Impulse and Momentum

PES 116 Advanced Physics Lab I

Purpose of the experiment

- Measure a cart's momentum change and compare to the impulse it receives.
- Compare average and peak forces in impulses.
- To put the impulse-momentum theorem to an experimental test.
- [FYI]

[FYI] The Main Library at Indiana University sinks over an inch every year because when it was built, engineers failed to take into account the weight of all the books that would occupy the building.

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Equipment List

- LabPro Interface
- Force Sensor with hook
- Motion Sensor
- 295 g calibration mass
- String
- Collision cart
- Dynamics track (with leveling foot)
- C-clamp
- 2 rubber bands (1 single band, 1 double band) and paper clip
- String
- Level
- Small Ring stand
- Ring stand clamp

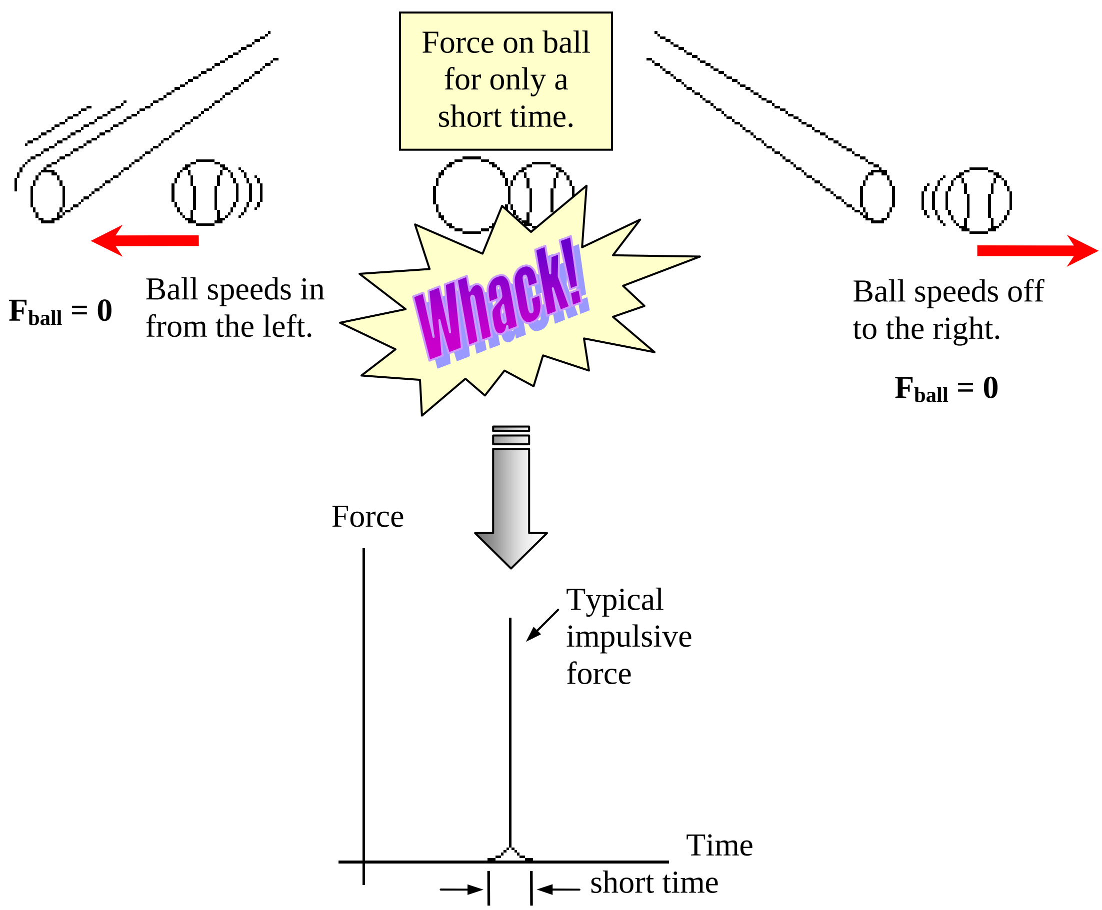
Background

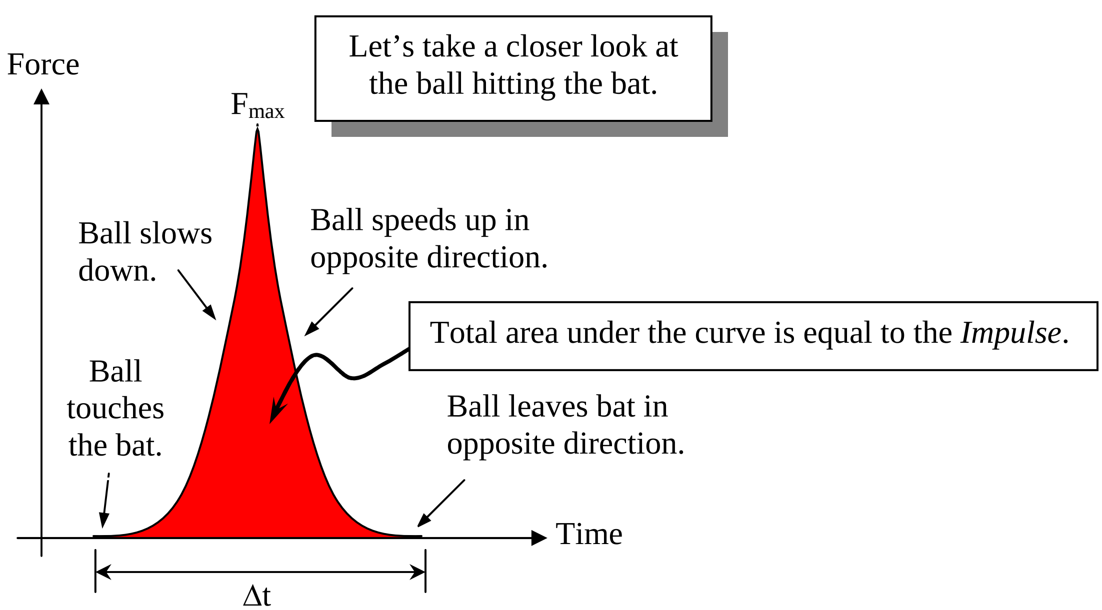
In a previous experiment we studied the momentum of objects before and after a collision. We however neglected to study the collision itself. We shall remedy that right now.

All the forces that you have studied to this point have either been steady (static) or have changed slowly in a predictable manner. An impulsive force acts just as its name implies, it will come and go very quickly (typically much less than a second) and will not remain constant.

Imagine hitting a ball with a bat. The ball has some given momentum and direction before being hit. After the ball has been hit, it will have some new momentum and direction. To get the ball to change in this way, a force had to be applied. The force comes from contact with the bat. If you have ever hit a ball or at least seen one hit (pretty standard for any blue-blooded American), you know that the ball will be in contact with the bat for only a very short time. This is the only time when any force can be applied which can change the ball's direction by 180°. Therefore, the force will not be constant or last for a *long* time.

