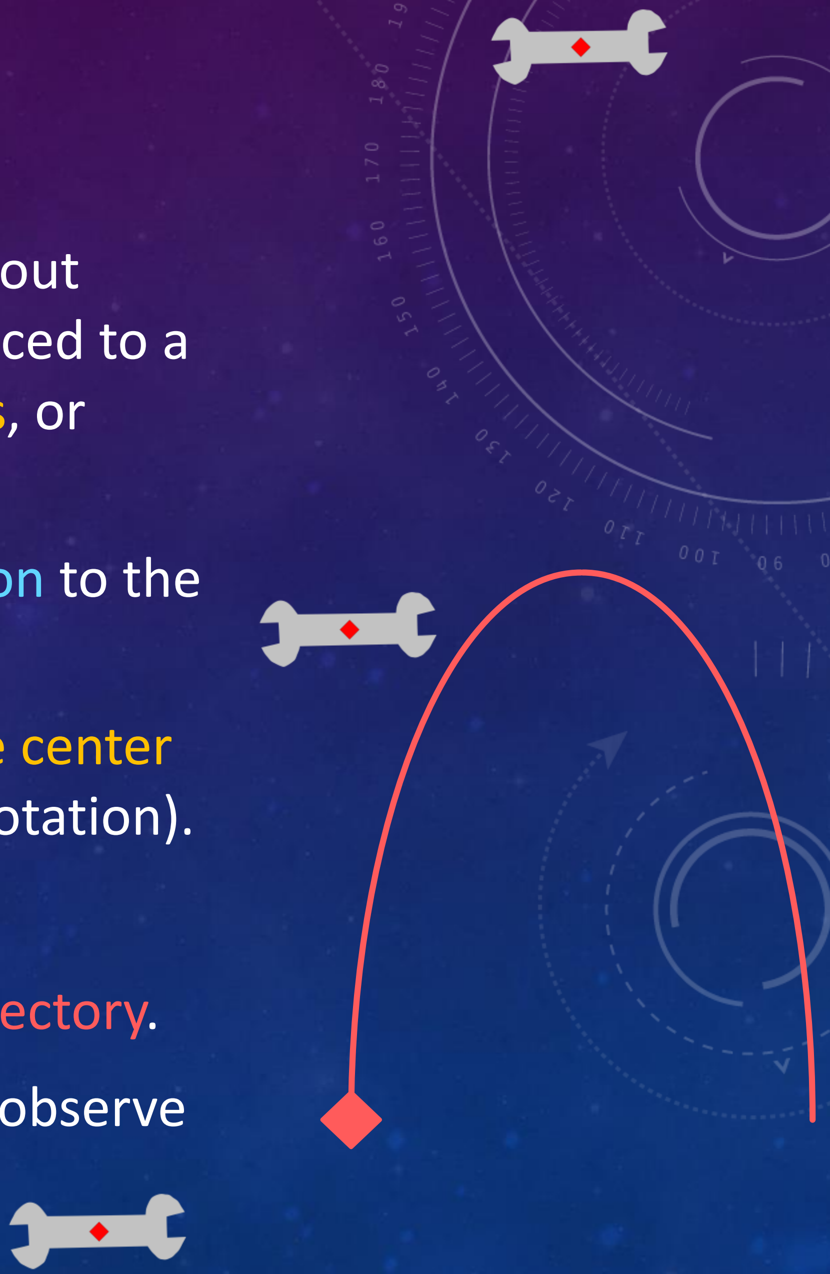
ROTATIONAL MOTION: ROTATIONAL VARIABLES & UNITS

PES 1000 – PHYSICS IN EVERYDAY LIFE

LINEAR AND ROTATIONAL MOTION

- Until now, we have only considered motion of an object without rotation. The whole object, regardless of its shape, was reduced to a single point which could be represented by its center of mass, or balance point.
- Now we consider adding the complication of rotational motion to the linear (or translational) motion
- This complicated motion can be separated into motion of the center of mass (translation) and motion about the center of mass (rotation).
- Thrown wrench example
  - The center of mass of the wrench follows a parabolic trajectory.
  - A camera following along with the center of mass would observe pure rotational motion.



ROTATIONAL MOTION DEFINITIONS

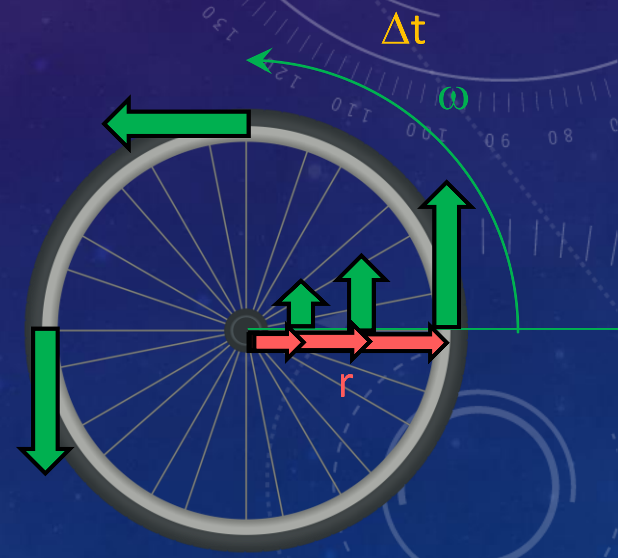
Linear Quantity	Rotational Quantity	Rotational Units
Position	Angular Position	Radians

