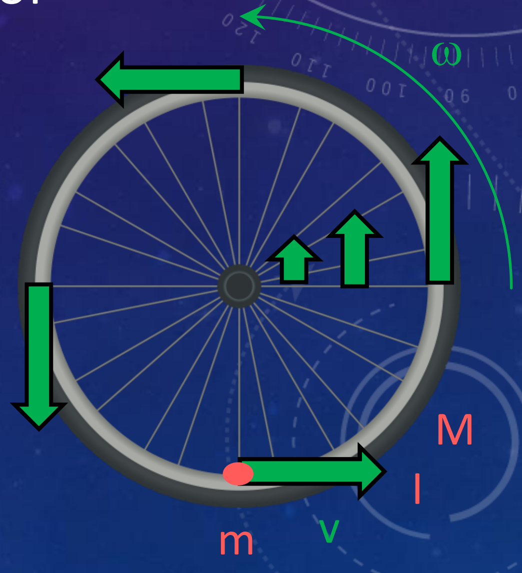
ROTATIONAL MOTION: ROTATIONAL ENERGY & ANGULAR MOMENTUM

PES 1000 – PHYSICS IN EVERYDAY LIFE

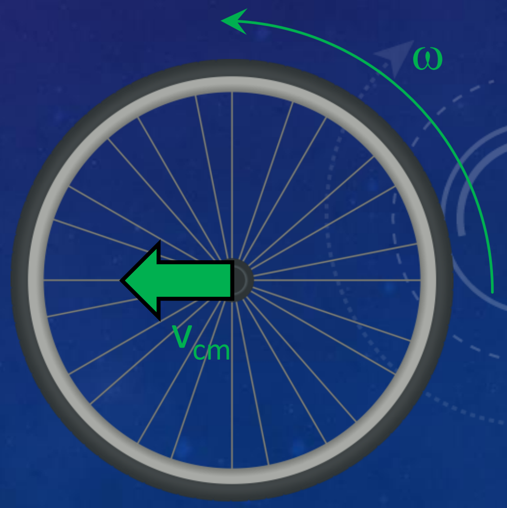
KINETIC ENERGY

- When an object is spinning, each point of mass on it has a velocity depending on its distance from the spin axis.
- Any mass that moves has kinetic energy ($K_{point} = \frac{1}{2} * m * v^2$), regardless of its direction.
- The total rotational kinetic energy is the sum over all of these points of mass.
  - The shape of the mass is described by its rotational inertia, I
  - The total kinetic energy due to an object's rotation turns out to be:
    - $K_{rotation} = \frac{1}{2} * I * \omega^2$
  - Note the similarity of this formula to the kinetic energy of a point mass.
  - The units are still energy units, *Joules*.



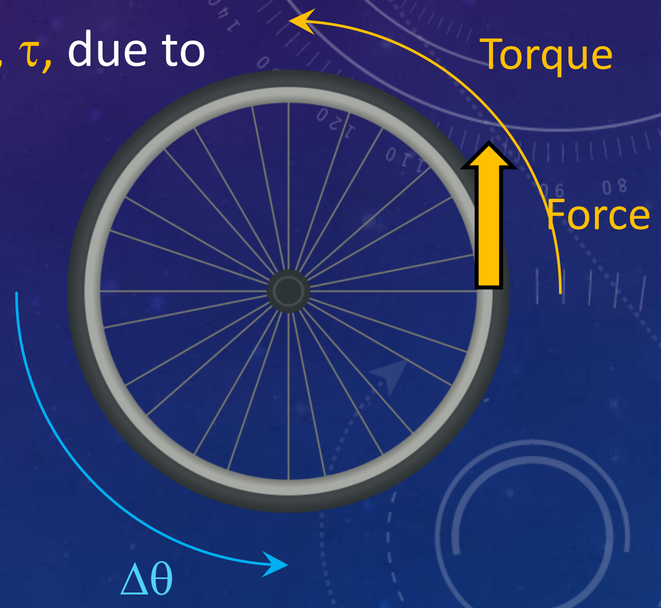
KINETIC ENERGY

- If the object is also translating, then the rotational kinetic energy is added to the translational kinetic energy due to the center of mass speed, v_{cm}
  - $K_{linear} = \frac{1}{2} * M * v_{cm}^2$
  - $K_{rotation} = \frac{1}{2} * I * \omega^2$
  - $K_{total} = K_{linear} + K_{rotation}$
  - Kinetic energy *of* the center of mass
    plus
    kinetic energy *about* the center of mass



