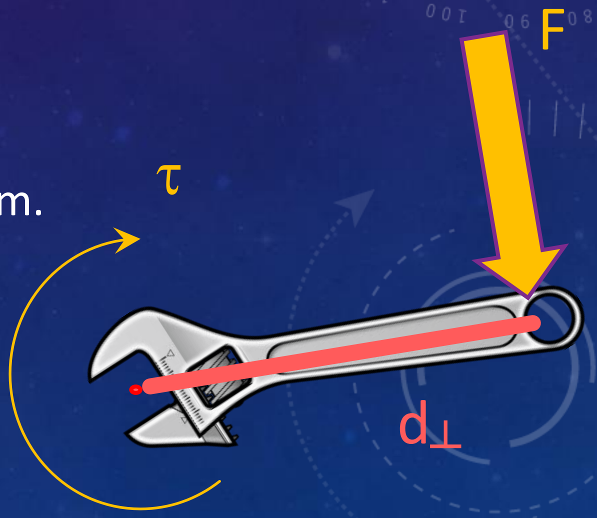
ROTATIONAL MOTION: TORQUE, ANGULAR INERTIA AND NEWTON'S LAWS

PES 1000 – PHYSICS IN EVERYDAY LIFE

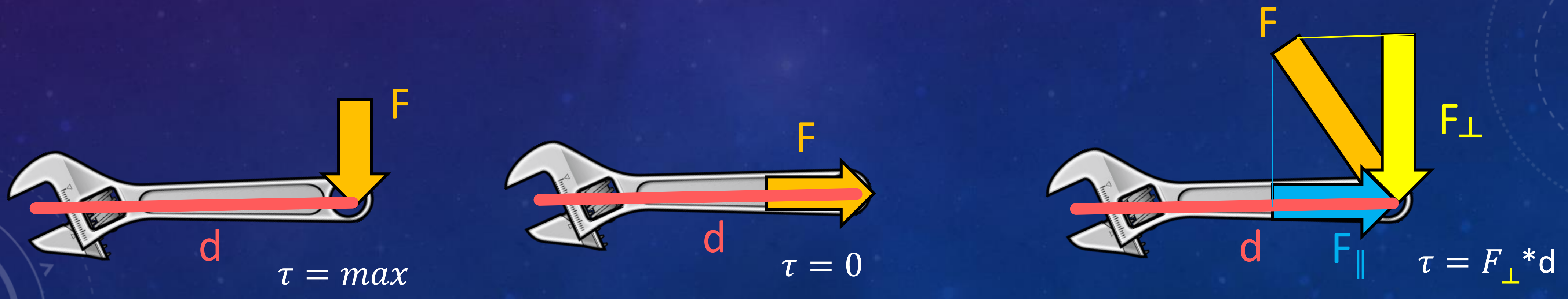
TORQUE: FORCE AND LEVER ARM

- For translational motion, forces cause changes to the velocity. The equivalent of force in rotational motion is called torque. Torque changes the angular velocity.
- Torque is the rotational effect of a perpendicular force acting at some distance from the spin axis of an object.
  - The variable we often use is τ (Greek tau)
  - Torque has a direction (clockwise or counterclockwise), so it is a *vector*.
  - The distance is sometimes called the perpendicular lever arm, $d_{\perp}$, or just lever arm.
  - The *longer the lever arm*, the *more torque* a force will have.
  - More force causes more torque, as well.
- The equation is: $\tau = F * d_{\perp}$
- The units are *N*m* in the SI system, and *foot-pounds* in the US Customary system



TORQUE: ANGLED FORCE

- If the force is not applied perpendicularly, but at an angle to the lever arm, then only the part of the force that is perpendicular causes a torque. The part of the force along the lever arm has no turning effect (though it could translationally accelerate the object if it is free to move.)
- The maximum torque of a force occurs when it is at 90º to the lever arm.
- No torque is generated when the angle is 0º, (i.e. it is parallel to the lever arm).
- We can separate the force into perpendicular and parallel components, and the torque is then: $\tau = F_{\perp}$*d



