

CHAPTER 2

DESCRIBING MOTION: KINEMATICS IN ONE DIMENSION

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- state from memory the meaning of the key terms and phrases used in kinematics.
- list the SI unit and its abbreviation associated with displacement, velocity, acceleration, and time.
- describe the motion of an object relative to a particular frame of reference.
- differentiate between a vector quantity and a scalar quantity and state which quantities used in kinematics are vector quantities and which are scalar quantities.
- state from memory the meaning of the symbols used in kinematics: x , x_0 , v , v_0 , a , y , y_0 , v_y , v_{y0} , g , t .
- write from memory the equations used to describe uniformly accelerated motion.
- complete a data table using information both given and implied in word problems.
- use the completed data table to solve word problems.
- use the methods of graphical analysis to determine the instantaneous acceleration at a point in time and the distance traveled in an interval of time.

KEY TERMS AND PHRASES

kinematics is the study of the motion of objects and involves the study of distance, speed, acceleration, and time.

average speed of an object is determined by dividing the distance that the object travels by the time required to travel that distance.

instantaneous speed is the speed of an object at a particular point in time.

acceleration is the rate of change of speed in time.

average acceleration is the change of velocity divided by the time required for the change.

instantaneous acceleration is the change of speed that occurs in a very small interval of time.

uniformly accelerated motion occurs when the rate of acceleration does not change, i.e., the rate of acceleration is constant.

free fall occurs when air resistance on an object is negligible and the only force acting on it is gravity.

gravitational acceleration for all objects in free fall is approximately 9.8 meters per second per second or 9.8 m/s^2 .

vector is a quantity that has both magnitude and direction. Examples of vector quantities are velocity, acceleration, displacement, and force.

scalar is a quantity that has magnitude but has no direction associated with it. Examples of scalar quantities include speed, distance, mass, and time.

SUMMARY OF MATHEMATICAL FORMULAS

average acceleration	$\bar{a} = \Delta v / \Delta t$	The average acceleration equals the change in speed divided by the change in time.
kinematics equations for uniformly accelerated motion	$v = v_0 + a t$ $x = x_0 + v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^2 + 2 a (x - x_0)$ $x - x_0 = \bar{v} t$ $\bar{v} = \frac{1}{2}(v + v_0)$	<p>Speed as related to initial speed, acceleration and time.</p> <p>Distance as related to initial distance, initial speed, acceleration, and time.</p> <p>Speed as related to initial speed, acceleration, and distance.</p> <p>Distance traveled equals the product of the average speed and the time.</p> <p>Average speed as related to the initial speed and final speed.</p>

CONCEPT SUMMARY

Kinematics

Kinematics is the study of the motion of objects and involves the study of the following concepts: distance, speed, acceleration, and time. This chapter is restricted to the motion of an object along a straight line. This is known as one-dimensional or linear motion.

The **average speed** of an object is determined by dividing the distance that the object travels by the time required to travel that distance. The **instantaneous speed** refers to the speed of an object at a particular point in time. In this study guide the SI system of units will be used. The SI unit of speed is meters per second (m/s) or kilometers per hour (km/h). SI is an abbreviation of the French words *Système International*. This system was formerly referred to as the MKS (meter-kilogram-second) system.

Acceleration refers to the rate of change of speed in time. The SI unit of acceleration is meters per second per second or m/s^2 . The **average acceleration** is defined as the change of velocity divided by the time required for the change. The **instantaneous acceleration** refers to

the change of speed that occurs in a very small interval of time ($\Delta t \rightarrow 0$).

Uniformly Accelerated Motion

Uniformly accelerated motion occurs when the rate of acceleration does not change, i.e. the rate of acceleration is constant. The following equations apply to this type of motion:

$$v = v_0 + a t$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2 a (x - x_0)$$

$$x - x_0 = \bar{v} t \quad \text{where } \bar{v} = \frac{1}{2}(v + v_0)$$

v_0 = initial speed of the object (at $t = 0$ s)

v = speed of the object after time t

a = rate of acceleration

x = position of the object after time t

\bar{v} = average speed

x_0 = initial position of the object

Note: $x_0 = 0$ unless otherwise specified.

t = time interval during which the motion has occurred; unless otherwise specified, the time at the start of the motion will be zero seconds.

EXAMPLE PROBLEM 1. A car accelerates from 10.0 m/s to a speed of 30.0 m/s in 10.0 s. If the rate of acceleration is uniform, determine the a) rate of acceleration and b) distance traveled during the 10.0 s of motion.

