CHAPTER 3

KINEMATICS IN TWO DIMENSIONS; VECTORS

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- represent the magnitude and direction of a vector using a protractor and ruler.
- multiply or divide a vector quantity by a scalar quantity.
- use the methods of graphical analysis to determine the magnitude and direction of the vector resultant in problems involving vector addition or subtraction of two or more vector quantities. The graphical methods to be used are the parallelogram method and the tip to tail method.
- use the trigonometric component method to resolve a vector components in the x and y directions.
- use the trigonometric component method to determine the vector resultant in problems involving vector addition or subtraction of two or more vector quantities.
- use the kinematics equations of chapter two along with the vector component method of chapter three to solve problems involving two dimensional motion of projectiles.

KEY TERMS AND PHRASES

resultant vector is the arithmetic sum (or difference) of the magnitudes and the directions of two or more vectors.

tip to tail method is a graphical method used to determine the vector sum of two or more vectors. The two vectors are drawn to scale and then moved parallel to their original direction until the tail of one vector is at the tip of the next vector. Once all of the vectors are joined in this manner the resultant vector can be determined. The resultant is drawn from the tail of the first vector to the tip of the last vector. The angle of the resultant above (or below) the x axis is determined by using a protractor.

parallelogram method is a graphical method useful if two vectors are to be added. The two vectors are drawn to scale and joined at the tails. Dotted lines are then drawn from the tip of each vector parallel to the other vector. The finished diagram is a parallelogram. The resultant is along the diagonal of the parallelogram and extends from the point where the tails of the original vectors touch to the point where the dotted lines cross. The angle of the resultant above (or below) the x axis is determined by using a protractor.

vector component method is used to replace each vector with components in the x and y directions. The arithmetic sum of the x components (ΣX) and y components (ΣY) are then determined. Since ΣX and ΣY are at right angles, the Pythagorean theorem can be used to determine the magnitude of the resultant. The definition of the tangent of an angle can be used to determine the angle of the resultant above (or below) the x axis.

relative velocity refers to the velocity of an object with respect to a particular frame of reference.

projectile motion is the motion of an object fired (or thrown) at an angle θ with the horizontal. The only force acting on the object during its motion is gravity.

SUMMARY OF MATHEMATICAL FORMULAS

projectile motion equations	$v_{yo} = v_o \sin \theta$	Initial vertical component of a projectile's velocity where θ is measured from the horizontal direction.
	$v_{xo} = v_o \cos \theta$	Initial horizontal component of a projectile's velocity. Again, θ is measured from the horizontal direction.
	$y = v_{yo}\, t - \frac{1}{2} g\, t^2$	Vertical component of an object's position (y) as related to its initial speed (v_{yo}), gravitational acceleration (g), and time of motion (t).
	$v_y^2 = v_{yo}^2 - 2\, g\, y$	Vertical component of an object's velocity (v_y) as related to its initial speed (v_{yo}), gravitational acceleration (g), and position (y).
	$v_y = v_{yo} - g\, t$	Vertical component of an object's velocity (v_y) as related to its initial speed (v_{yo}), gravitational acceleration (g), and time (t).
	$v_x = v_{xo}$	Horizontal component of velocity (v_x) remains constant during the projectile's flight.
	$x = x_o + v_{xo}\, t$	horizontal component of an object's position (x) as related to the initial horizontal speed (v_{xo}), and time (t)

CONCEPT SUMMARY

Addition or Subtraction of Vectors - Graphical Methods

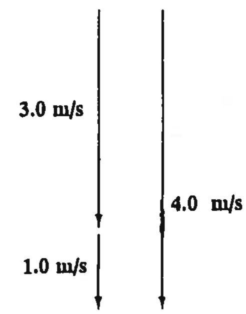
If two vectors are in the same direction and along the same line, the **resultant vector** will be the arithmetic sum of the magnitudes and the direction will be in the direction of the original vectors. If the vectors are in opposite directions and along the same line, the resultant vector will have a magnitude equal to the difference in the magnitudes of the original vectors and the direction of the resultant will be in the direction of the vector which has the greater magnitude.

If the vectors are at some angle to each other than 0° or 180°, then special methods must be used to determine the magnitude and direction of the resultant. Two graphical methods used are the 1) tip to tail method and 2) parallelogram method.