TORQUE AND WORK

- For linear motion, we found that when a force, F, moves through a distance, Δx, work is done: $W = F * \Delta x$
- When that force makes an object turn through an angle, Δθ, then the torque, τ, due to that force does *work* on the object.
- The work is $W = \tau * \Delta\theta$
  - Note the similarity to the linear work formula.
  - The work units are still energy units, *Joules*.
- For linear motion, we found the Principle of Work and Energy
  - Net work changes the kinetic energy: $W_{net} = \Delta K$
- If rotational motion is included, then the net work can now also contain work done by a torque turning the object.
- ΔK now can include the rotational kinetic energy *in addition to* the linear kinetic energy.

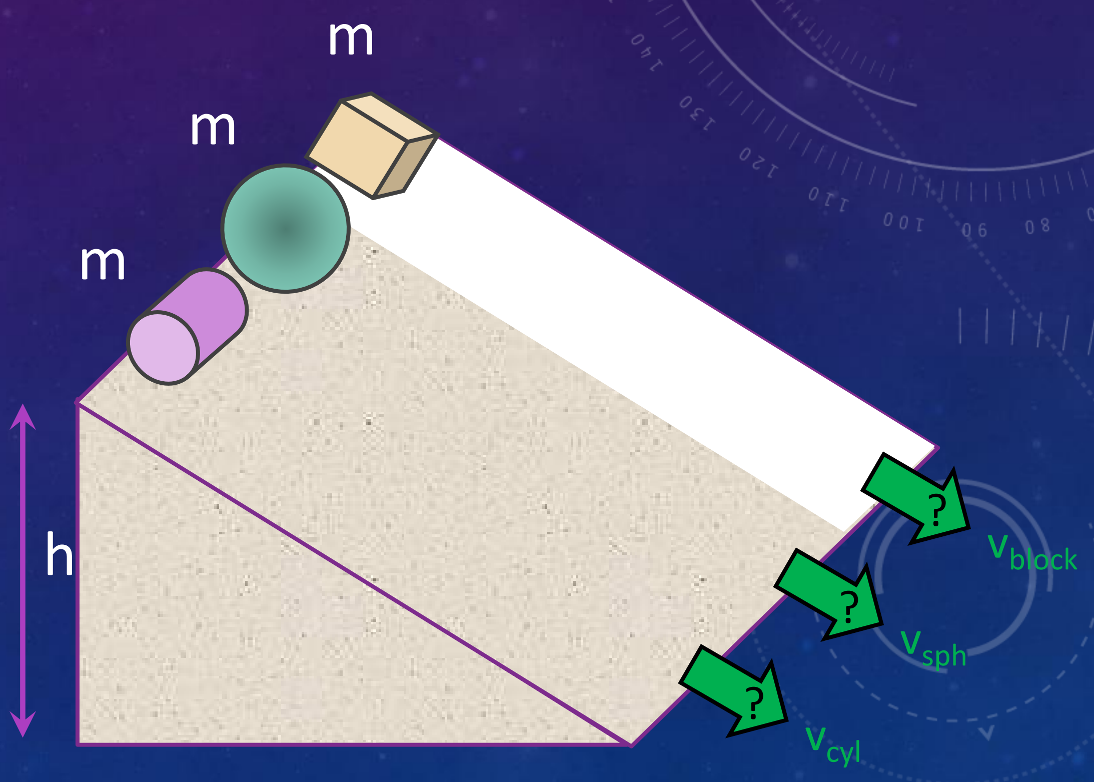


CONSERVATION OF ENERGY

- Conservation of Energy
  - Recall that if the only forces involved in a situation are gravity and springs, then the total mechanical energy is constant:
  - $PE_{total} + KE_{total} = constant$
  - Recall that gravitational potential energy depends on mass, gravity, and height:
    - $PE_{gravity} = m * g * h$
  - Recall that spring potential energy depends on the spring constant, k, and the stretch or compression, s:
    - $PE_{spring} = \frac{1}{2} * k * s^2$
  - With rotational motion now considered, the total kinetic energy is the sum of linear and rotational energy of motion:
    - $K_{total} = K_{linear} + K_{rotation} = \frac{1}{2} * M * v^2 + \frac{1}{2} * I * \omega^2$