- When an object rotates about a spin axis through some angle, it changes its angular position (relative to some reference line).
  - The direction of rotations is important. So angular position is a vector quantity.
  - The variable often used is θ (Greek theta).
- Angles are commonly measured in degrees. But for rotational physics, the unit more commonly used is the radian.
  - 1 full revolution = 360° = 2π radians, so 1 rad = 57.3°
  - Technically, radians are unitless, so the ‘rad’ sometimes isn’t used.
- When viewed in 2-D, the spin axis is a point; in 3-D, it is a line.

ANGULAR VELOCITY

Linear Quantity	Rotational Quantity	Rotational Units
Velocity	Angular Velocity	Radians/sec or 1/s or s^{-1}

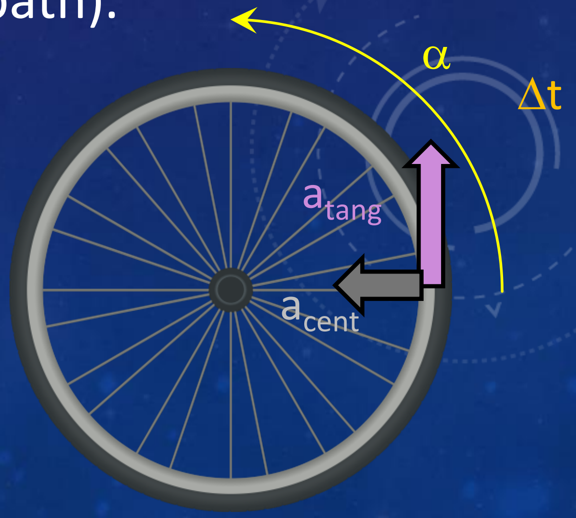
- Angular velocity is defined as change in angular position divided by time for that change.
  - Its units are rad/s (or 1/s or s^{-1}).
  - The variable used is usually ω (Greek lower-case omega). $\omega=\Delta\theta/\Delta t$.
- Direction is important, so angular velocity is a *vector quantity*.
- Another common angular speed unit is *RPM* (revolutions per minute).
  - Rad/s and RPM differ by about a factor of 10, so 1000 RPM is about 100 rad/s.
- Each point has a linear velocity (if a grain of sand left the wheel at that point, it would be the sand's velocity).
  - Every point has the <u>same</u> angular velocity, but a <u>different</u> linear velocity from other points, which depends on its radial distance, r, from the spin axis.
  - The equation is $v=\omega * r$



ANGULAR ACCELERATION

Linear Quantity	Rotational Quantity	Rotational Units
Acceleration	Angular Acceleration	Radians/sec^2 or 1/s^2 or s^{-2}

- Angular acceleration is defined as change in angular velocity divided by the time for that change.
  - Its units are rad/s^2 (or 1/s^2 or s^{-2}).
  - The variable used is usually α (Greek lower-case alpha).
  - Direction is important, so angular acceleration is a *vector quantity*.
- Every point has the <u>same</u> angular acceleration, but
  - Each point has a <u>different</u> tangential acceleration (along the circular path).
    - It depends on its radial distance, r, from the spin axis.
    - The equation is $a_{tang}=\alpha*r$
  - Every point also has a <u>different</u> centripetal acceleration
    - it depends on its radial distance and angular velocity.
    - The equation is $a_{cent}=\omega^2*r$

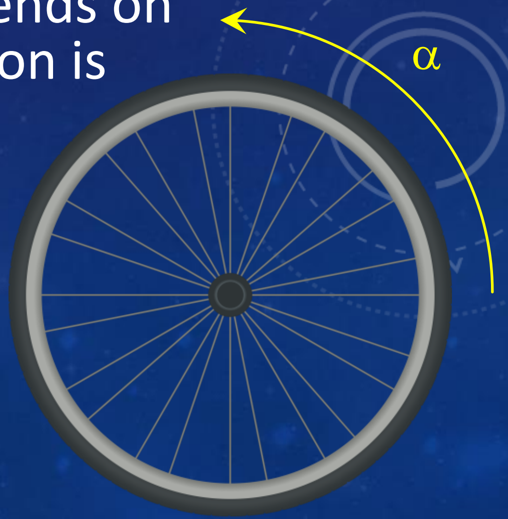


SPECIAL CASE: ZERO ACCELERATION

- Recall this case for linear motion. If there is no acceleration (like not braking or hitting the gas in a car), then the speed is constant. Distance traveled is speed times time.
  - $v = constant$
  - $d = v * t$
- Likewise, if there is no angular acceleration, then angular velocity will not change. (For a wheel, it will spin at a constant rate.) Angle traveled is angular speed times time.
  - $\omega = constant$
  - $\theta = \omega * t$