Hitting a Ball with a Bat





From this discussion you should see that we need to hunt for an expression that relates the amount of force applied to the change in the momentum.

Momentum is a fairly intuitive concept. Momentum is simply the product of an object's mass times its velocity: $\vec{p} = m\vec{v}$

Because of this dependence on mass and velocity, a slow moving tractor-trailer can have as much momentum as a less massive, fast moving sports car.

If we take a look at Newton's 2nd Law of motion, we can see that force and momentum are simply related:

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

By multiplying both sides of this equation by *dt*, we get the following expression:

$$\vec{F}dt = d\vec{p}$$

Then, upon integrating both sides from the initial state to the final state, we have:

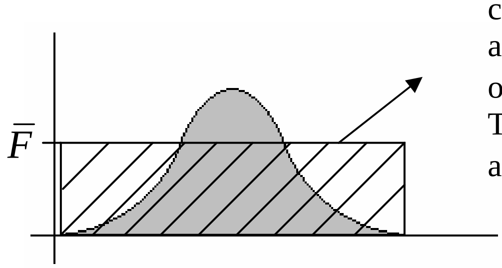
$$\boxed{\int_{t_1}^{t_2} \vec{F}dt = \Delta p}$$

We define the left hand side of this last equation as the *impulse* $\vec{J}$. So:

$\boxed{\vec{J} = \Delta\vec{p}}$ (Impulse-Momentum Theorem)

For those who may not know calculus:

There is a good approximation to ∫ F dt (which is just the area under the curve).



The area of this rectangle (for which calculating the area is very easy!) has a width of the same time as the original curve, but a height of $\overline{F}$. The result is an easy way to find the area if you know $\overline{F}$:

$$\int F\,dt = \overline{F}\,\Delta t$$

The impulse-momentum theorem becomes, the average force applied to an object times the length of time the force is applied is equal to the change in momentum of the object:

$$\boxed{\overline{F}\Delta t = mv_f - mv_O}$$

(At this time we will only consider motion and forces along a single direction, so I will drop the vector signs.)

