leftarrow$

$\boxed{x = 1.49\ldots}$

- $e^x = 7$

$\ln(e^x) = \ln(7) \leftarrow$

$\boxed{x = 1.95\ldots}$

- $\ln y = 2$

$e^{\ln y} = e^2 \leftarrow$

$\boxed{y = 7.39\ldots}$