SPECIAL CASE: CONSTANT ACCELERATION

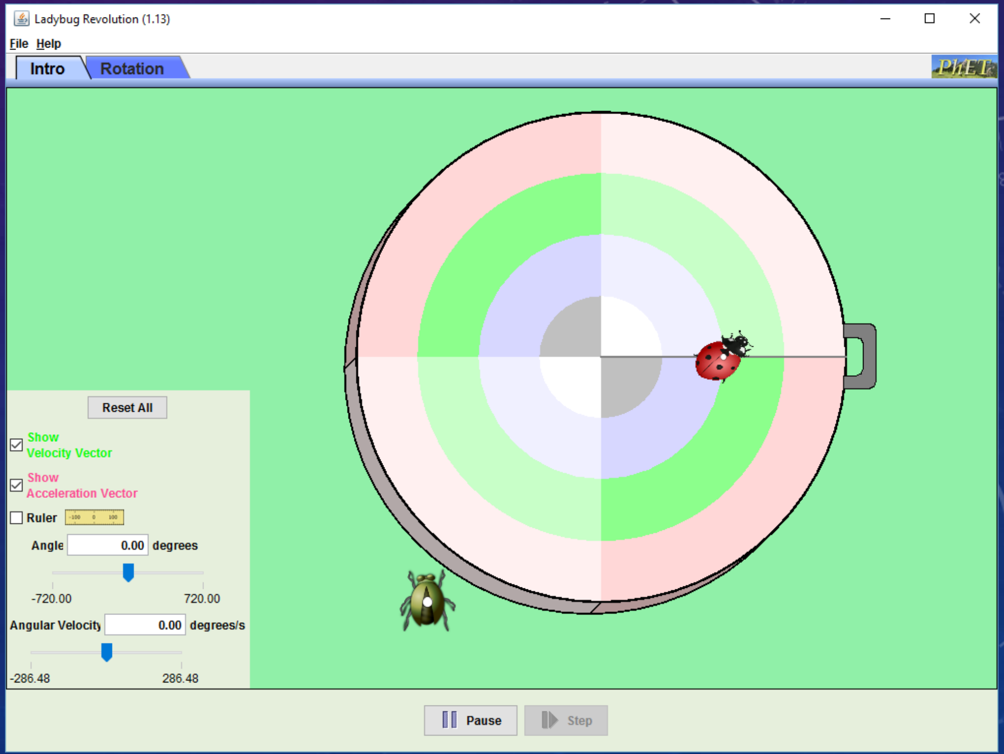
- Recall this case for linear motion. If there is constant acceleration (in a car, constant braking or hitting the gas), then the speed is changing at a constant rate. Distance travelled depends on initial speed, the acceleration, and how long the acceleration is applied.
  - $\Delta v = a * t$
  - $d = v_o * t + \frac{1}{2} * a * t^2$
- Likewise, if there is constant angular acceleration (like constant braking for a wheel), then the angular speed is changing at a constant rate. Angle travelled depends on initial angular speed, the angular acceleration, and how long the acceleration is applied.
  - $\Delta\omega = \alpha * t$
  - $\theta = \omega_o * t + \frac{1}{2} * \alpha * t^2$



ROTATIONAL MOTION SIMULATION

- Link to simulation: https://phet.colorado.edu/sims/rotation/rotation_en.jnlp

- Things to do:
  - Grab the handle and give the disk a spin.
  - Notice the acceleration vectors on the ladybug.
  - Put the beetle in a place where it has twice the tangential acceleration of the ladybug.
  - Put the beetle in a place where it has twice the centripetal acceleration of the ladybug.
  - Reverse the spin by moving the slider for angular velocity. What happens to the arrows?



CONCLUSION

- Motion of a moving, spinning object can be separated into pure rotation about a purely translating point (center of mass)l
- Each of the linear quantities of position, velocity, and acceleration have an angular equivalent.
    - Distance units are replaced with radians.
- The equations for rotation are similar in form to the equations for translational motion we've studied , with the variables replaced by the angular equivalents.

Linear Quantity	Linear Units	Rotational Quantity	Rotational Units
r=Position	m	θ=Angular Position	Radians
v=Velocity	m/sec or m/s or $m*s^{-1}$	ω=Angular Velocity	Radians/sec or 1/s or s^{-1}
a=Acceleration	m/sec^2 or m/s^2 or $m*s^{-2}$	α=Angular Acceleration	$Radians/sec^2$ or $1/s^2$ or s^{-2}