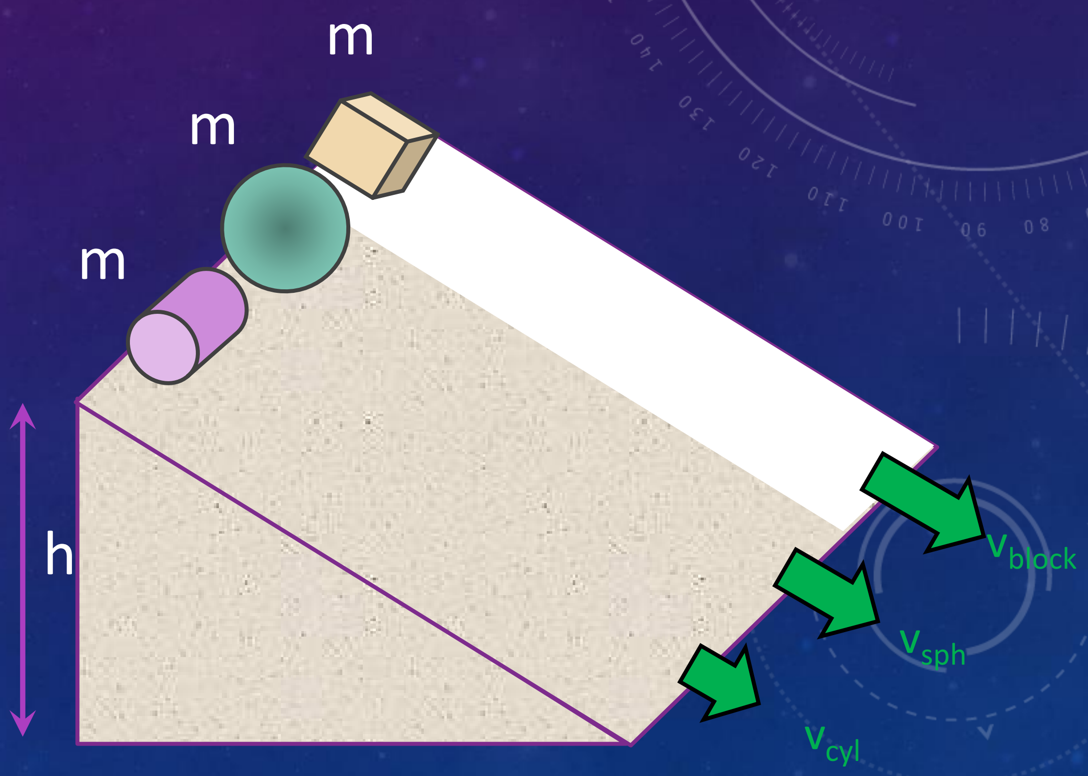
EXAMPLE: CONSERVATION OF ENERGY

- Imagine a downhill race between three shapes, a cylinder, a sphere, and a block. All have the same mass, and the round objects have the same radius.
  - The ramp under the block is ice (no friction) so that *the block slides*.
  - The ramp under the other two shapes is rough enough that *the other two shapes roll*.
- If they all have the same mass and start at the same height, then they all have the same *gravitational potential energy*.
- Whichever shape has the greatest translational speed will win the race. So, who will win the race?