Part a. Step 1. List each symbol and complete a data table based on the information given in the problem.	Solution: (Sections 2-5 and 2-6) $v_0 = 10.0 \text{ m/s}$ $t = 10.0 \text{ s}$ $v = 30.0 \text{ m/s}$ $a = ?$ $x = ?$ $x_0 = 0$
Part a. Step 2. Determine the rate of acceleration.	$v = v_0 + a t$ $30 \text{ m/s} = 10.0 \text{ m/s} + a (10.0 \text{ s})$ $a = 2.0 \text{ m/s}^2$
Part b. Step 1. Determine the distance traveled in 10.0 seconds.	$x = x_0 + v_0 t + \frac{1}{2} a t^2$ $x = 0 \text{ m} + (10.0 \text{ m/s})(10.0 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s}^2)(10.0 \text{ s})^2$ $x = 200 \text{ m}$

EXAMPLE PROBLEM 2. A car is traveling at the posted speed limit of 15.0 miles per hour (6.70 m/s) in a school zone on dry pavement. The driver applies maximum braking force and the car decelerates at a constant rate of 7.92 m/s^2 until coming to a halt. a) Calculate the distance the car travels while decelerating. b) Calculate the distance required to stop if the car is initially traveling at 30.0 miles per hour (13.4 m/s).

Part a. Step 1.	Solution: (Sections 2-5 and 2-6)
Complete a data table for the deceleration.	$x = ?$ $x_0 = 0 \text{ m}$ $a = -7.92 \text{ m/s}^2$ $v_0 = 6.70 \text{ m/s}$ $v = 0 \text{ m/s}$ (car comes to a halt) $t = ?$
Part a. Step 2.	$2 a (x - x_0) = v^2 - v_0^2$
Calculate the distance the car would travel during the deceleration.	$2 (-7.92 \text{ m/s}^2)(x - x_0) = (0 \text{ m/s})^2 - (6.70 \text{ m/s})^2$ $(-15.8 \text{ m/s}^2)(x - x_0) = -44.9 \text{ m}^2/\text{s}^2$ $x - x_0 = 2.83 \text{ m}$ or 9.30 feet
Part b. Step 1.	$x = ?$ $x_0 = 0 \text{ m}$ $a = -7.92 \text{ m/s}^2$ $v_0 = 13.4 \text{ m/s}$ $v = 0 \text{ m/s}$ (car comes to a halt) $t = ?$
Part b. Step 2.	$2 a (x - x_0) = v^2 - v_0^2$
Calculate the distance the car would travel during the deceleration.	$2 (-7.92 \text{ m/s}^2)(x - x_0) = (0 \text{ m/s})^2 - (13.4 \text{ m/s})^2$ $(-15.8 \text{ m/s}^2)(x - x_0) = -180 \text{ m}^2/\text{s}^2$ $x - x_0 = 11.3 \text{ m}$ or 37.4 feet At 30.0 mph, the car requires approximately four times more distance to come to a complete stop as compared to 15.0 mph. Therefore, in order to protect children, a low speed limit is set when traveling through a school zone. Using the same method it can be shown that at 60 miles per hour the stopping distance is 16 times further than at 15 mph, i.e., 149 feet. On wet pavement the stopping distance is even greater.

EXAMPLE PROBLEM 3. A car traveling at a constant speed of 15.0 m/s (approximately 34 miles per hour) in a zone where the posted speed limit is 25.0 miles per hour. As the motorist passes a stationary police car, the police car accelerates at a constant rate of 3.00 m/s² and maintains this rate of acceleration until the police car pulls next to the speeding car. Determine the (a) time required for the police officer to catch the speeder and (b) distance traveled during the chase. Hint: In order to catch the speeder the police car must exceed 15.0 m/s.

Part a. Step 1.	Solution: (Sections 2-5 and 2-6)								
Complete a data table for both vehicles using information both given and implied in the statement of the problem.	<table border="0"> <thead> <tr> <th>motorist's car</th> <th>police car</th> </tr> </thead> <tbody> <tr> <td>$v_0 = 15.0 \text{ m/s}$</td> <td>$v = 0 \text{ m/s}$</td> </tr> <tr> <td>$v = 15.0 \text{ m/s}$</td> <td>$v = ?$</td> </tr> <tr> <td>$a = 0 \text{ m/s}$</td> <td>$a = 3.00 \text{ m/s}^2$</td> </tr> </tbody> </table>	motorist's car	police car	$v_0 = 15.0 \text{ m/s}$	$v = 0 \text{ m/s}$	$v = 15.0 \text{ m/s}$	$v = ?$	$a = 0 \text{ m/s}$	$a = 3.00 \text{ m/s}^2$
motorist's car	police car								
$v_0 = 15.0 \text{ m/s}$	$v = 0 \text{ m/s}$								
$v = 15.0 \text{ m/s}$	$v = ?$								
$a = 0 \text{ m/s}$	$a = 3.00 \text{ m/s}^2$								

