PLANETARY MOTION

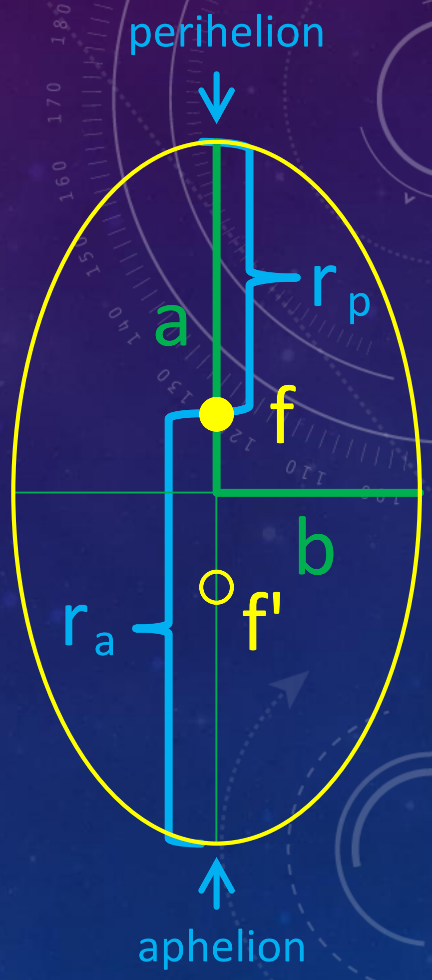
PES 1000 – PHYSICS IN EVERYDAY LIFE

HISTORY

- Ptolemy (100-170 AD)
  - Believed the Earth was the center of the universe (geocentric)
  - To him, the sun, moon, planets, and other celestial bodies orbit Earth in perfect circles.
  - He proposed epicycles, or circles upon the circular orbits, to explain an odd behavior of the planets
- Nicolaus Copernicus (1473-1543 AD)
  - Proposed that the Sun was the center of the universe (heliocentric)
  - The planets, including Earth, orbited the Sun in perfect circles (didn't resolve Ptolemy's dilemma).
- Johannes Kepler (1571-1630 AD)
  - Modified Copernicus' heliocentric model by proposing planetary orbits were elliptical. This resolved Ptolemy's dilemma.
- Isaac Newton (1642-1726)
  - Proved Kepler's Laws starting from his own 3 Laws of Motion and Universal Law of Gravity.

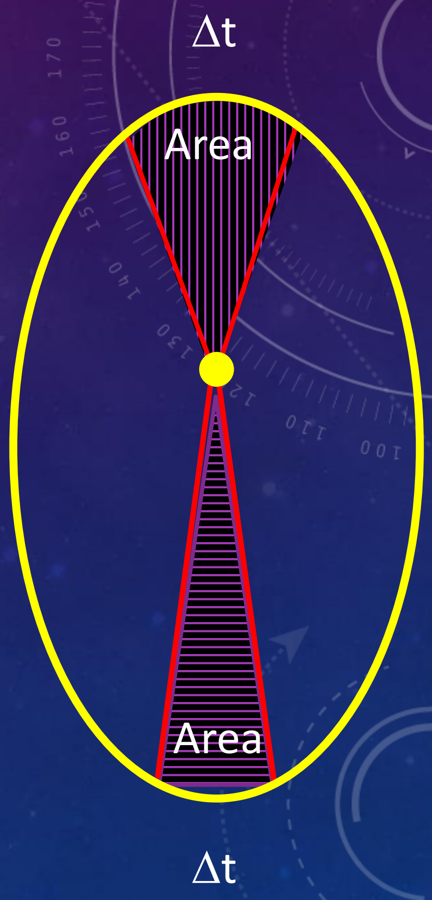
KEPLER'S LAWS: I. LAW OF ELLIPSES

- Statement - "The orbit of the planets are ellipses, with the sun at one focus."
- Ellipse
  - The sun is at one focus, labeled f. The other focus, f', is empty.
  - The longest dimension is called the major axis. Half this length is the semi-major axis which we designate with 'a'.
  - The shortest dimension is called the minor axis. Half this length is the semi-minor axis which we designate with 'b'.
  - The nearest point of the orbit to the sun is the perihelion. The distance from the sun to this point is labeled r_p.
  - The farthest point of the orbit from the sun is the aphelion. The distance from the sun to this point is labeled r_a.