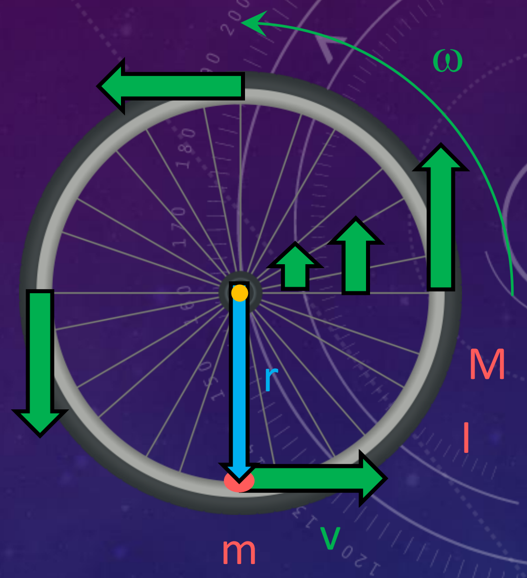


EXAMPLE: CONSERVATION OF ENERGY

- Conservation of energy says:
  - $PE_{gravity} = KE_{linear} + KE_{rotation}$
- Note than whichever shape transfers the least potential energy into rotational energy will have the most left in translational energy (which wins the race.)
  - The block transfers no potential energy into rotational energy, so the block wins the race.
  - Since $KE_{rotation} = \frac{1}{2} * I * \omega^2$, the shape with the least value of I will come in second.
  - The sphere's is $I = \frac{2}{5} * MR^2$, while the cylinder's is $I = \frac{1}{2} * MR^2$
  - So the sphere is second, and the cylinder is last.



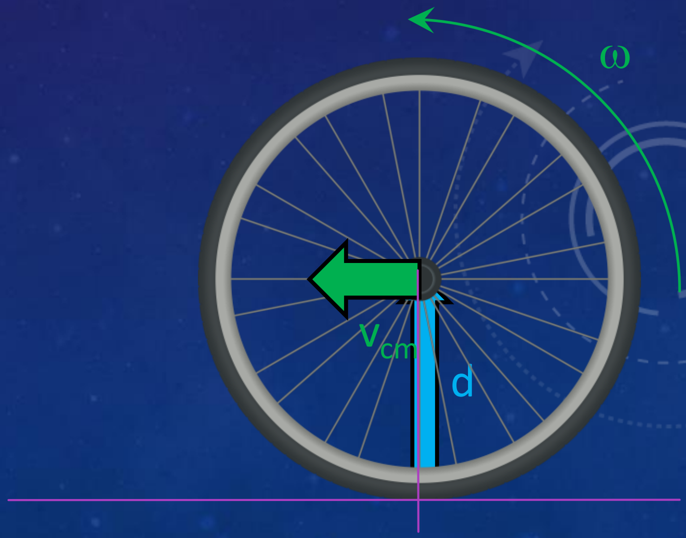
ANGULAR MOMENTUM



- When an object is spinning, each point of mass on it has a velocity depending on its distance from the spin axis.
- Any mass that moves has linear momentum ($p_{point} = m * v$).
- Its angular momentum (called *orbital angular momentum*) is defined as a vector:
  - The direction is along the spin axis (all points have the spin axis in common)
  - Its magnitude is $L_{orbital} = m * v * r$
- The total orbital angular momentum is the sum over all of these points of mass.
  - The shape of the mass is contained in the rotational inertia, I
  - The combined angular momentum due to an object's spin turns out to be:
    - $L_{spin} = I * \omega$
  - Note the similarity of this formula to the linear momentum of a point mass.