	$t = ?$ $t = ?$ $x = ?$ $x = ?$ $x_o = 0 \text{ m}$ $x_o = 0 \text{ m}$
<p>Part a. Step 2.</p> <p>Write an equation for the motorist's car as a function of time.</p>	<p style="text-align: center;">motorist</p> $x - x_o = \bar{v} t$ but $x_o = 0$, therefore $x = \frac{1}{2}(15.0 \text{ m/s} + 15.0 \text{ m/s})t$ and $x = (15.0 \text{ m/s})t$
<p>Part a. Step 3.</p> <p>Write an equation for the position of the police car as a function of time.</p>	<p style="text-align: center;">police car</p> $x = v_o t + \frac{1}{2} a t^2 + x_o$ $x = (0 \text{ m/s}) t + \frac{1}{2} (3.00 \text{ m/s}^2) t^2 + 0 \text{ m}$ $x = (1.50 \text{ m/s}^2) t^2$
<p>Part a. Step 4.</p> <p>Determine the time required for the police to catch the motorist. Note: when the police car pulls alongside the motorist, it is traveling faster than the motorist's car. However, both cars have traveled the same distance.</p>	<p>Since $x_{\text{motorist}} = x_{\text{police car}}$</p> $(15.0 \text{ m/s}) t = (1.50 \text{ m/s}^2) t^2$ writing the expression as an algebraic equation and solving for t, $15.0 t = 1.50 t^2$ $0 = 1.5 t^2 - 15.0 t$ $0 = (t - 10.0)(1.50 t)$ Either $0 = t - 10.0$ or $0 = 1.50 t$ $t = 10.0 \text{ s}$ or $t = 0 \text{ s}$ The equation is a quadratic equation and two values are obtained for the time of motion. However, $t = 0 \text{ s}$ merely corresponds with our initial assumption that the two cars were at the same position at $t = 0$. The correct solution is $t = 10.0 \text{ s}$.
<p>Part b. Step 1.</p> <p>Determine the distance each vehicle travels during the time interval.</p>	<p>Add $t = 10.0 \text{ s}$ to the data table and solve for the distance traveled during the chase.</p> $x - x_o = \bar{v} t$ $x - 0 \text{ m} = (15.0 \text{ m/s})(10.0 \text{ s})$ $x = 150 \text{ m}$

Free Fall

One application of uniformly accelerated motion is to the problem of objects in **free fall**. When an object is in free fall, we assume that air resistance is negligible and that the only force acting on it is gravity. Assuming air resistance is negligible, the rate of acceleration (g) of all objects in free fall is approximately 9.8 meters per second per second or 9.8 m/s^2 .

The equations for uniformly accelerated motion can be applied to free fall. Since the motion is vertical, y replaces x and y_0 replaces x_0 . Also, v_y replaces v and v_{y0} replaces v_0 while g replaces the symbol a .

TEXTBOOK QUESTION 14. How would you estimate the maximum height you could throw a ball vertically upward? How would you estimate the maximum speed you could give it?

ANSWER: Suppose the total time for the ball to return to your hand is measured to be 4.0 s. Then the time to reach maximum height is $\frac{1}{2}(4.0 \text{ s}) = 2.0 \text{ s}$. Also, as the ball rises it is decelerating, i.e. $g = -9.8 \text{ m/s}^2$ and at maximum height its speed is zero, i.e., $v_y = 0$. The ball's initial speed can be calculated as follows:

$$v_y = g t + v_{oy}$$

$$0 = (-9.8 \text{ m/s}^2)(2.0 \text{ s}) + v_{oy} \quad \text{and} \quad v_{oy} = 19.6 \text{ m/s}$$

The maximum height reached by the ball can be determined as follows:

$$y = v_0 t + \frac{1}{2} a t^2$$

$$y = (19.6 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(2.0 \text{ s})^2$$

$$y = 19.6 \text{ m}$$

EXAMPLE PROBLEM 4. A stone is thrown vertically downward with an initial speed of 9.80 m/s from the top of a building 29.4 m high. Determine the a) velocity of the stone just before it strikes the ground and b) time that the stone is in the air.