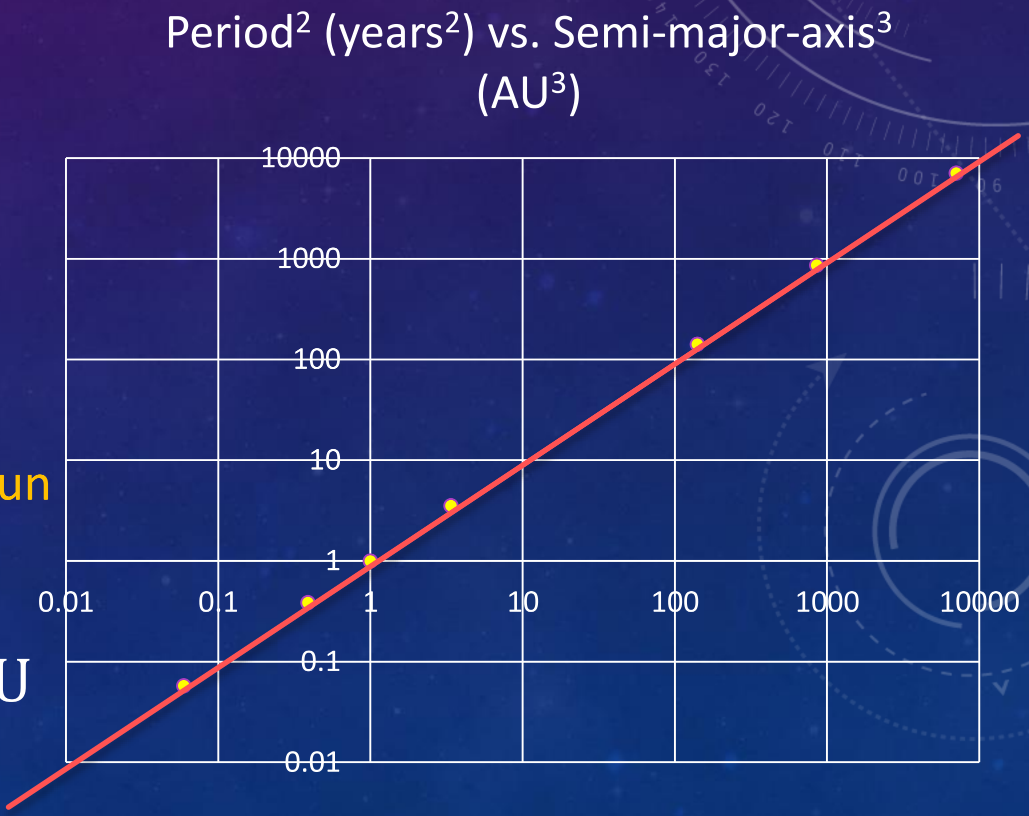
KEPLER'S LAWS: II. LAW OF AREAS

- Statement – "A line from planet to sun sweeps out *equal area in equal time*."
- Re-statement – "When a planet is nearer to the sun along its orbit, it moves faster than when it is farther away."
- This is similar to the behavior of objects in a whirlpool. Objects near the vortex move faster.
- Video of a simulated body orbit using weights on a stretched sheet



KEPLER'S LAWS: III. LAW OF PERIODS

- Statement – "A planet's period squared is proportional to the cube of its semi-major axis."
- In equation form: $T^2=a^3$
- Units
  - Period in Earth Years
  - Semi-major-axis in Astronomical Units
    - 1 AU is the average distance from Earth to the Sun
    - 1 AU is 150 million km, or 93 million miles
- Venus: T=0.62 years, so $a = \sqrt[3]{0.62^2} = 0.72$ AU
- Pluto: a=39.5 AU, so $T = \sqrt{39.5^3} = 248$ years