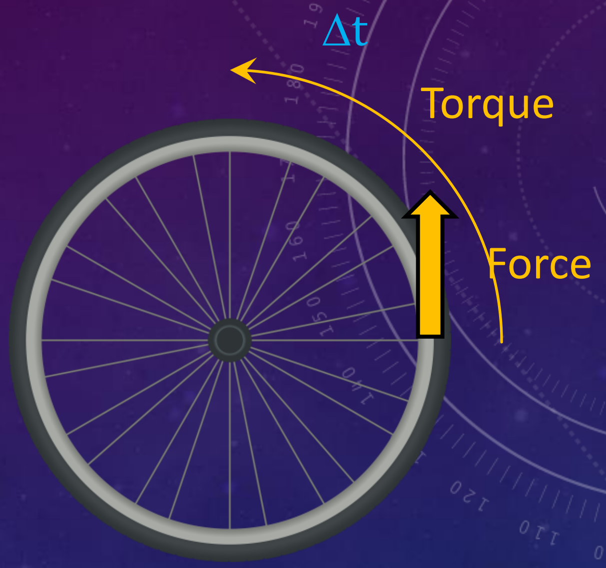
ANGULAR MOMENTUM

- If the object is also translating, then the spin angular momentum is added to the orbital angular momentum of the center of mass
  - $L_{orbital} = M * v_{cm} * d$ where M is the total mass, v_{cm} is the velocity of the center of mass, and d is the distance from center of mass from the origin.
  - $\vec{L}_{total} = \vec{L}_{orbital} + \vec{L}_{spin}$ (these are added as vectors, tip-to-tail)
  - Angular momentum *of* the center of mass
    plus
    angular momentum *about* the center of mass



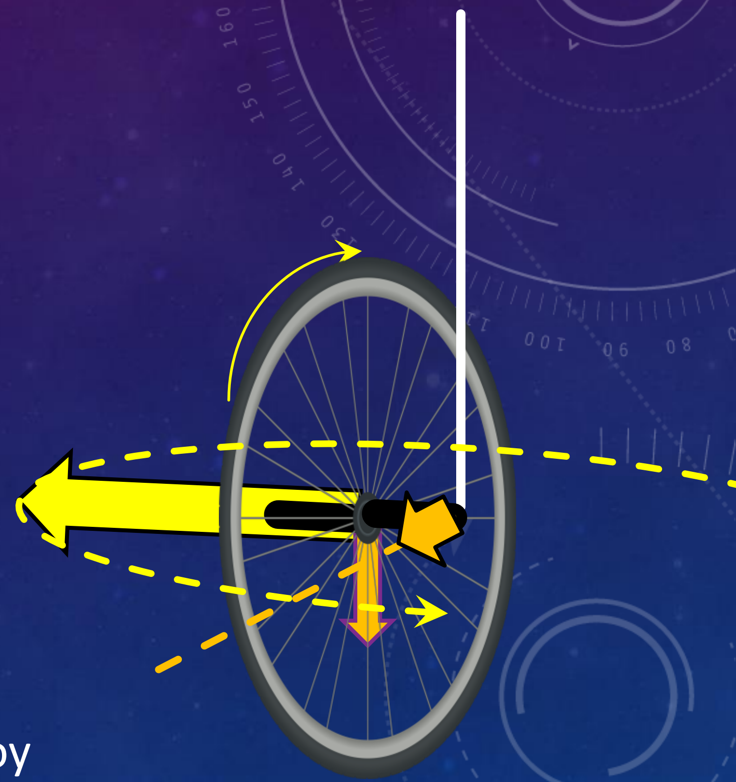
TORQUE AND ANGULAR IMPULSE

- For linear motion, we found that when a force, F, moves acts over a time interval, Δt, impulse is transferred:
  $\Delta p = F * \Delta t$
- When that force makes an object turn, then the torque, τ, due to that force transfers *angular impulse* to the object:
  $\Delta L = \tau * \Delta t$
  - Note the similarity to the linear impulse formula.
- Forces can cause both a change in linear momentum and a change in angular momentum (if it also causes a torque).



TORQUE AND ANGULAR MOMENTUM

- Recall Newton's 2nd Law: F=m*a
  - If mass is constant, then mass*acceleration = m*(change in velocity)
  - force = m*(change in velocity) = (change in momentum)
  - So force causes a change in momentum over time, in the direction of the force.
- Newton's 2nd Law applied to spinning objects is similar: $\tau = I * \alpha$
  - If rotational inertia, I, is constant, then $I * \alpha$ = I *(change in ω)
  - torque = I *(change in ω) =(change in angular momentum)
  - So torque causes a change in angular momentum over time
    - This leads to the gyroscopic effect that makes spinning wheels and toy tops behave in such a complicated way.
    - The spinning wheel turns sideways instead of falling down due to its weight.
    - Here is a video of this phenomenon: https://www.youtube.com/watch?v=NeXIV-wMVUk