Part a. Step 1.

Because the motion is downward, let the downward direction be positive. Complete a data table based on the information given.

Solution: (Section 2-7)

$v_0 = 9.80 \text{ m/s}$	$t = ?$
$v = ?$	$y = 29.4 \text{ m}$
$g = 9.80 \text{ m/s}^2$	$y_0 = 0 \text{ m}$

Part a. Step 2.

Determine the velocity of the stone just before it strikes the ground.

$$v^2 = v_0^2 + 2 g (y - y_0)$$

$$v^2 = (9.80 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(29.4 \text{ m} - 0 \text{ m})$$

$$v^2 = 96.0 \text{ m}^2/\text{s}^2 + 576 \text{ m}^2/\text{s}^2$$

Note: after it leaves the person's hand it is in free fall.

$$v^2 = 672 \text{ m}^2/\text{s}^2$$

$$v = 25.9 \text{ m/s}$$

Part b. Step 1.

Determine the time that the stone is in the air.

The velocity of the stone just before it strikes the ground can now be included in the data table.

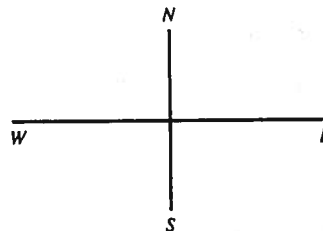
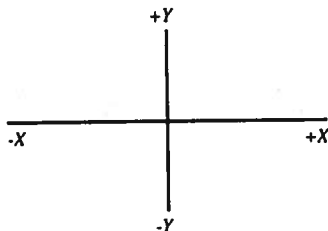
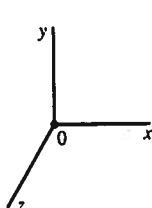
$$v = v_0 + a t$$

$$25.9 \text{ m/s} = 9.80 \text{ m/s} + (9.80 \text{ m/s}^2) t$$

$$t = 1.64 \text{ s}$$

Frames of Reference

The description of motion of any object must always be given relative to a **frame of reference** or **reference frame**. The reference frame is usually specified by using **Cartesian Coordinates**. The x, y, and z axes shown in the figures below can be used to locate the position of an object with respect to a fixed point o. At times, the x and y directions will be used to represent the direction of cardinal points: north (N), south (S), east (E), and west (W), with "up" above the plane of the paper and "down" below the plane of the paper. In certain problems, it will be convenient to use the x axis to represent the horizontal direction while the y axis represents the vertical direction.



Vectors and Scalars

A **vector** is a quantity that has both magnitude and direction. An example of a vector quantity is velocity. A car traveling at 20 m/s must be traveling in a specified direction, e.g., due north. Displacement, velocity, and acceleration are examples of vector quantities. A vector is designated by the symbol for the vector quantity in bold face type as in the textbook. For example, the velocity vector will be represented by a bold face \mathbf{v} with an arrow placed above it, i.e., $\vec{\mathbf{v}}$.

A **scalar** is a quantity that has magnitude but has no direction associated with it. A scalar is specified by giving its magnitude and units (if any). Speed, distance, mass, and time are examples of scalar quantities. As an example, speed refers to the magnitude of an object's motion but not the direction in which it is traveling.

TEXTBOOK QUESTION 1. Does a car speedometer measure speed, velocity, or both?

ANSWER: A car's speedometer indicates the magnitude of the car's motion. It does not indicate the car's direction. Since velocity is both magnitude and direction, the speedometer measures speed but does NOT measure velocity.

TEXTBOOK QUESTION 2. Can an object have varying speed if its velocity is constant? If yes, give examples.

ANSWER: Velocity is a vector quantity while speed is a scalar quantity. In order for the velocity to be constant both the magnitude, i. e. speed, as well as the direction must be constant. Therefore, if the speed changes then the velocity is not constant.

TEXTBOOK QUESTION 7. Can an object have a northward velocity and a southward acceleration? Explain.