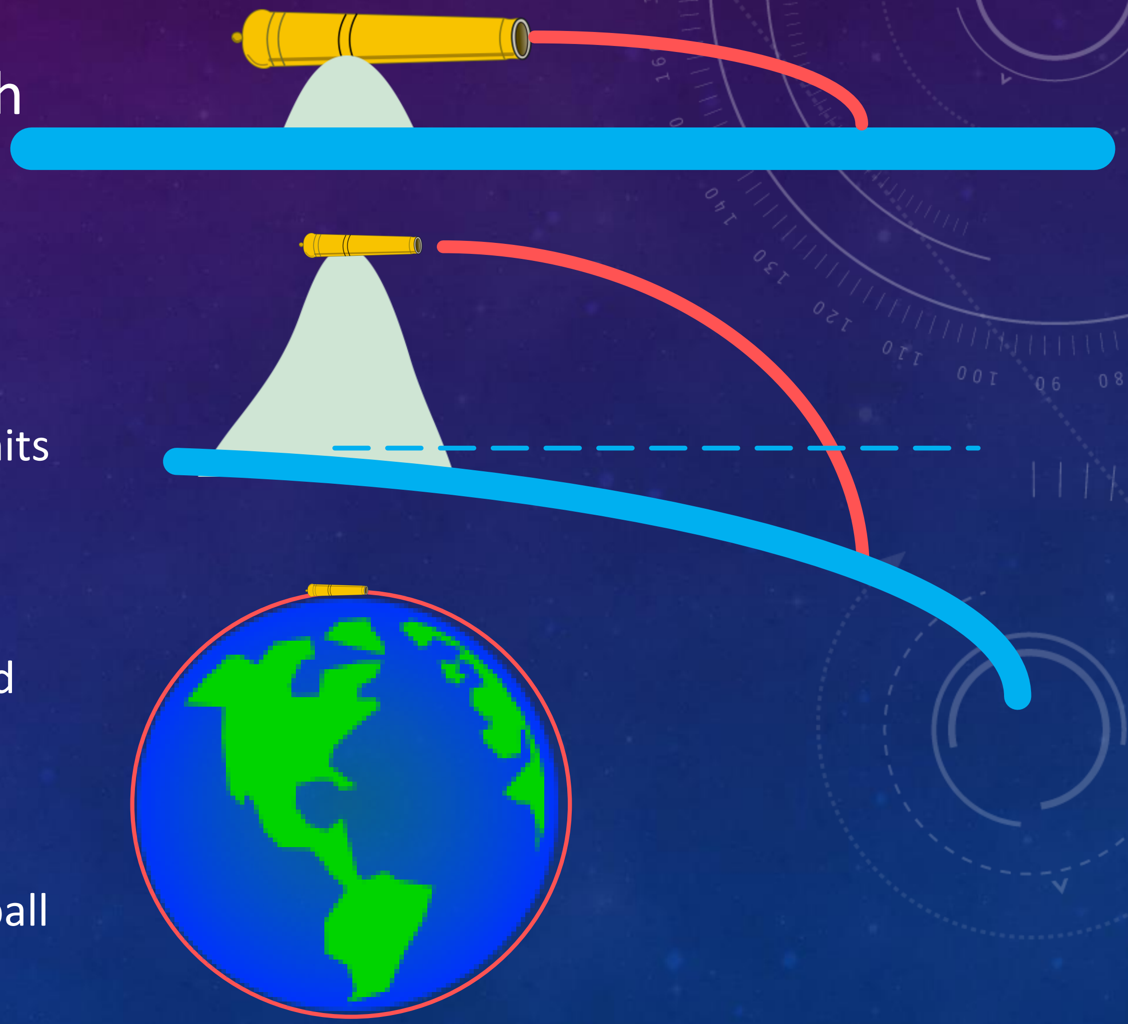


KEPLER'S LAWS FOR OTHER SYSTEMS

- Kepler derived his laws specifically for the planets orbit the sun in the solar system, but they also apply to other cases:
  - Comets and asteroids also behave according to Kepler's Laws as he phrased them.
- For cases where a small satellite orbits a larger body other than the sun, Kepler's Laws still apply with the modification that the satellite orbit is an ellipse, with the large body at one focus. The units in the Law of Periods must be changed, as well, but the relationship is the same.
  - Examples: Man-made satellites around the Earth & Exo-planets around other stars
- For systems in which *both* objects are large, Kepler's Laws apply, except both objects follow elliptical orbits about their common center of mass.
  - Examples: Earth/Moon & Binary Stars