ROTATIONAL INERTIA

- For translational motion, inertia (mass) resists the acceleration of an object.
- For rotational motion, an object's rotational inertia resists rotational acceleration.
  - Rotational inertia is sometimes called 'moment of inertia'.
- Rotational inertia depends not just on the mass, but <u>where</u> the mass is located relative to the spin axis.
  - The *farther away* the mass is, the *more rotational inertia* it causes.
  - Waggle your pencil about its center of mass and sense the torque required.
  - Then hold it near the eraser and waggle it again. You can sense the extra torque required. The pencil has the same mass as before, but more of its mass is farther from your fingers at the spin axis. In fact, the rotational inertia increases with the <u>square</u> of the distance.
- The variable we use for rotational inertia is 'I'.
- The units are $kg*m^2$.

CALCULATING ROTATIONAL INERTIA

- The rotational inertia of a point mass, m, at some distance, r, from a spin axis has rotational inertia, $I=m*r^2$.
- Physical objects are made up of many point masses, and their distances from the spin axis depend on the shape of the object and where the spin axis is placed.
- Calculating this for a random shape is difficult, but if the shape is regular, then the math is simplified, and we get easy-to-use formulas.
- Examples: thin rod, hoop, disk, cylinder, sphere

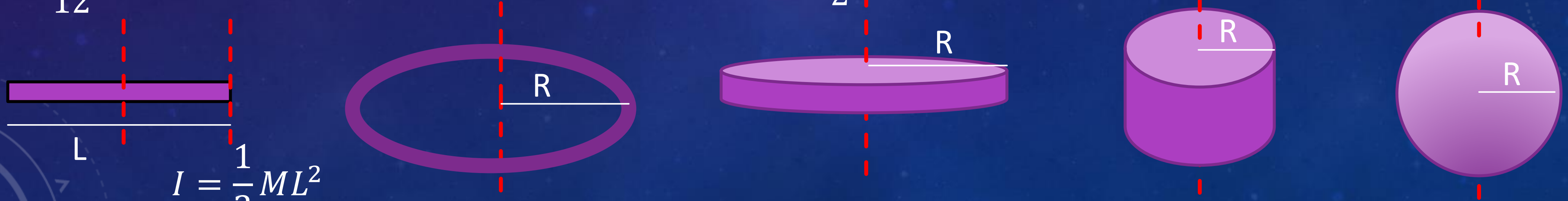
$$I = \frac{1}{12}ML^2$$

$$I = MR^2$$

$$I = \frac{1}{2}MR^2$$

$$I = \frac{1}{2}MR^2$$

$$I = \frac{2}{5}MR^2$$



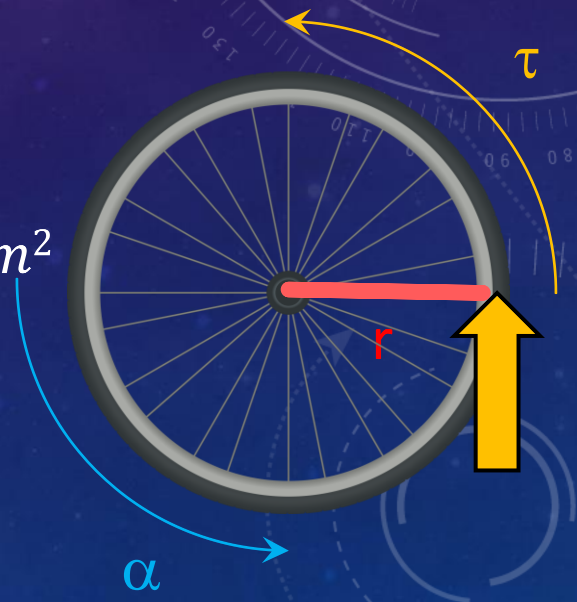
$$I = \frac{1}{3}ML^2$$

NEWTON'S 2ND LAW

- For translational motion, Newton's 2nd Law relates the acceleration to the net force and the mass (inertia):
  - $F_{net} = m * a$
  - The acceleration is in the direction of the net force.
- For rotational motion, Newton's 2nd Law relates the angular acceleration to the net torque (rotational effect of the force) and the rotational inertia:
  - $\tau_{net} = I * \alpha$
  - The angular acceleration is in the direction of the net torque.