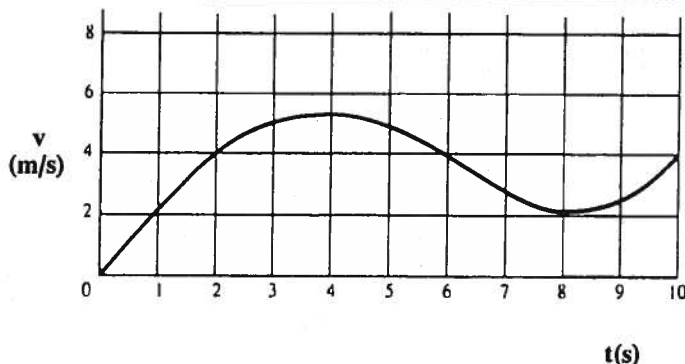
ANSWER: If the object is traveling northward, then the direction of the velocity vector is northward. If the object slows, but is still traveling northward, then the velocity vector is northward but the car is decelerating. Acceleration (or deceleration) is a vector quantity. If the object is accelerating, then the acceleration vector is in the same direction as the velocity vector. In the instance where the object is decelerating, the acceleration vector is directed opposite from the velocity vector. Therefore, for an object traveling northward but decelerating, the velocity vector is northward but the acceleration vector is southward.

Graphical Analysis

Graphs can be used to analyze the straight line motion of objects. Although **graphical analysis** can be used for uniformly accelerated motion, the method is especially useful when dealing with the motion of an object that is not undergoing uniform acceleration.

In a distance versus time graph the instantaneous velocity at any point can be determined from the slope of a tangent line drawn to the point in question. In a velocity versus time graph the slope of the tangent line represents the instantaneous acceleration while the area under the curve represents the distance traveled.

EXAMPLE PROBLEM 5. The graph shown below represents the motion of a car over a period of 10.0 s. Use graphical analysis to determine the a) rate of acceleration of the car at 2.0 s, 5.0 s, and 8.0 s and b) distance traveled during the 10.0 s of motion.



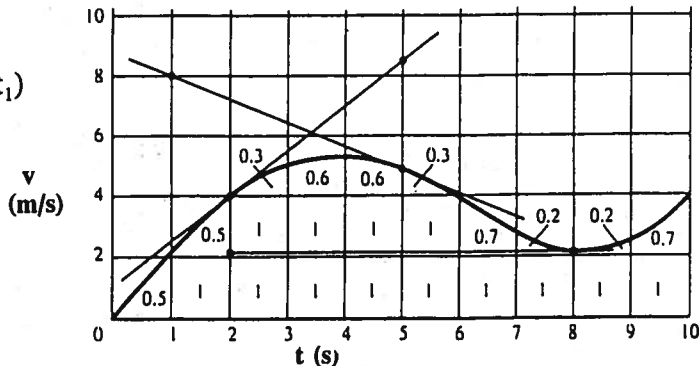
Part a. Step 1.

Determine the acceleration of the car at 2.0 s, 5.0 s, and 8.0 s.

Solution: (Section 2-8)

To determine the rate of acceleration, a tangent line is drawn at each point in question. Two data points are selected from each tangent line and the rate of acceleration is determined by using the following formula:

$$a = (v_2 - v_1)/(t_2 - t_1)$$



at $t = 2.0$ s: $a = (8.5 \text{ m/s} - 4.0 \text{ m/s})/(5.0 \text{ s} - 2.0 \text{ s}) = +1.5 \text{ m/s}^2$

at $t = 5.0$ s: $a = (8.0 \text{ m/s} - 5.0 \text{ m/s})/(1.0 \text{ s} - 5.0 \text{ s}) = -0.75 \text{ m/s}^2$

at $t = 8.0$ s: $a = (2.2 \text{ m/s} - 2.2 \text{ m/s})/(8.0 \text{ s} - 2.0 \text{ s}) = 0 \text{ m/s}^2$

There is a certain amount of judgment required in drawing the tangent lines. The actual value of the acceleration at each point may be different from the values obtained above. Your knowledge of algebra will help in checking whether or not the answers are reasonable. The tangent line at $t = 2.0$ s has a positive slope and therefore the car should be accelerating. The car's speed is increasing at this point; therefore, it is accelerating. At $t = 5.0$ s, the car is slowing down and is therefore decelerating. The slope of the tangent line at $t = 5.0$ s is negative and therefore agrees with observation. The slope of the tangent line at $t = 8.0$ s is zero. This indicates that the car is neither accelerating nor decelerating. Looking at the graph, we can see that the car has stopped slowing down and has not yet begun to increase in speed; thus its rate of acceleration is indeed zero.

Part b. Step 1.

Use graphical integration to determine the distance traveled during the 10.0 s motion.