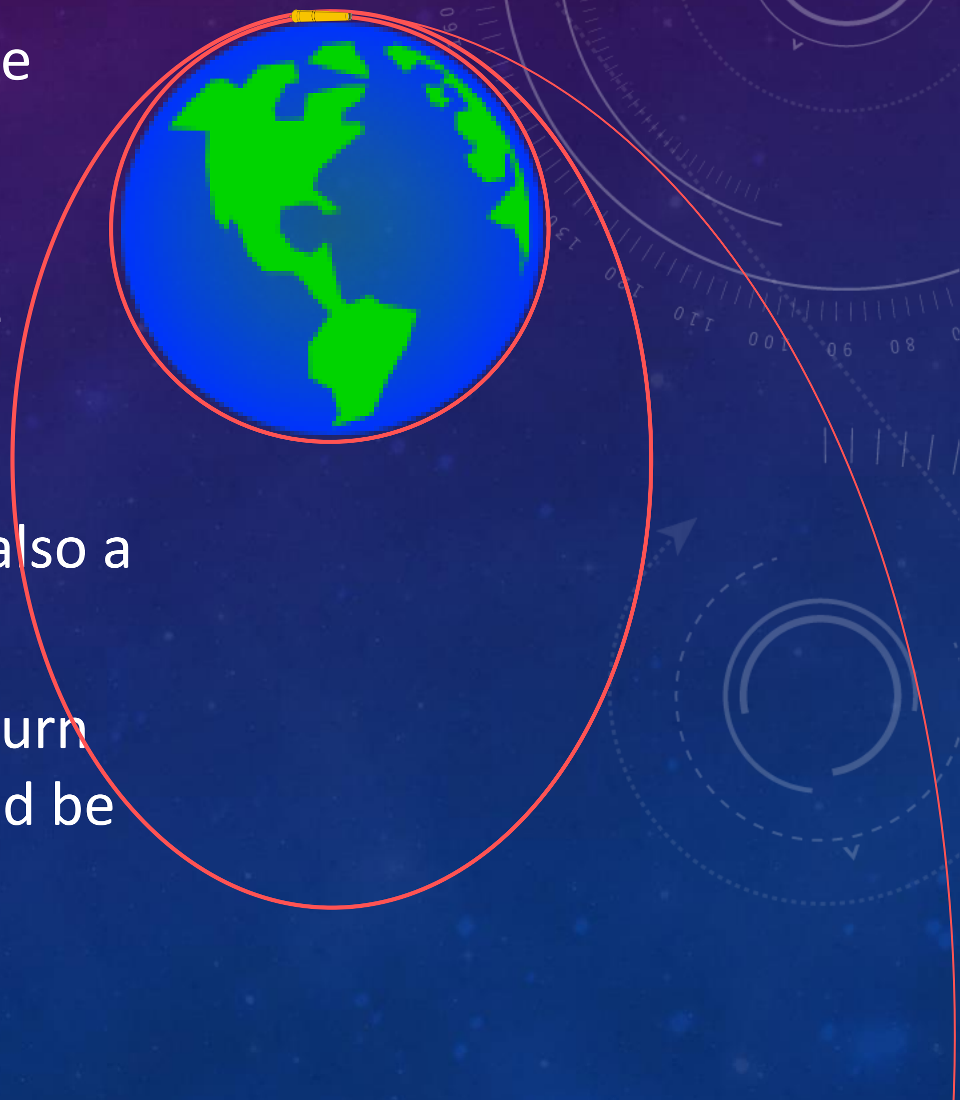
NEWTON'S CANNON EXAMPLE

- Newton explained objects that orbit the Earth using an idealized cannon-on-a-hill example
  - He ignored drag
  - If the ball is fired with some speed,
    - It follows a parabolic trajectory which eventually hits the flat ground
  - If the ball is fired with a higher speed,
    - The ball hits the ground farther away than it would have if the ground was flat because Earth curves
  - If the ball is fired with a high enough speed,
    - The Earth will curve away exactly as much as the ball curves along its trajectory
    - This forms a circular orbit. The ball is always falling.



