Circular Motion

PES 1160 Advanced Physics Lab I

Purpose of the experiment

- To empirically verify that both centripetal acceleration and centripetal force are directed radially inward toward the center of a circular path.
- To verify that centripetal accelerations have a magnitude of v^2/r.
- To verify that centripetal forces have a magnitude of mv^2/r.
- Determine the dependence of mass, radius and angular velocity on centripetal force.
- [FYI]

[FYI] The Earth orbits the sun at an average speed of 107,220 kilometers per hour (66,623 mph).

Background

For this laboratory exercise, we turn our attention to circular motion. Like projectile motion, circular motion is an important case to examine because it appears so many times in science and engineering. Cars on highway off-ramps, amusement park rides, centrifuges for separating samples in chemistry and biology, and stars and planets all exhibit circular motion (or something very close to it).

In this lab, you will first develop the simple mathematical model that describes circular motion. This model includes both the kinematic description, which is centripetal acceleration, as well as the dynamic description, centripetal force. Then, you will put both centripetal acceleration and force to an experimental test to see if the mathematical model corresponds to the actual phenomenon of circular motion that you will see in the lab.

Centripetal Acceleration

As you should be familiar with, acceleration is the rate of change of velocity. Since acceleration is a vector, it has both a magnitude and a direction. Consequently, acceleration includes not only changes in an object's speed, such as speeding up or slowing down, but also changes in direction. For circular motion, it is these changes in direction we will be interested in.

By this time, it should be fairly intuitive to you that an object in motion that is turning (i.e. changing direction) has an acceleration. An object turning to the left must have a left-directed acceleration, while an object turning to the right must have a right-directed acceleration, as seen in the figure below:

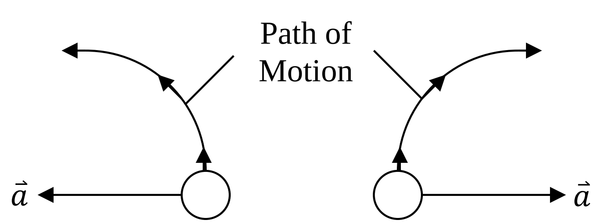


Figure 1: Left and right-turning objects and the accelerations involved in their change of direction.

So, to continue this train of thought, what would happen if an object continuously accelerated to its left (or right) with an acceleration that was constant in magnitude? *It would travel in a circle*! The following diagram shows this circular motion:

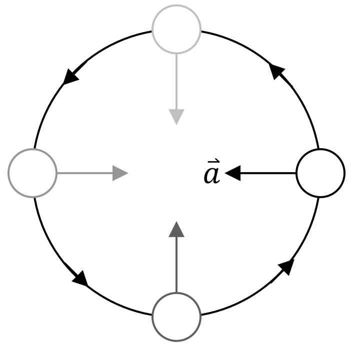


Figure 2: An object with a constant acceleration that is perpendicular to its motion will undergo uniform circular motion.

Since the object in the figure above is continuously turning to its left, this means that its acceleration is always pointing *inward* on the circle. This is an important feature of the accelerations involved in circular motion:

> For uniform circular motion (i.e. circular motion where the speed is constant), the acceleration must point *inward* exactly toward the center of the circle. This acceleration is called a *centripetal acceleration*.

In fact, *centripetal* means "center-seeking" and refers to the direction the acceleration must point for an object to move on a circle.

It is at this point that some people's intuition trips them up. They will argue, "Yeah, but when I go in a circle, it clearly feels like the acceleration is outward." The problem with this argument is the "feels like" part of the argument. What you feel when you accelerate is your body's inertia, which is your body's resistance to acceleration. Sometimes, this is called *centrifugal* ("center-fleeing") *acceleration* because your body's inertia wants to carry you in a straight, rather than curved, line which would move you further from the circle's center. Thus, centrifugal acceleration is not a real acceleration, and consequently what you feel when you accelerate is exactly opposite to the direction of the real acceleration.

Still don't believe me? Look at it this way: When you are at a stop light and push on the gas pedal in your car, you begin to speed up and move forward. Clearly, your real acceleration is *forward* in this case. But what does it feel like? It feels like you are being pushed *backward* in your seat. The reverse is also true: if you are traveling quickly in your car and you step hard on the brake, you are slowing down. Consequently, your real acceleration is *backward*. However, your body's own inertia tends to want to keep you moving *forward* in a straight line and at the speed you were traveling before you hit the brake (this is why we have air bags in cars). Here again, your real acceleration is opposite to what you feel. Thus, centripetal acceleration is inward.