The graphical methods involve the use of a ruler and a protractor. An appropriate scale factor must be used to represent the vectors. For example, a velocity vector which has a magnitude of 3.0 m/s and directed due east can be represented by a line 3.0 cm long directed along the +x axis. 3.0 cm +x

Tip to Tail Method

The tip to tail method can be conveniently used for the addition or subtraction of two or more vectors. The method consists of moving the vectors parallel to their original direction until the tail of one vector is at the tip of the next vector. Once all of the vectors are joined in this manner the resultant vector can be determined. The resultant is drawn from the tail of the first vector to the tip of the last vector. For example, the resultant velocity relative to the bank of a river for a person swimming at 3.0 m/s downstream in a river with a current of 1.0 m/s can be determined as shown in the figure. The magnitude of the resultant can be determined by measuring the length of the vector and multiplying by the scale factor. The scale used in the diagram is 1.0 cm = 1.0 m/s.



Using a ruler it can be determined that the resultant has a length of 4.0 cm. Therefore, the magnitude is (4.0 cm) x (1.0 m/s)/(1.0 cm) = 4.0 m/s. The direction of the resultant vector is downstream.

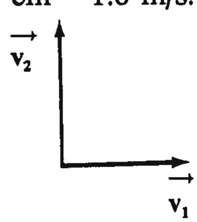
Parallelogram Method

The parallelogram method is useful if two vectors are to be added. If more than two vectors are involved, the method becomes cumbersome and an alternate method should be used.

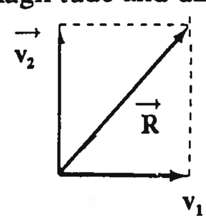
The two vectors are drawn to scale and joined at the tails. Dotted lines are then drawn from the tip of each vector parallel to the other vector. A protractor can be used to ensure that the lines are drawn parallel. The finished diagram is a parallelogram. The resultant is along the diagonal of the parallelogram and extends from the point where the tails of the original vectors touch to the point where the dotted lines cross. As in the tip to tail method, the resultant is determined by measuring the length of the resultant and multiplying by the scale factor used to represent the vectors. The angle of the resultant above (or below) the x axis is determined by using a protractor.

For example, determine the sum of the following vectors: $\vec{v}_1$ = 3.0 m/s due east, $\vec{v}_2$ = 4.0 m/s due north.

step 1. Draw a diagram representing each with the tail of each joined at a point. Scale: let 1.0 cm = 1.0 m/s.



step 2. Complete the parallelogram. Draw the resultant vector across the diagonal. Determine the magn tude and direction of the resultant vector.



Using a ruler, it can be determined that the resultant has a length of 5.0 cm. Therefore, the magnitude of the resultant is (5.0 cm) x (1.0 m/s)/(1.0 cm) = 5.0 m/s. Using a protractor, the angle θ can be determined to be 53° north of east. Therefore, $\vec{v}_R$ = 5.0 m/s ∠53° N of E.

Subtraction of Vectors

The negative of a vector is a vector of the same magnitude but in the opposite direction. Thus, if vector $\vec{v}$ = 5 m/s due east, then $-\vec{v}$ = 5 m/s due west.

In order to subtract one vector from another, rewrite the problem so that the rules of vector addition can be applied. For example, $\vec{A} - \vec{B}$ can be rewritten as $\vec{A} + (-\vec{B})$. Determine the magnitude and direction of $-\vec{B}$ and apply the rules of vector addition to solve for the resultant vector.

Multiplication of a Vector by a Scalar

The product of a vector times a scalar has the same direction as the vector and a magnitude equal to the product of the magnitude of the scalar times the magnitude of the vector. For example, if c is a scalar while $\vec{V}$ is a vector, then the product has a magnitude cV and the same direction as $\vec{V}$. If c is a negative scalar, the magnitude of the resultant is still $c\vec{V}$ but the direction of the resultant is directly opposite that of $\vec{V}$.

Analytic Method for Adding Vectors

In the trigonometric component method, each of the original vectors is expressed as the vector sum of two other vectors. The two vectors are chosen to be in directions which are perpendicular to another one. At first, this method may seem long and tedious. However, with some practice this method is by far the most useful for this course.