The distance traveled can be determined by calculating the area under the graph. This can be done by determining the distance represented by the area of one block and then multiplying this value by the total number of full and partial blocks that lie between the curve and the time axis. Note: a major source of error in determining the distance traveled is the judgment required in estimating the value of a partial block. A more accurate value can be obtained by using graph paper that contains a fine grid.

2.0 m/s distance represented by one block = (2.0 m/s)(1.0 s)
 1.0 s distance = 2.0 m

sum of complete blocks = 13.0

sum of partial blocks = 0.5 + 0.5 + 0.3 + 0.6 + 0.6 + 0.3 + 0.7 + 0.2
 + 0.2 + 0.7 = 4.6

$$\text{total number of blocks} = 13.0 + 4.6 = 17.6$$

$$\text{total distance} = (17.6 \text{ blocks})(2.0 \text{ m/block}) = 35 \text{ m}$$

PROBLEM SOLVING SKILLS

For problems involving uniformly accelerated motion:

1. Obtain a mental picture by drawing a diagram that reflects the motion of the object in question. This is especially useful in the free fall problems where the initial motion may be vertically upward or downward.
2. Complete a data table using information both given and implied in the wording of the problem.
3. Use the proper sign for the quantity represented by the symbol in the data table. For example, if a car is slowing down, then the rate of acceleration is negative. If an object in free fall was initially thrown downward, then the downward direction is taken to be positive and both the initial velocity and the rate of acceleration are positive. If the object in free fall was given an initial upward motion, then the upward direction is taken to be positive. This means that the initial upward velocity is positive but the rate of acceleration is negative because it is slowing down as it travels upward.
4. Memorize the formulas for uniformly accelerated motion. It is also necessary to memorize the meaning of each symbol in each formula. Using the data from the completed data table, determine which formula or combination of formulas can be used to solve the problem.

For problems related to graphical analysis where velocity is a function of time:

1. Determine the area of one block. This area represents the distance represented by a single block.
2. Count the number of blocks in the time interval being considered. Multiply the total number of blocks by the distance represented by one block. The product is the total distance traveled during the time interval. This technique is known as graphical integration.
3. The instantaneous acceleration at a particular moment of time is determined as follows:
 - a) draw a tangent line to the graph at the point in question, and
 - b) determine the magnitude of the slope of the tangent line. The magnitude of the slope of the line represents the instantaneous value of the acceleration at that moment in time. If the slope is positive, the object is accelerating. If the slope is negative, the object is decelerating. If the slope is zero, the object is traveling at constant speed.

For problems related to graphical analysis where distance is a function of time:

1. The instantaneous velocity at a particular moment in time equals the slope of the tangent line drawn to the curve at the point in question. If the slope is positive, the object's speed is positive. If the slope is negative, the object's speed is negative. If the slope is zero, the object is not moving.

SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 9. A person jogs 8 complete laps around a quarter-mile track in a total time of 12.5 min. Calculate (a) the average speed and (b) the average velocity in m/s.

Part a. Step 1. Determine the total distance traveled in meters.	Solution: (Sections 2-1 through 2-3) $(8)(\frac{1}{4} \text{ mile})(1609 \text{ m/mile}) = 3220 \text{ m}$
Part a. Step 2. Determine the average speed.	average speed = total distance/total time $\bar{v} = (3220 \text{ m})/[(12.5 \text{ min})(60 \text{ s/1 min})] = 4.3 \text{ m/s}$
Part b. Step 1. Determine the runner's total displacement from the starting point.	While distance is a scalar quantity, displacement is a vector quantity. Assuming the runner ended up at the exact spot where he/she started, the runner's displacement is zero.
Part b. Step 2. Determine the runner's average velocity.	As stated in Part b. Step 1, the runner's displacement from the starting point is zero. The runner's average velocity equals the displacement/time. Therefore, the average velocity equals zero. $\bar{v} = \Delta x/\Delta t = (0 \text{ m})/[(12.5 \text{ min})(60 \text{ s/ min})] = 0 \text{ m/s}$

TEXTBOOK PROBLEM 18. At highway speeds, a particular automobile is capable of an acceleration of about 1.6 m/s^2 . At this rate, how long would it take to accelerate from 80 km/h to 110 km/h ?