So, we have established the direction of the centripetal acceleration. What about its magnitude? As it turns out, the magnitude of centripetal acceleration depends on (1) the radius of the circle r that the object is traveling on, and (2) the speed of object around the circle v.

$$a_C = \frac{v^2}{r} \qquad \text{(Magnitude of Centripetal Acceleration)}$$

First, consider the dependence on radius. For an object with a very small radius of motion, say for example a sports car going around a hairpin turn, the object will need a large centripetal acceleration to keep it turning on such a small radius. Now, consider the other extreme: traveling in a straight line. By definition, straight line motion cannot have an acceleration that causes a change in direction, and therefore the centripetal acceleration is zero. The equation above bears this out: a straight line is a circle with infinite radius. So if $r = \infty$, then $a_C = 0$. These cases are shown below:

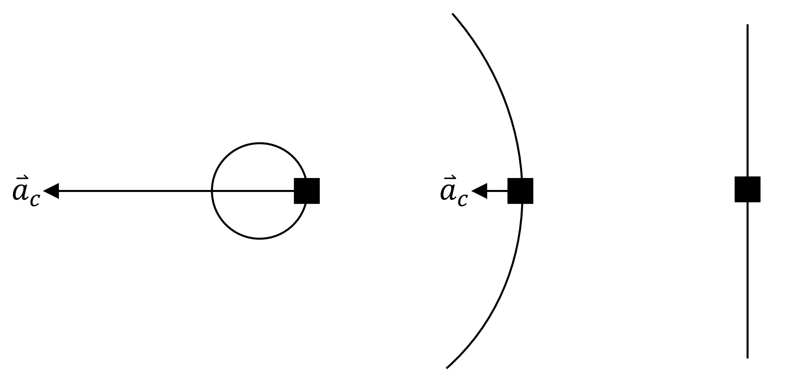
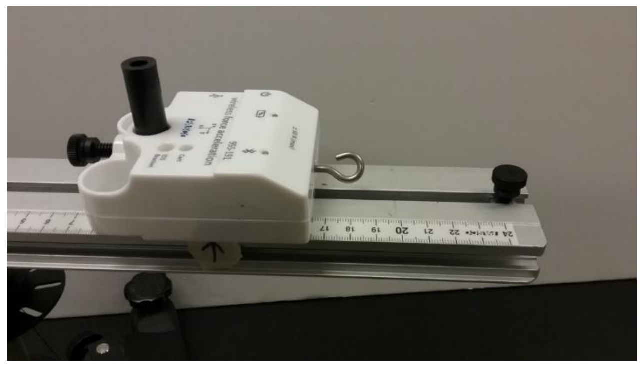


Figure 3: The objects in the above figure have the same speed. The object traveling on a circle with a small radius (left) must have a large acceleration. Conversely, the object traveling on a very large circle (center) is only barely turning and therefore only requires a small acceleration. Lastly, the object at right travels on a straight line and consequently has no centripetal acceleration.

Consequently, the centripetal acceleration has an inverse relationship (i.e. $a_C \propto 1/r$) with the radius of motion.

Second, consider the dependence on speed. It stands to reason that an object that is traveling quickly around a circle would require a large acceleration in order to change its direction quickly enough to keep it on its curved path. A slower-moving object, however, would only require a smaller acceleration. Our previous example demonstrates this quite nicely: it is safer to go around a hairpin turn at low speed than to go around the same hairpin turn at high speed because the lower speed will require a much smaller centripetal acceleration. It turns out that centripetal acceleration is proportional to the square of the speed of the object (i.e. $a_C \propto v^2$).

For this lab, the motion we will be looking at is that of a rotating platform. As such, while the motion is circular motion, it is not *uniform* circular motion. This means that the speed is constantly changing, as the platform slows down. Since the speed is constantly changing, so will the centripetal acceleration. But, no matter what the speed, the centripetal acceleration will still be v^2/r. This situation is a nice feature of this lab: we are able to check that the centripetal acceleration is v^2/r for many values of v, and not just for the one value of v that uniform circular motion would give us.



Centripetal Force

Kinematics is the branch of physics that *describes* motion. As such, acceleration is a kinematic quantity because it describes whether the object is speeding up, slowing down, or changing direction. Therefore, in our preceding discussion of centripetal acceleration, we really just looked at the kinematic description (i.e. how objects move) of circular motion, but nowhere did we mention what *causes* the acceleration (i.e. why objects move). The branch of physics that talks about the *causes* of motion is *dynamics*. And according to dynamics, the cause of motion is *force*.