For problems involving the addition of two or more vectors lying in the x-y plane, the method reduces to the following steps:

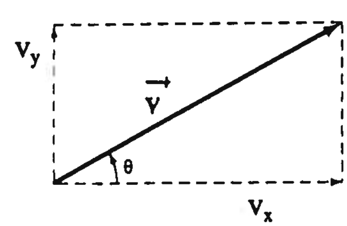
Step 1. Resolve each vector into x and y components.

If the angle is measured from the x axis to the vector, then the x component is equal to the product of the magnitude of the vector and the cosine of the angle. The y component is equal to the product of the magnitude of the vector and the sine of the angle. For example, for

$\vec{v}$ = 5.0 m/s ∠30° N of E

$v_x = v \cos\theta$ = (5.0 m/s)(cos 30°) = 4.3 m/s

$v_y = v \sin\theta$ = (5.0 m/s)(sin 30°) = 2.5 m/s



Step 2. Sign Convention

Assign a positive value to the magnitude if the component is in the +x or +y direction and a negative value if the component is in the -x or -y direction. Thus, for the vector used as an example in Step 1, v_x = + 4.3 m/s and v_y = + 2.5 m/s

Step 3. Reduce the problem to the sum of two vectors.

Determine the sum of the x components (ΣX) and the sum of the sum of the y components (ΣY), where Σ is the upper case Greek letter sigma which is designated to mean "the sum of." Since the x components are along the same line, their magnitudes are added arithmetically. This is also true for the y components.

Step 4. Determine the magnitude and direction of the resultant.

Since ΣX and ΣY are at right angles, the Pythagorean theorem can be used to determine the magnitude of the resultant. The definition of the tangent of an angle can be used to determine the direction of the resultant.

EXAMPLE PROBLEM 1. A man walks 100 meters due east, then 100 meters ∠60.0° north of east. Determine the magnitude and direction of the resultant displacement using the a) tip to tail method, b) parallelogram method, and c) vector component method.

Part a. Step 1.
Displacement is a vector quantity. Write each vector in vector notation.

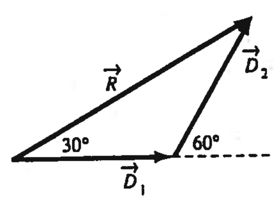
Solution: (Sections 3-2 and 3-4)

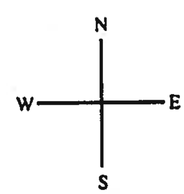
$\vec{\mathbf{D}}_1$ = 100 m ∠due east

$\vec{\mathbf{D}}_2$ = 100 m ∠60.0° north of east

Part a. Step 2.

Draw an accurate diagram locating each vector. Let the scale factor used in the drawing be 1.0 cm ≈ 50 m. A protractor must be used to measure the angles.

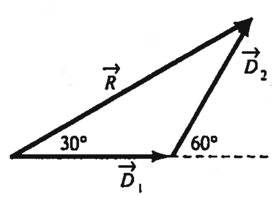




Part a. Step 3.

Determine the magnitude and direction of the resultant vector using the tip to tail method.

This method consists in moving one of the vectors parallel to its original direction until its tail is at the tip of the other vector. The resultant is drawn from the unattached tail of the second vector to the unattached head of the first.

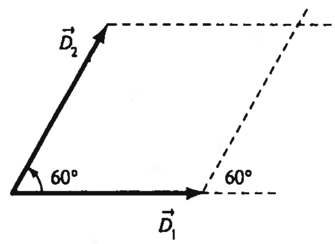


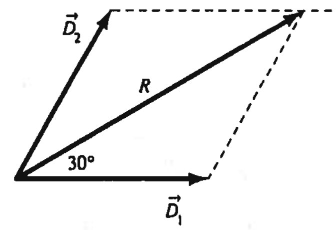
Based on careful measurement with a ruler, the magnitude of the resultant is approximately 3.4 cm. Using the scale factor, (3.4 cm)(50 m/1 cm) ≈ 170 m, the magnitude in meters can be determined. The direction of the resultant is determined by use of a protractor and is 30° N of E. Thus $\vec{\mathbf{R}}$ ≈ 170 m ∠30° N of E.

Part b. Step 1.

Draw the vectors and complete the parallelogram. Note: in the diagram shown at the right the scale factor is 1.0 cm ≈ 43 m.

The two vectors are drawn tail to tail. In order to complete the parallelogram, the dotted lines must be drawn parallel to the opposite sides. A protractor should be used to ensure that this is done properly.





Part b. Step 2.