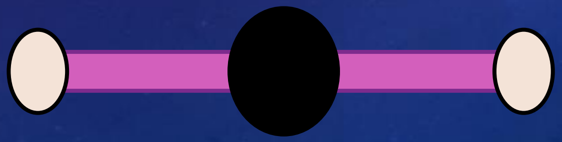
CONSERVATION OF ANGULAR MOMENTUM

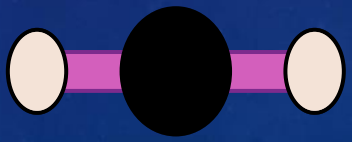
- Conservation of Linear Momentum
  - Recall that if the only significant forces involved in a situation are internal, then the total linear momentum is constant.
  - For multiple objects, this says: $m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = m_A\vec{v}_{A2} + m_B\vec{v}_{B2}$
  - For a single object, this is trivial: $m\vec{v}_1 = m\vec{v}_2$ or, in other words, velocity remains constant if no net force acts on it. This is just a restatement of Newtons 1st Law.
- Conservation of Angular Momentum
  - If the only significant torques involved in a situation are internal, then the total angular momentum is constant.
  - For multiple objects (about some axis), this says: $I_A\omega_{A1} + I_B\omega_{B1} = I_A\omega_{A2} + I_B\omega_{B2}$
  - This becomes interesting when a single object can change its shape, however, thus changing its rotational inertia, I.
  - $I_1\omega_1 = I_2\omega_2$ or $\omega_2 = (I_1/I_2)\omega_1$

EXAMPLES OF CONSERVATION OF ANGULAR MOMENTUM

A single object: an ice skater going into a spin

- The skater has an initial angular velocity, ω_1, and a spread-out mass distribution, I_1.
- Part-way into the spin, she brings her arms in, nearer to the spin axis. This reduces her rotational inertia to I_2.
- There is no net external torque on the skater, and the shape-changing forces are internal to the system, so angular momentum is conserved.
- Since the product of $I * \omega$ is constant before and after, and if I_2 reduces, then ω_2 must increase.





EXAMPLES OF CONSERVATION OF ANGULAR MOMENTUM

Two objects: Earth and Moon

- The angular momentum of the Earth/Moon system is a combination of the Moon's orbital angular momentum and the Earth's spin angular momentum.
  - (We can ignore the relatively slow spin of the Moon's spin and Earth's small orbital motion about the Earth/Moon center of mass.)
- The total angular momentum of the Earth/Moon system is constant; there are no external torques that change it.
- The moon pulls on the water of Earth, causing tidal bulges on the Earth/Moon line.