ORBITS ACCORDING TO CANNON EXAMPLE

- At circular orbit speed, the cannonball would orbit the Earth in a circle. This is an example of a *bound orbit*, which closes back on itself.
- At an even higher speed, the cannonball would move farther from the Earth, but would eventually reach a farthest point, at which it would fall back toward the cannon. This would form an elliptical orbit, which is also a bound orbit.
- Given enough speed, the cannonball would never return to Earth. The path would be a hyperbola, and it would be an unbound orbit, or an escape orbit.
- These paths are all examples of conic sections.

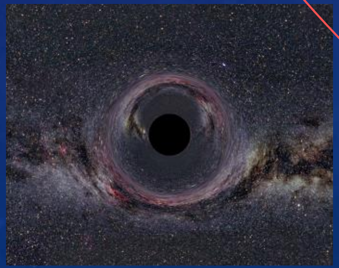


SPEEDS

- The speed needed to enter a circular orbit around a body can be easily derived from Newton's Law of Universal Gravitation ($F_G = G * M * m/d^2$) and the equation for centripetal force ($F_{cent}=m*v^2/r$).
  - For a circular orbit at distance R from the central body, circular orbit speed is $v_{circ}=\sqrt{G * M/R}$
  - For Earth at 100 km altitude, the orbital speed is v_{circ}= 7.9 km/s.
  - To orbit the Moon, the speed needed is v_{circ}= 1.7 km/s.

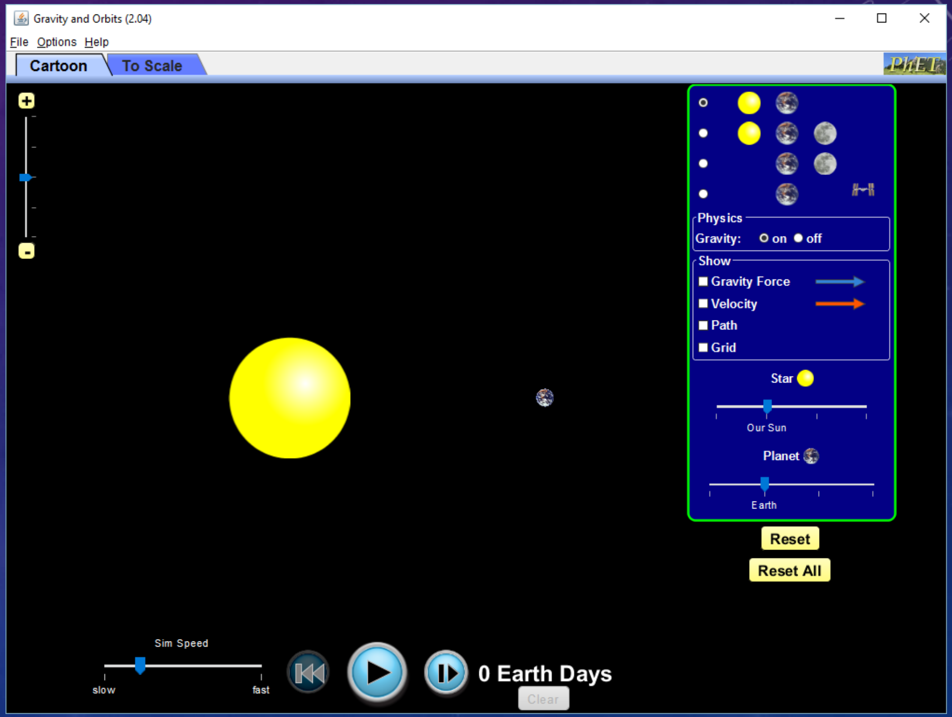
SPEEDS

- It is also possible to calculate the speed needed to permanently leave the central body. This is called escape velocity.
  - The equation is $v_{esc}=\sqrt{2G * M/R}$ (about 40% faster than orbit speed).
  - From an orbit around Earth, it is v_{esc} = 11.2 km/s.
  - For the Moon, it is v_{esc} = 2.4 km/s.
  - The fastest speed possible is the speed of light, *c*. We can work backwards from the escape velocity equation to find the distance from a black hole where light cannot escape. It is $R=2*G*M/c^2$.