Part a. Step 1. Convert km/h to m/s .	Solution: (Section 2-5 and 2-6) $v_o = (80 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 22.2 \text{ m/s}$ $v = (110 \text{ km/h})(1000 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 30.5 \text{ m/s}$
Part a. Step 2. Complete a data table.	$v_o = 22.2 \text{ m/s}$ $t = ?$ $a = 1.6 \text{ m/s}^2$ $v = 30.6 \text{ m/s}$ $x - x_o = ?$
Part a. Step 3. Select the appropriate equation and solve for the time.	$v = v_o + a t$ $30.5 \text{ m/s} = 22.2 \text{ m/s} + (1.6 \text{ m/s}^2) t$ $t = 5.2 \text{ s}$

TEXTBOOK PROBLEM 26. In coming to a stop, a car leaves a skid mark 92 m long on the highway. Assuming a deceleration of 7.00 m/s^2 , estimate the speed of the car just before braking.

Part a. Step 1. Complete a data table based on information both given and implied in the problem.	Solution: (Section 2-5 and 2-6) $v_o = ?$ $t = ?$ $a = - 7.00 \text{ m/s}^2$ $v = 0 \text{ m/s}$ (car comes to a halt) $x - x_o = 92 \text{ m}$
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Part a. Step 2.

Calculate the initial speed.

$$v^2 = v_0^2 + 2 a (x - x_0)$$

$$(0 \text{ m/s}^2) = v_0^2 + 2 (- 7.00 \text{ m/s}^2)(92 \text{ m})$$

$$v_0^2 = 1290 \text{ m}^2/\text{s}^2$$

$$v_0 = 36 \text{ m/s}$$

TEXTBOOK PROBLEM 32. A person driving her car at 45 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning red, and she is 28 m from the near side of the intersection (Fig. 2-31). Should she try to stop, or should she speed up to cross the intersection before the light turns red? The intersection is 15 m wide. Her car's maximum deceleration is -5.8 m/s^2 , whereas it can accelerate from 45 km/h to 65 km/h in 6.0 s. Ignore the length of her car and her reaction time.

Part a. Step 1.

Complete a data table if she stops. Convert the initial speed to m/s.

Solution: (Sections 2-5 and 2-6)

$$x = ? \quad x_0 = 0 \text{ m} \quad t = ?$$

$$a = - 5.8 \text{ m/s}^2 \quad v = 0 \text{ m/s (car comes to a halt)}$$

$$v_0 = (45 \text{ km/h})[(1000 \text{ m})/(1 \text{ km})][(1 \text{ h})/(3600 \text{ s})] = 12.5 \text{ m/s}$$

Part a. Step 2.

Calculate the distance the car would travel during the deceleration.

$$2 a (x - x_0) = v^2 - v_0^2$$

$$2 (- 5.8 \text{ m/s}^2)(x - x_0) = (0 \text{ m/s})^2 - (12.5 \text{ m/s})^2$$

$$x - x_0 = (-156 \text{ m}^2/\text{s}^2)/(- 11.6 \text{ m/s}^2) \approx 13 \text{ m}$$

The start of the intersection is 28 m from the car's initial position. Based on the calculation, the student could easily stop before reaching the intersection.

Part a. Step 3.

Determine the rate of acceleration if she decides to make it across before the light changes.

$$v_0 = (45 \text{ km/h})[(1000 \text{ m})/(1 \text{ km})][(1 \text{ h})/(3600 \text{ s})] = 12.5 \text{ m/s}$$

$$v = (65 \text{ km/h})[(1000 \text{ m})/(1 \text{ km})][(1 \text{ h})/(3600 \text{ s})] = 18.1 \text{ m/s}$$

$$v = a t + v_0$$

$$18.1 \text{ m/s} = a (6.0 \text{ s}) + 12.5 \text{ m/s}$$

$$a = 0.926 \text{ m/s}^2$$

Part a. Step 4.

Determine the distance that the accelerating car would travel in 2.0 s.

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$x - x_0 = (12.5 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2} (+0.926 \text{ m/s}^2)(2.0 \text{ s})^2$$

$$x - x_0 = 25 \text{ m} + 1.85 \text{ m} \approx 27 \text{ m}$$

The intersection is 28 m from her car at the start of the acceleration.

After 2.0 seconds, the car would be just about to enter the intersection traveling at high speed. This means that she would pass through the intersection against a red light. Therefore, she should adjust the rate of deceleration, stop, and wait for the next green light.

TEXTBOOK PROBLEM 36. A baseball is hit nearly straight up into the air with a speed of 22 m/s. (a) How high does it go? (b) How long is it in the air?

Part a. Step 1.

Complete a data table based on information both given and implied in the problem.