Recap of Mechanics

Let us pause for a moment and look back on several ideas from mechanics. We have three different but equivalent ways of explaining why objects move. First, we have Newton's 2nd Law:

$$\vec{F} = m\vec{a}$$

which states that an object accelerates because a force acts on it. Second, we have the work-energy theorem:

$$W = \Delta K$$

which tells us that an object changes its kinetic energy (and therefore accelerates) because energy is transferred to the object in the form of work. Lastly, we have the impulse-momentum theorem:

$$\vec{J} = \Delta\vec{p}$$

which says that an object changes its momentum (and therefore accelerates) because momentum is transferred to the object in the form of an impulse.

It is also worthy to note the similarities between the work-energy theorem and the impulse momentum theorem:

$$W = \Delta K \quad \Rightarrow \quad F\Delta x = \tfrac{1}{2}\, m\, \Delta v^2 \qquad \text{(Work-Energy Theorem)}$$

$$\vec{J} = \Delta\vec{p} \quad \Rightarrow \quad \overline{F}\, \Delta t = m\, \Delta v \qquad \text{(Impulse-Momentum Theorem)}$$

where, work is force acting over a distance:

$$W = F\Delta x$$

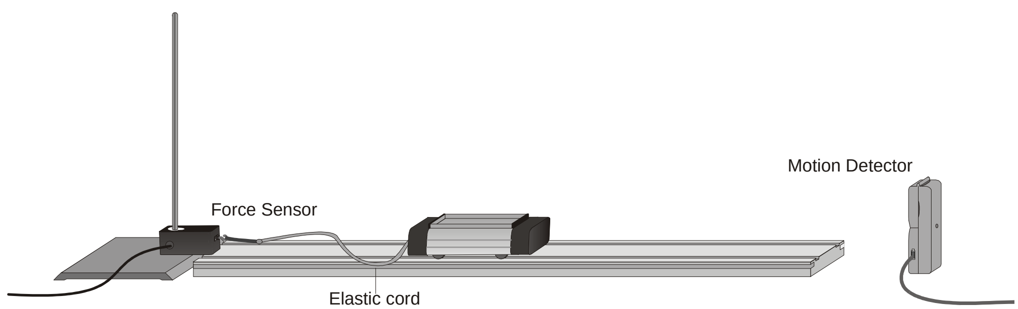
and impulse is force acting over time:

$$\vec{J} = \vec{\overline{F}}\Delta t$$

The Lab

The goal: Measure a cart's momentum change and compare to the impulse it receives. Put the impulse-momentum theorem to the test.

1. For this experiment we will test the impulse-momentum theorem using a dynamics cart rolling along a track. Its momentum will change as it reaches the end of an initially slack elastic tether, much like a horizontal bungee jump. The tether will stretch and apply an increasing force until the cart stops. The cart then changes direction and the tether will soon go slack. A Force Sensor measures the force applied by the cord. The cart's velocity throughout the motion is measured with a Motion Detector.



<u>Figure:</u> Experimental Setup

2. Record the total mass of your dynamics cart.
3. Connect the Motion Detector to DIG/SONIC 1 of the LabPro Interface. Connect the Force Sensor to CH 1 of the box. Set the Force Sensor to the 10 N range. Set the Motion Detector to the "Track" setting.
4. Open the file *PES 116/Impulse and Momentum/Impulse and Momentum.cmbl*. Logger *Pro* will plot the cart's position and velocity *vs*. time, as well as the force applied by the Force Sensor *vs*. time. Data are collected for 5 s.
5. Calibrate the Force Sensor.
6. Place the track on a level surface. Confirm that the track is level by using the level provided. If necessary, adjust the track.

7. Attach the single (not double) rubber band to the cart and then the rubber band to the string. Attach the string to the Force Sensor a short distance away. Clamp the Force Sensor to the ring stand (using the *thumb screw*) so that the string and cord, when taut, are horizontal and in line with the cart's motion. Make sure the cart can roll freely with the cord slack for most of the track length, but be stopped by the cord before it reaches the end of the track. Clamp the ring stand to the table.
8. Place the Motion Detector at the other end of the track so that the detector has a clear view of the cart's motion along the entire track length. When the cord is stretched to maximum extension the cart should not be closer than 20 cm to the detector.
9. Click Zero, then Zero Force, to zero the Force Sensor.
10. Practice releasing the cart so it rolls toward the Motion Detector, bounces gently, and returns to your hand. The Force Sensor must not shift and the cart must stay on the track. Arrange the cord and string so that when they are slack they do not interfere with the cart motion. You may need to guide the string by hand, but be sure that you do not apply any force to the cart or Force Sensor. Keep your hands away from area between the cart and the Motion Detector.
11. Click Collect to take data; roll the cart and confirm that the Motion Detector detects the cart throughout its travel. Inspect the force data. If the peak is flattened, then the applied force is too large. Roll the cart with a lower initial speed. If the velocity graph has a flat area when it crosses the x-axis, the Motion Detector was too close and the run should be repeated.
12. Once you have made a run with good distance, velocity, and force graphs, analyze your data. To test the impulse-momentum theorem, you need the velocity before and after the impulse. Choose a time interval just before the bounce when the speed was approximately constant, and drag the mouse pointer across the interval. Click the Statistics button, , and read the average velocity. Record the value in your data table. In the same manner, determine the average velocity just after the bounce and record the value in your data table.