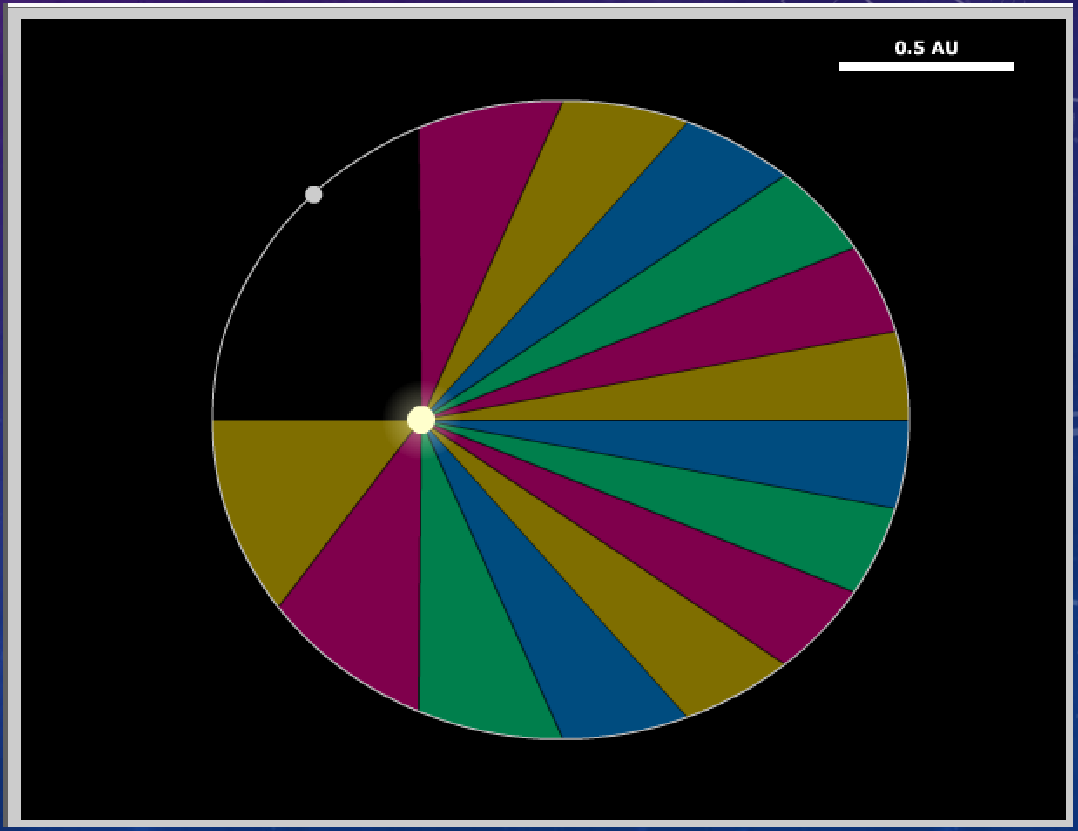
PLANETARY ORBIT SIMULATION

- Link to simulation: https://phet.colorado.edu/en/simulation/legacy/gravity-and-orbits
- Some things to try:
  - Check the ‘Show Path’ box
  - Change the mass of Earth. How does the path change?
  - Move Earth to a different location. How does the path change?
  - Change the mass of the Sun. How does the path change?