Solution: (Section 2-7)

$$v_0 = 22 \text{ m/s} \quad t = ? \quad a = -9.80 \text{ m/s}^2$$

$$v = 0 \text{ m/s (at top of motion)} \quad y_0 = 0 \text{ m} \quad y = ?$$

The initial direction of motion is upward, so let the ball's initial speed be positive. The ball decelerates as it rises; therefore, $a = -9.80 \text{ m/s}^2$. The ball's speed when it reaches maximum height above the ground is zero.

Part a. Step 2.

Use the appropriate equation to determine the maximum height reached by the ball.

$$2 a (y - y_0) = v^2 - v_0^2$$

$$2(-9.8 \text{ m/s}^2)(y - 0 \text{ m}) = (0 \text{ m/s})^2 - (22 \text{ m/s})^2$$

$$(-19.6 \text{ m/s}^2)(y - 0 \text{ m}) = -480 \text{ m}^2/\text{s}^2$$

$$y = 25 \text{ m}$$

Part b. Step 1.

Use the appropriate equation to determine the total time the ball is in the air.

The ball started at $y_0 = 0 \text{ m}$ and returns to the same position. Therefore, $y = 0 \text{ m}$.

$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$0 \text{ m} - 0 \text{ m} = (22 \text{ m/s}) t + \frac{1}{2} (-9.80 \text{ m/s}^2) t^2$$

$$0 \text{ m} = [(22 \text{ m/s}) + (-4.90 \text{ m/s}^2) t] t$$

Solving for the time gives $t = 0 \text{ s}$ or $t = 4.5 \text{ s}$

$t = 0 \text{ s}$ is the time at which it was struck by the bat. Therefore, the time in the air is 4.5 s.

TEXTBOOK PROBLEM 47. A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 70.0 m high (Fig. 2-34). (a) How much later does it reach the bottom of the cliff? (b) What is its speed just before hitting? (c) What total distance did it travel?

<p>Part a. Step 1.</p> <p>Complete a data table based on information both given and implied in the problem.</p>	<p>Solution: (Section 2-7)</p> $v_o = +12.0 \text{ m/s} \quad t = ? \quad a = -9.80 \text{ m/s}^2$ $v = ? \quad y = -70.0 \text{ m}$ <p>The initial direction of motion is upward, so let the stone's initial speed be positive. The stone decelerates as it rises, therefore, $a = -9.80 \text{ m/s}^2$. The stone falls to a point 70.0 m below its starting point; therefore $y = -70.0 \text{ m}$.</p>
<p>Part a. Step 2.</p> <p>Use the appropriate equation to determine the time of flight.</p>	$y = v_o t + \frac{1}{2} a t^2$ $-70.0 \text{ m} = (12.0 \text{ m/s}) t + \frac{1}{2}(-9.80 \text{ m/s}^2) t^2$ $0 = (-4.90 \text{ m/s}^2) t^2 + (12.0 \text{ m/s}) t + 70.0 \text{ m}$ <p>Using the quadratic formula, it can be shown that either</p> $t = 5.20 \text{ s} \quad \text{or} \quad t = -2.74 \text{ s}$ <p>Since time cannot be negative, the answer is $t = 5.20 \text{ s}$.</p>
<p>Part b. Step 1.</p> <p>The time of flight can now be added to the data table. Solve for the velocity of the stone just before it strikes the ground.</p>	$v = v_o + a t$ $v = 12.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.20 \text{ s})$ $v = -38.9 \text{ m/s}$ <p>Note: the negative value for the velocity indicates that the stone is traveling downward. As you recall, the upward direction was selected as the positive direction.</p>
<p>Part c. Step 1.</p> <p>Complete a data table based on information both given and implied in the problem. Then determine the height.</p>	$v_o = +12.0 \text{ m/s} \quad t = ? \quad a = -9.80 \text{ m/s}^2$ $v = 0 \quad y = ?$ <p>The initial direction of motion is upward, so let the stone's initial speed be positive. The stone decelerates as it rises; therefore, $a = -9.80 \text{ m/s}^2$. At maximum height the stone momentarily comes to a halt, i.e., $v = 0 \text{ m/s}$.</p> $v^2 = v_o^2 + 2 a (y - y_o)$ $0^2 = (12.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(y - y_o)$ $y - y_o = 7.35 \text{ m}$
<p>Part c. Step 2.</p> <p>Determine the total distance traveled.</p>	<p>The stone travels upward 7.35 m. At that point it is 7.35 m + 70.0 m = 77.35 m above the ground. Therefore, the total distance that the stone travels is 7.35 m (upward) + 77.35 m (downward) = 84.7 m (total).</p>