Then, what causes an object to exhibit centripetal acceleration? The answer is that a *centripetal force* causes a centripetal acceleration. So, centripetal force is the force required to keep an object moving on a circular path. By Newton's second law, centripetal force is mass times centripetal acceleration ($F_c = ma_c$). Therefore, centripetal force has a magnitude of:

$$F_C = m\frac{v^2}{r} \qquad \text{(Magnitude of Centripetal Force)}$$

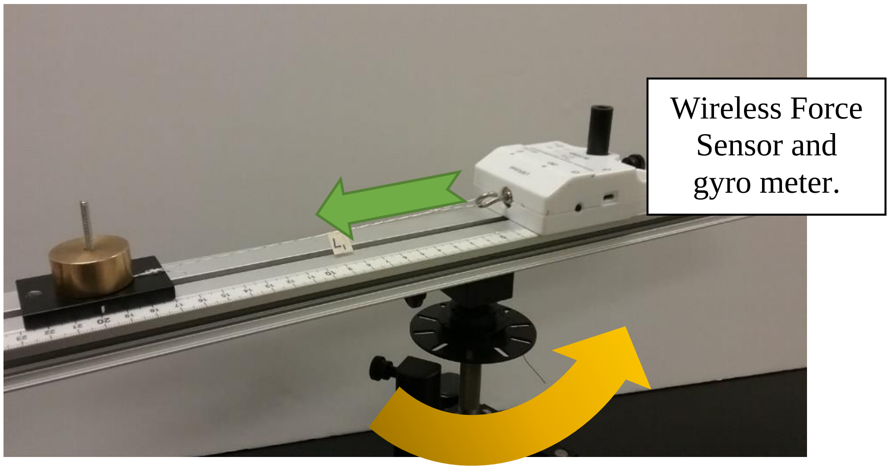
And since centripetal acceleration always points inward on the circle, so does centripetal force.

One thing to keep in mind is that centripetal force is not a force in the same way gravity, normal forces, friction, or tension are forces. A nice analogy is that gravity, normal forces, friction, tension, etc. are "actors" that play the "role" of centripetal force. More precisely, after drawing a force diagram for the object in question, the force (or combination of forces) that acts as the centripetal force can be identified from the actual forces (like friction or whatever) acting on the object. The centripetal force is readily identified as the net inward force that points toward the center of the circle:

$$\boxed{F_C = F_{inward} - F_{outward}}$$

In the case of the rotating platform we will be examining in this lab, the centripetal force is a combination of the tension in the string and the radial component of the weight of the mass.

There is one last important note: you may hear the term *centrifugal force* referring to an outward force. It is important to emphasize that *there is no such thing as a centrifugal force*. A centrifugal force, similar to centrifugal acceleration, is a fictional force that arises when an observer, who is moving on a circle, mistakes the effects of his or her own inertia for a real force. Again, your inertia makes it "feel" like you are moving outward, even though the true acceleration and force point inward. This "feeling" is confusing and is the source of the term "centrifugal force". Now, this is not to say that some of the forces that make up a centripetal force can't point outward (they can, but they would not be called "centrifugal forces"). It is simply that for a centripetal force, the inward pointing components of forces exceed the outward pointing components of forces by the exact amount mv^2/r needed to keep the object moving on a circle. In this lab, you will measure the centripetal force F_C and check to see whether it does in fact have a magnitude of mv^2/r.



Force sensor is riding on a "sled" with little friction. The sensor and the sled act as the spinning mass. The radius is changed by moving the pin with which the string in looped around. *Note:* the radius is measured from the center "0" mark on the rotating platform and **NOT** the length of the string.

Use the mark on the sled minus the "offset" value given in the lab procedure to record the radius.

Relationship between Linear and Angular Variables

For any object that moves on a circular path or even a section of a circular path, there exists a simple relationship between the angle the object sweeps through (θ) and the distance the object travels (d).

Consider the wheel of radius r shown in the diagram below, which is rotating in the clockwise direction:

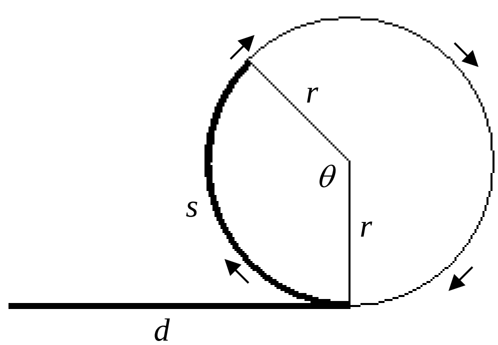


Figure 4: A wheel rotating in the clockwise direction along a horizontal surface.

For purposes of illustration, assume that the edge of the wheel has wet paint on it, and that wet paint covers a length s of edge of the wheel and subtends an angle θ as measured from the center of the wheel (see the figure above). If we allow the wheel to roll through this angle θ, then the wheel will leave a leave a trail of paint on the surface of length d. Further, if the wheel rolls without slipping along the surface, the distance travelled by the wheel d will be the same length as the painted section of the wheel s.