KEPLER'S LAWS SIMULATION

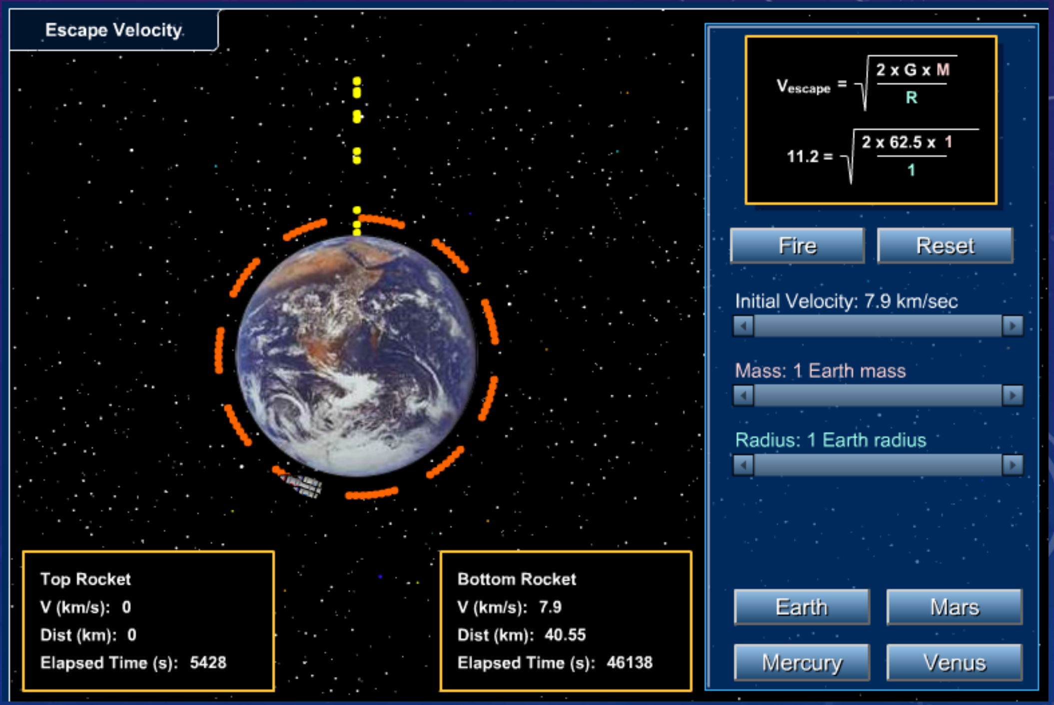
- Link to simulation: http://media.wwnorton.com/college/astronomy/animations/interactive/kepler.html
- Some things to try:
  - Select Kepler's 1st Law
    - Click all the boxes and experiment with various orbit sizes and shapes
  - Select Kepler's 2nd Law
    - Turn on 'sweep continuously' and start the animation
  - Select Kepler's 3rd Law
    - With plot type 'logarithmic', choose planets from Orbit Settings and see them fall on the straight line



ESCAPE VELOCITY SIMULATION

- Link to simulation: http://highered.mheducation.com/olcweb/cgi/pluginpop.cgi?it=swf::800::600::/sites/dl/free/0072482621/78778/Escape_Nav.swf::Escape%20Velocity%20Interactive

- Some things to try:
  - Press the ‘Earth’ button then ‘Fire’ to simulate a circular orbit (v_{circ} = 7.9 km/s)
  - Slide the initial velocity left of 7.9 and press ‘Fire’ to see a sub-orbital path.
  - Slide the initial velocity between 7.9 and 11.2 and press ‘Fire’ to see an elliptical orbit.
  - Slide the initial velocity beyond 11.2 and press ‘Fire’ to see an escape orbit.
  - Try the same with different planets



CONCLUSION

- Kepler formed three laws of planetary motion around the sun.
  - Law of Ellipses, Law of Equal Areas, Law of Periods
  - Kepler's Laws, with modification, also apply to any two-body orbit.
- Newton described the shapes of orbits using a cannon analogy.
  - The equations of the orbits can be derived from his Laws of Motion and his Law of Universal Gravity.
  - The orbits are either circles or ellipses, (bound orbits) or parabolas or hyperbolas (escape orbits).
- The speed needed to go into circular orbit can be found for any celestial body.
- The speed needed to escape a celestial body can be calculated.

IMAGE ATTRIBUTION

- Black hole: https://commons.wikimedia.org/wiki/File:Black_Hole_Milkyway.jpg