Use a protractor and a ruler to determine the magnitude and direction of the resultant. Note: remember to multiply by the scale factor.

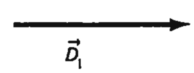
The resultant is along the diagonal of the parallelogram. The direction is from the point where the tails are joined to the point where the dotted lines cross. As in the tip to tail method, the resultant is determined by measuring the length and multiplying by the scale factor. The direction is determined by using a protractor.

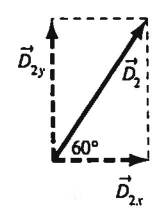
The length of R is measured to be approximately 4.0 cm; therefore, (4.0 cm)(43 m/l cm) ≈ 170 m. Using a protractor, the angle is measured to be 30°. Thus, $\vec{\mathbf{R}} \approx 170$ m ∠30° N of E.

Vector Component Method: This method does not require an accurate diagram as part of the solution. However, a diagram reflecting the relative magnitudes and directions of the vectors is helpful.

Part c. Step 1.

Make a drawing and resolve each vector into x and y components.





Part c. Step 2.

Use trigonometry to determine the magnitude of each component.

If the angle is measured from the east-west axis (x axis) to the vector, then the x component is equal to the product of the magnitude of the vector and the cosine of the angle. The y component is equal to the product of the magnitude of the vector and the sine of the angle.

x components

$\vec{\mathbf{D}}_{1x}$ = (100 m)(cos 0°) = + 100 m $\vec{\mathbf{D}}_{2x}$ = (100 m)(cos 60.0°) = + 50.0 m

y components

$\vec{\mathbf{D}}_{1y}$ = (100 m)(sin 0°) = 0 $\vec{\mathbf{D}}_{2y}$ = (100 m)(sin 60.0°) = +86.6 m

Part c. Step 3. Determine the vector sum of the x components ($\Sigma\vec{X}$) and y components ($\Sigma\vec{Y}$).	The x components are in the +x direction; therefore, both are assigned positive values. The y-component of D_2 is in the +y direction; therefore, it has been assigned a positive value. $\Sigma\vec{X} = \vec{D}_{1x} + \vec{D}_{2x} = 100\text{ m} + 50.0\text{ m} = 150\text{ m}$ $\Sigma\vec{Y} = \vec{D}_{1y} + \vec{D}_{2y} = 0\text{ m} + 86.6\text{ m} = 86.6\text{ m}$
Part c. Step 4. Complete the parallelogram using ΣX and ΣY as the vectors to be added. Solve for the magnitude and direction of the resultant vector.	Since ΣX and ΣY are at right angles, the Pythagorean theorem and trigonometry can be used to determine the magnitude and direction of the resultant. $R = [(\Sigma X)^2 + (\Sigma Y)^2]^{1/2} = [(150\text{ m})^2 + (86.6\text{ m})^2]^{1/2} = 173\text{ m}$ $\tan\theta = (86.6\text{ m})/(150\text{ m}) = 0.577$; $\theta = 30.0°$ N of E Thus, $\vec{R} = 173\text{ m} \angle 30.0°$ N of E.

TEXTBOOK QUESTION 1. One car travels due east at 40 km/h, and a second car travels due north at 40 km/h. Are their velocities equal? Explain.

ANSWER: Velocity is a vector quantity, i.e., it has both magnitude and direction. The two cars have the equal speed but they are traveling in different directions. Therefore, their velocities are not equal.

TEXTBOOK QUESTION 4. During baseball practice, a batter hits a very high fly ball, and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball?

ANSWER: The distance traveled by the ball is greater than the distance traveled by the player. But distance is a scalar quantity not a vector quantity. Displacement is a vector quantity and is the same for each. The displacement vector is the distance and straight line direction between the starting point and the ending point.

TEXTBOOK QUESTION 6. Two vectors have length V_1 = 3.5 km and V_2 = 4.0 km. What are the maximum and minimum magnitudes of their vector sum?

ANSWER: The maximum magnitude occurs if the two vectors are in the same direction. The resultant velocity is the arithmetic sum of their magnitudes, i.e., V_R = 3.5 km + 4.0 km = 7.5 km.

The minimum magnitude occurs if the two vectors are in opposite directions. The resultant velocity is the difference of their magnitudes, i.e., V_R = 4.0 km - 3.5 km = 0.5 km in the direction of the vector which has the greater magnitude.