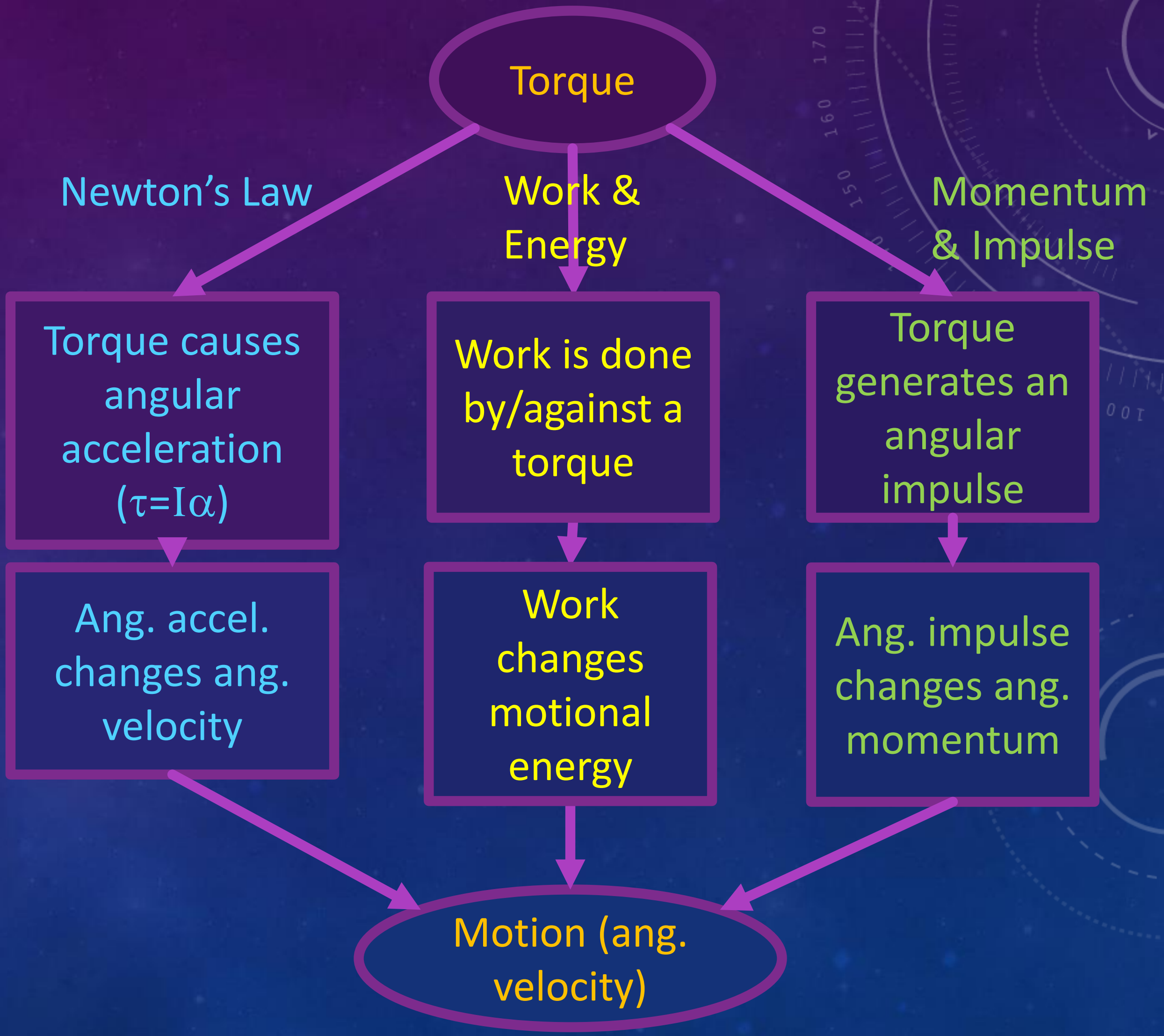
EXAMPLES OF CONSERVATION OF ANGULAR MOMENTUM

Two objects: Earth and Moon

- The Moon makes Earth's water bulge toward itself. The bulge causes friction on the ocean beds as Earth turns underneath it, slowing the Earth's rotation, and therefore lowering Earth's spin angular momentum.
- Since the total angular momentum is constant, the Moon's orbital momentum must increase. The Moon does this by moving farther from the Earth.
- This phenomenon is observable using laser reflection from mirrors left on the Moon during the Apollo program
  - The Earth's rotation is slowing by 2.3 milliseconds/century, and the Moon is receding at 3 cm/year.

SUMMARY

- Newton's Law approach:
  - Torque causes angular acceleration
  - Angular acceleration changes angular velocity
- Work and energy approach:
  - Torque can do work
  - Work can change the amount of rotational kinetic energy of an object, and thus its angular velocity.
- Momentum and impulse approach:
  - Torque generates angular impulse
  - Angular impulse changes angular momentum, and thus its angular velocity.



CONCLUSION

- Rotational motion may be analyzed using energy methods:
  - Torque can do work, which changes total kinetic energy.
  - Kinetic energy can be due to *linear motion*, *rotational motion*, or *both*.
  - Conservation of Energy includes rotational kinetic energy, too.
- Rotational motion may be analyzed using momentum methods:
  - Torque generates angular impulse, which changes angular momentum.
  - Angular momentum can be due to *orbital motion*, *spin motion*, or *both*.
  - Conservation of Angular Momentum includes *orbital motion*, *spin motion,* or *both*.