NEWTON'S LAWS APPLIED TO ROTATION

- Here is a numerical example with units to show how this all works together.
- A 20 N force is applied at the edge of a wheel that has a fixed axis. The wheel (essentially a hoop) has a mass of 4 kg and a radius of 0.2 m.
- Let's calculate the angular acceleration.
  - The rotational inertia is $I = MR^2 = (4\text{ kg}) * (0.2\, m)^2 = 0.16\, kg * m^2$
  - The torque due to the force is $(20\, N) * (0.2\, m) = 4\, N * m$
  - The angular acceleration of the wheel is $\alpha = \frac{4N*m}{0.16\, kg*m^2} = 25\frac{N}{kg*m}$
  - Remembering that 1 N = 1 kg*m/s^2, our answer is in units of 1/s^2, or **25 rad/s^2**



NEWTON'S 1ST AND 3RD LAWS

- For translational motion, Newton's 1st Law states that if there are no net forces, the objects velocity will remain unchanged:

  "An object will maintain a constant velocity unless acted upon by a net force."

  - Rotational equivalent:

    "An object will maintain a constant angular velocity unless acted upon by a net torque."

- For translational motion, Newton's 3rd Law states that objects interact with equal and opposite forces.
  - Rotational equivalent:

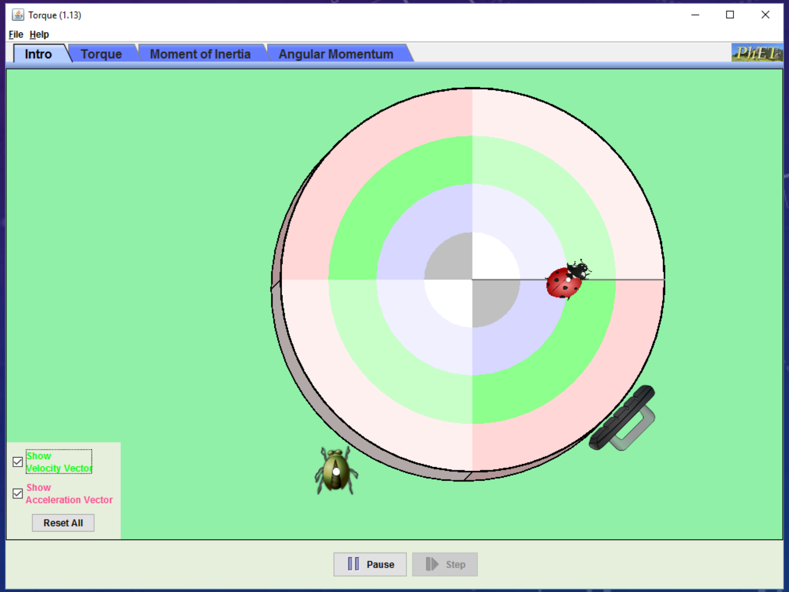
    "If one object exerts a torque on another, the other object exerts an equal and opposite torque on the first object."

TORQUE SIMULATION

- Link to simulation: https://phet.colorado.edu/en/simulation/legacy/torque

Some things to do:

- In the ‘Intro’ tab:
  - Click and drag somewhere on the disk. A perpendicular force generates a torque as long as you hold your finger down. Watch the disk accelerate.
  - Apply the brake at the edge of the wheel. What effect does it have?
- In the ‘Moment of Inertia’ tab
  - Adjust the inner radius and apply a torque like you did before. What is different?
  - Increase the mass of the platform and apply a torque again. What is different now?



CONCLUSION

- Forces can generate a twisting effect, called torque.
- Torque depends on the force, the angle, and the distance from the spin axis.
- Torque *motivates* an object to accelerate rotationally.
- The mass and shape of an object give it a rotational inertia that *opposes* acceleration.
- The resulting angular acceleration is the balance between torque (motivation) and rotational inertia (opposition).