Relative Velocity

Relative velocity refers to the velocity of an object with respect to a particular frame of reference. As in chapter 2, the reference frame is usually specified by using Cartesian coordinates, i.e., x, y, and z axes, relative to which the position and/or motion of an object can be determined. As stated in the textbook, the velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference and the relative velocity of the two reference frames are known.

EXAMPLE PROBLEM 2. The current in a river is 1.00 m/s. A woman swims 600 m downstream and then back to her starting point without stopping. If she can swim 2.00 m/s in still water, determine the time required for the round trip.

Part a. Step 1.

Determine her velocity relative to the river bank as she swims downstream.

Solution: (Section 3-8)

Let $\vec{v}_{pw}$ = velocity of the person relative to the river water

$\vec{v}_{ws}$ = velocity of the water relative to the shore of the river, i.e., the river current

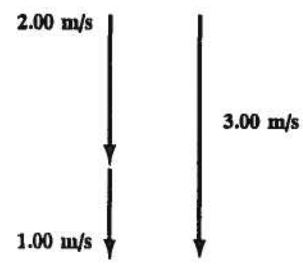
$\vec{v}_{ps}$ = velocity of the person relative to the shore of the river

The tip to tail method can be used to solve for v_{ps}. The vectors are along the same line; therefore, the resultant vector is the arithmetic sum of the magnitudes.

$\vec{v}_{ps} = \vec{v}_{pw} + \vec{v}_{ws}$

$\vec{v}_{ps}$ = 2.00 m/s + 1.00 m/s

$\vec{v}_{ps}$ = 3.00 m/s (downstream)



Part a. Step 2.

Determine the time to swim downstream.

time = displacement/resultant velocity

$t = \vec{D}/\vec{v}_{ps}$ = (600 m)/(3.00 m/s) = 200 s

Part a. Step 3.

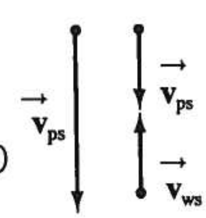
Determine the time required to swim back to the starting point.

The tip to tail method can again be used to solve for $\vec{v}_{ps}$. The woman is now swimming against the current. Thus

$\vec{v}_{ps} = \vec{v}_{pw} - \vec{v}_{ws}$

$= \vec{v}_{pw} + (-\vec{v}_{ws})$

$\vec{v}_{ps}$ = 2.00 m/s + (-1.00 m/s) = 1.00 m/s (upstream)



time = displacement/resultant velocity

$t = \vec{D}/\vec{v}_{ps}$ = (600 m)/(1.00 m/s) = 600 s

Part a. Step 4.	$t_{total} = t_{downstream} + t_{upstream}$
Determine the total time for the round trip.	$t_{total} = 200\ s + 600\ s = 800\ s$

Projectile Motion

Projectile motion is the motion of an object fired at an angle θ with the horizontal. This motion can be discussed by analyzing the horizontal component of the object's motion independent of the vertical component of motion. If air resistance is negligible, then the horizontal component of motion does not change; thus $a_x = 0$ and $v_x = v_{xo}$ = constant.

The vertical component of motion is affected by gravity and is described by the equations for an object in free fall discussed in the textbook in chapter 2. The following equations are used to describe the motion of a projectile.

vertical component of motion	horizontal component of motion
$v_y = v_{yo} - gt$	$x = x_o + v_{xo}\, t$
$y = y_o + v_{yo}\, t - ½\, g\, t^2$	where $v_x = v_{xo}$
$v_y^2 = v_{yo}^2 - 2\, g\, y$	

v_y is the vertical component of velocity at time t.

v_{yo} is the initial vertical component of velocity, $v_{yo} = v_o \sin\theta$.

g is the acceleration due to gravity, $g = 9.8\ m/s^2$.

t is the time interval of the motion, $t_o = 0$ s.

y is the vertical displacement from the release point. The origin of the coordinate system is the release point (y_o) with the upward direction taken as positive.

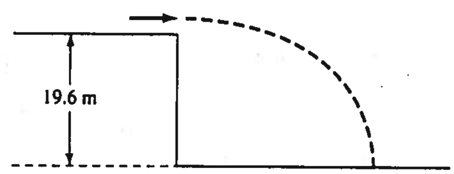
x is the horizontal displacement from the release point. The release point (x_o) is usually taken to be the zero point of motion in the horizontal direction, i.e., $x_o = 0$.

v_{xo} is the initial horizontal component of velocity. Assuming air resistance to be negligible, the horizontal component of velocity does not change during the motion. Therefore, $v_x = v_{xo} = v_o \cos\theta$.