It should be clear from the diagram above that if the wheel rolls through exactly one revolution without slipping, the wheel will have moved a distance equal to its circumference, (that is $d = 2\pi r$). And, if the wheel rolls through half a revolution, it will travel a distance d equal to half of its circumference ($d = \frac{1}{2}(2\pi r) = \pi r$). Finally, if the wheel does not roll at all, then both s and d will both be zero. In fact, for <u>any</u> fraction of a revolution f (f can be bigger than 1 too!) that the wheel rolls through, it will move a distance equal to:

$$d = (f \cdot 2\pi)r$$

This realization allows us to define a new unit of angular measure, called the *radian*, which is more natural to use than the degree. The following table shows how an angle in radians translates to both degrees and revolutions:

Fraction of a Revolution f	θ (in revolutions)	θ (in degrees)	θ (in radians)
0	0	0^O	$0 \cdot 2\pi = 0$
1/12	1/12	30^O	$\frac{1}{12} \cdot 2\pi = \pi/6$
1/8	1/8	45^O	$\frac{1}{8} \cdot 2\pi = \pi/4$
1/6	1/6	60^O	$\frac{1}{6} \cdot 2\pi = \pi/3$
1/4	¼	90^O	$\frac{1}{4} \cdot 2\pi = \pi/2$
1/2	½	180^O	$\frac{1}{2} \cdot 2\pi = \pi$
3/4	¾	270^O	$\frac{3}{4} \cdot 2\pi = 3\pi/2$
1	1	360^O	$\frac{1}{1} \cdot 2\pi = 2\pi$
2	2	720^O	$2 \cdot 2\pi = 4\pi$

<u>Table 1:</u> Angle θ measured in revolutions, degrees and radians.

Returning to our wheel example, if we know the angle θ the wheel rotates through, we can quickly calculate the distance d the wheel moves. If we know θ in revolutions, the distance travelled by the wheel is:

$$d = 2\pi\theta_{rev}r \qquad \text{(revolution measure)}$$

If we know θ in degrees, the distance travelled by the wheel becomes:

$$d = \frac{2\pi}{360^O}\theta_{deg}r \qquad \text{(degree measure)}$$

But, the equation for distance travelled is simplest when measured in radians:

$$d = \theta_{rad}r \qquad \text{(radian measure, Equation 1)}$$

With this Equation 1, we can also determine the linear speed of the wheel. Assume the wheel rotates through an angle θ (again measured in radians) and moves a corresponding distance d in some known time Δt. We divide both sides of the Equation 1 above by Δt:

$$\frac{d}{\Delta t} = \frac{\theta_{rad}}{\Delta t}r \qquad \text{(Equation 2)}$$

On the left hand side of Equation 2, we have the distance travelled by the wheel divided by the time. This is exactly the linear speed of the wheel. So:

$$v = \frac{d}{\Delta t}$$

On the right hand side of the Equation 2 is the angle the wheel rotates through divided by the time it takes to do so. This new quantity of angle per time is called the *angular speed*. Angular speed is measured in radians/second and denoted by the Greek lower case omega ω.

$$\omega = \frac{\theta_{rad}}{\Delta t}$$

So, substituting v and ω into Equation 2, we have the final relationship between linear speed and angular speed:

$$v = \omega r \qquad \text{(Equation 3)}$$

The table below summarizes these important relationships:

Angular Quantity	Linear Quantity	Relationship
θ (Angle in radians)	d (Distance in meters)	$d = r\theta$
ω (Angular speed in rad/s)	v (Speed in m/s)	$v = r\omega$

Table 2: Relationship between linear and angular quantities.

It is important to notice that linear quantity is related to the angular quantity by the radius r of the wheel. The reason is simple: If you double the radius of the wheel, then it will travel twice the distance for the same rotation θ, because its circumference is twice as big. Similarly, if two wheels have the same angular speed ω, the bigger wheel will travel faster because of the larger radius (and circumference).

These equations hold for any object moving on a circular path. This includes the circular path of the rotating arm you will examine in this lab. We can rewrite the results we found earlier for centripetal acceleration and centripetal force in terms of this new (more useful) angular form.

$$v = \omega r \qquad \text{(Equation 3)}$$

Therefore:

$$a_C = \frac{v^2}{r}$$

Becomes

$$\boxed{a_C = r\,\omega^2}$$

$$F_C = m\frac{v^2}{r}$$

Becomes

$$\boxed{F_C = (mr)\,\omega^2}$$