4-26

SYMBOLIC LOGIC

STATEMENTS → SYMBOLS

TRUTH TABLES

LOGICAL OPERATORS

SIMPLE STATEMENT HAS WORDS REPLACED WITH A LOGICAL VARIABLE THAT HAS THE POSSIBLE VALUES 'TRUE' (T) OR 'FALSE' (F)

"IT WILL RAIN TODAY." → r

"IT WILL SNOW TODAY." → s

COMPOUND STATEMENT: A STATEMENT MADE UP OF SIMPLE STATEMENTS CONNECTED BY LOGICAL OPERATORS. IT HAS A SINGLE 'TRUE' OR 'FALSE' VALUE.

LOGICAL OPERATORS:

			MNEMONIC ↓
BINARY OPERATORS	AND	___ AND ___ ∩	(A)
	OR	___ OR ___ ∪	(UNION)
	IF... THEN...	IF ___, THEN ___ →	
		___ IMPLIES ___	
UNARY OPERATOR	NOT	NOT ___ ~	

OTHERS (WE WON'T USE) ⊃, ↔

EXAMPLES OF COMPOUND STATEMENTS

"IT WILL RAIN AND SNOW TODAY." $r \cap s$

"IT WILL RAIN OR SNOW TODAY." $r \cup s$

"IF IT RAINS, THEN IT WILL SNOW." } $r \to s$

"RAIN IMPLIES SNOW." }

"IT WON'T RAIN TODAY." $\sim r$

INCLUSIVE 'OR': ONE OR THE OTHER OR BOTH

EXCLUSIVE 'OR': ONE OR THE OTHER, BUT NOT BOTH

ON AIRPLANE: "WOULD YOU LIKE CHICKEN OR FISH?"
(EXCLUSIVE OR)

FOR THIS CLASS 'OR' IS ALWAYS INCLUSIVE.

"ONLY IF"

"... ONLY IF..." USED AS A SUBSTITUTE FOR "IF..., THEN..."

r = "THE ENGINE IS RUNNING."

f = "THE ENGINE HAS FUEL."

"THE ENGINE IS RUNNING ONLY IF IT HAS FUEL."

"IF THE ENGINE IS RUNNING, THEN IT HAS FUEL."

$$r \to f$$

"IT IS NOT THE CASE THAT..."

"IT IS NOT THE CASE THAT IT WILL RAIN AND SNOW TODAY."

$$\sim(r \cap s)$$

"IT CAN'T BOTH RAIN AND SNOW."

$$\sim(r \cap s)$$

RULE FOR PARENTHESES: EVALUATE STATEMENTS IN PARENTHESES FIRST.

BUT

"IT RAINED BUT IT DIDN'T SNOW TODAY."

IN LOGIC, "BUT" = "AND"

$$r \cap (\sim s) \quad \text{OR} \quad r \cap \sim s$$

"IT DIN'T RAIN BUT IT DID SNOW."

$$(\sim r) \cap s$$

TRUTH TABLES

CAPTURES ALL POSSIBLE PERMUTATIONS OF THE SIMPLE STATEMENTS OF A COMPOUND STATEMENT BEING TRUE OR FALSE. THE LAST COLUMN IN THE TABLE IS THE TRUE OR FALSE VALUE OF COMPOUND STATEMENT IN EACH CASE.

ORDER OF THE ROWS DOESN'T MATTER.
ORDER OF COLUMNS DOES.

BASIC TRUTH TABLES ARE FORMED FOR: $\cap, \cup, \rightarrow, \sim$

USE THESE TO BUILD MORE COMPLICATED TRUTH TABLES.

BASIC TRUTH TABLES

p (p = "I WILL EAT PIZZA FOR LUNCH.")

q (q = "I WILL EAT QUICHÉ FOR LUNCH.")

AND "I WILL EAT PIZZA AND QUICHÉ FOR LUNCH."

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(MEMORIZE THIS TABLE)

OR "I WILL EAT PIZZA OR QUICHÉ FOR LUNCH."

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(MEMORIZE THIS)

IF...THEN... "IF I EAT PIZZA, THEN I WILL EAT QUICHÉ."

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(MEMORIZE THIS)

} CAN'T CHECK IF STATEMENT IS TRUE OR NOT, SO ASSUME TRUE

COMBINED BASIC TRUTH TABLE — MEMORIZE

p	q	$p \cap q$	$p \cup q$	$p \to q$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

NOT "I WILL NOT EAT PIZZA FOR LUNCH."

MEMORIZE

p	$\sim p$
T	F
F	T

TRUTH TABLES FOR MORE COMPLICATED STATEMENTS

a (a = I WILL EAT APPLES.)

b (b = I WILL EAT BANANAS.)

EXAMPLE: $a \cap (\sim b)$

SIMPLE STATEMENTS		HELPER COLUMNS	ANSWER
a	b	$(\sim b)$	$a \cap (\sim b)$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F