TEXTBOOK QUESTION 19. A projectile is launched at an angle of 30° to the horizontal with a speed of 30 m/s. How does the horizontal component of its velocity 1.0 s after launch compare with its horizontal component of velocity 2.0 s after launch?

ANSWER: The motion of a projectile at any point in its motion is the vector sum of the horizontal component and the vertical component of its velocity. As a projectile travels upward the magnitude of the vertical component decreases, reaching zero at maximum height. On the way down, the vertical component of velocity again increases. However, in the absence of air resistance, the magnitude of horizontal component does **not** change. The magnitude of horizontal component $v_x = v_{xo} = (30\ m/s)(\cos 30^\circ) = 26\ m/s$ at each point in the motion.

EXAMPLE PROBLEM 3. A stone is thrown horizontally outward from the top of a bridge. The stone is released 19.6 meters above the street below. The initial velocity of the stone is 10.0 m/s. Determine the a) total time that the stone is in the air and b) magnitude and direction of the velocity of the projectile "just" before it strikes the street.

Part a. Step 1. Draw an accurate diagram showing the trajectory of the projectile.	Solution: (Sections 3-5 and 3-6) 
Part a. Step 2. Determine the initial horizontal, and vertical components of velocity.	Since the object is thrown horizontally, the initial vertical component (v_{yo}) is zero. The initial horizontal component (v_{xo}) is 10.0 m/s. Note: the vertical component of motion is independent of the horizontal motion.
Part a. Step 3. Complete a data table using information both given and implied in the problem.	Using the technique discussed in the textbook, the upward direction will be designated as the positive. The top of the cliff is the zero point of position, i.e. $y_o = 0$. Therefore, the bottom, of the cliff is at $y = -19.6$ m. $g = 9.8\ m/s^2$ $v_{yo} = 0$ $t = ?$ $y = 19.6$ m $v_y = ?$ $x = ?$ $v_{xo} = 10.0$ m/s
Part a. Step 4. Determine the total time that the stone is in the air.	$y = v_{yo}\, t - \frac{1}{2}\, g\, t^2$ $-19.6\ m = (0\ m/s)\ t - \frac{1}{2}\ (9.8\ m/s^2)\ t^2$ $t^2 = 2(-19.6\ m)/(-9.8\ m/s^2) = 4.0\ s^2$ Either $t = 2.0$ s or -2.0 s. Since the time of flight cannot be negative, the answer is $t = 2.0$ s.
Part b. Step 1. Determine the vertical velocity just before it hits the street.	$v_y = v_{yo} - g\, t$ $= 0\ m/s - (9.8\ m/s^2)(2.0\ s)$ $v_y = -19.6\ m/s$

Part b. Step 2.

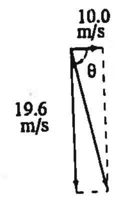
Determine the magnitude and direction of the velocity of the projectile just before it strikes the street.

The horizontal velocity remains constant; therefore, both components of motion are now known. Use the Pythagorean theorem and trigonometry to determine the magnitude and direction of the resultant velocity (v_R).

$$v_R = [(v_y) + (v_{xo})]^{1/2}$$

$$= [(-19.6\ \text{m/s})^2 + (10.0\ \text{m/s})^2]^{1/2}$$

$$v_R = 22.0\ \text{m/s}$$



$$\tan\theta = v_y/v_{xo} = (-19.6\ \text{m/s})/(10.0\ \text{m/s}) = 1.96; \quad \theta = -63.0^\circ$$

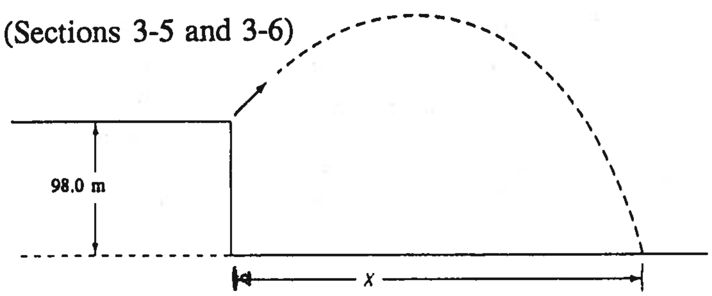
$$\vec{v}_R = 22.0\ \text{m/s}\ \angle 63.0^\circ \text{ below the horizontal}$$