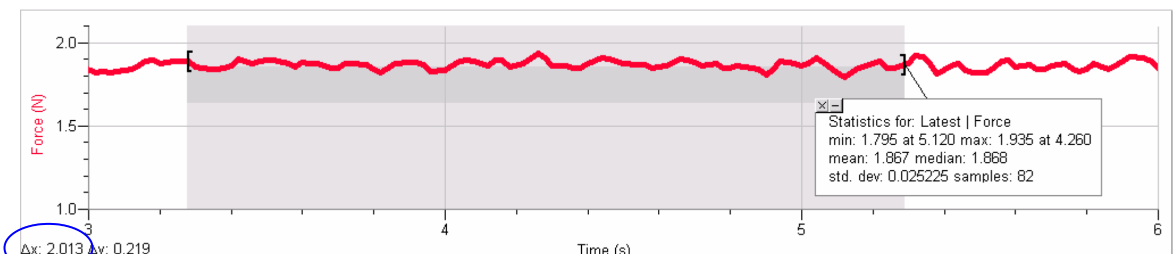
13. Now determine and record the impulse. There are two ways to do this. Do both.

<u>Method 1:</u> Calculus tells us that the expression for the impulse is equivalent to the integral of the force vs. time graph, or

$$\overline{F}\Delta t = \int_{t_{initial}}^{t_{final}} F(t)dt$$

On the force *vs*. time graph, drag across the impulse, capturing the entire period when the force was non-zero. Find the area under the force *vs.* time graph by clicking the integral button, . Record the value of the integral in the impulse column of your data table.

<u>Method 2:</u> On the force vs. time graph, drag across the impulse, capturing the entire period when the force was non-zero. Find the average value of the force by clicking the Statistics button, , and also read the length of the time interval over which your average force is calculated. Use the Examine button, to find the time interval Δt. Record the values in your data table. Further, record the maximum force under 'Peak Force' in your data table for later comparison.



Notice the value for Δx for the grayed-out area is the same as Δt.

14. Print your graph(s).
15. Perform a second identical experiment to verify your results. Record the information in your data table.
16. Attach the double rubber band to the cart. Repeat the experiments and record the information in your data table.
17. Attach the string alone to the cart. Repeat the experiments and record the data.

Additional Questions

1. Calculate the changes in velocity and record them in the data table. From the mass of the cart and change in velocity, determine the change in momentum as a result of the impulse. Make this calculation for each trial and enter the values in the data table below:

Method 1   Method 2

Trial	Impulse J (Area under curve) (N·s)	Impulse $\overline{F}\Delta t$ (N·s)	Change in momentum Δp (kg·m/s) or (N·s)	% difference between J and Δp (N·s)	% difference between $\overline{F}\Delta t$ and Δp (N·s)
Elastic 1					
1					
2					
Elastic 2					
1					
2					
String					
1					
2					

2. For Method 1, transfer your impulse (J) values from your data table into the table above. For Method 2, determine the impulse for each trial from the average force and time interval values. Record these values in your data table.

3. If the impulse-momentum theorem is correct, the change in momentum will equal the impulse for each trial. Experimental measurement errors, along with friction and shifting of the track or Force Sensor, will keep the two from being exactly the same. One way to compare the two is to find their percent difference. Do this for $\overline{F}\Delta t$ and Δp, and J and Δp. Does the data support the impulse-momentum theorem?

4. When you use different rubber bands, what changes occurred in the shapes of the graphs? Is there a correlation between the rubber band used and the shape of the graph?

5. When you used the double band as opposed to the single band, what effect did this have on the duration of the impulse? What affect did this have on the peak force? Can you develop a general rule from these observations?

6. How did the string change the shape of the graph? What happened to the peak force? What happened to the duration of the pulse? Would you rather bungee jump with an elastic cord or a rope? Support your answer with your graphs.

7. Look at the shape of the last force *vs*. time graph (string only). Is the peak value of the force significantly different from the average force? Could you create a situation that would deliver the same amount of impulse with a much smaller force? Explain.