EXAMPLE PROBLEM 4. A projectile is fired with an initial speed of 196 m/s at an angle of 30.0° above the horizontal from the top of a cliff 98.0 m high. Determine the a) time to reach maximum height, b) maximum height above the base of the cliff reached by the projectile, c) total time it is in the air, and d) horizontal range of the projectile.

Part a. Step 1.

Draw an accurate diagram showing the trajectory of the projectile.

Solution: (Sections 3-5 and 3-6)



Part a. Step 2.

Determine v_{yo} and v_{xo}.

$v_{yo} = v_o \sin 30.0^\circ = (196\ \text{m/s})(0.500) = 98.0\ \text{m/s}$

$v_{xo} = v_o \cos 30^\circ = (196\ \text{m/s})(0.866) = 170\ \text{m/s}$

Part a. Step 3.

Complete a data table using information both given and implied.

$v_{yo} = 98.0$ m/s	$t = ?$ (to reach max height)
$y = ?$ (at max height)	$v_y = 0$ (at maximum height)
$g = 9.80$ m/s^2	
$v_{xo} = 170$ m/s	$x = ?$

Part a. Step 4.

Determine the time to reach maximum height.

$v_y = v_{yo} - gt$

$0\ \text{m/s} = 98.0\ \text{m/s} - (9.80\ \text{m/s}^2)\, t$

$t = 10.0\ \text{s}$

Part b. Step 1. Determine the maximum height above the cliff reached by the projectile Now determine the maximum height above the ground reached by the projectile.	$y = -\frac{1}{2} g t^2 + v_{yo} t$ $= -\frac{1}{2}(9.80\ m/s^2)(10.0\ s)^2 + (98.0\ m/s)(10.0\ s)$ $= -490\ m + 980\ m$ $y = 490\ m$ The total height above the base of the cliff can now be determined: $y = 490\ m + 98.0\ m = 588\ m$
Part c. Step 1. Complete a data table and determine the time required for the projectile to fall from maximum height to the bottom of the cliff.	Using the technique discussed in the textbook, the upward direction is designated as positive. The top of the trajectory is the zero point of position, i.e., $y_o = 0$. Therefore, the bottom, of the cliff is at $y = -588$ m. $g = 9.8\ m/s^2$ $v_{yo} = 0$ (at maximum height) $t = ?$ $y = -588\ m$ $y = v_{yo} t - \frac{1}{2} g t^2$ $-588\ m = (0\ m/s)\ t - \frac{1}{2}(9.80\ m/s^2)\ t^2$ $t^2 = 2(-588\ m)/(-9.80\ m/s^2) = 120\ s^2$ $t = 11.0\ s$
Part c. Step 2. Determine the total time the projectile is in the air.	total time = 10.0 s + 11.0 s = 21.0 s
Part d. Step 1. Use the equation for horizontal motion to determine the horizontal range.	$x = v_{xo} t = (170\ m/s)(21.0\ s)$ $x = 3570\ m$

PROBLEM SOLVING SKILLS

For problems involving vector addition or subtraction:

1. Use a protractor and ruler to accurately represent each vector involved in the problem. Make sure to use an appropriate scale factor in representing the vector.
2. Choose an appropriate method to solve the problem. If a graphical method is used, be sure to measure both the magnitude and direction of the resultant.

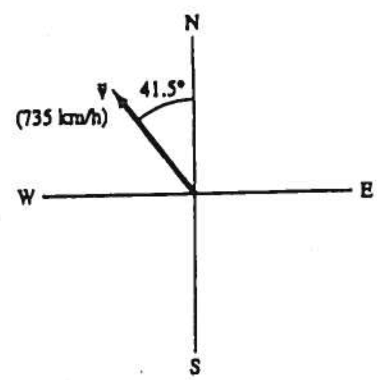
3. If the trigonometric component method is used, you must
   a) break each vector into x and y components.
   b) use the sign convention and assign a positive sign or a negative sign to the magnitude.
   c) determine the sum of the x components and repeat for the sum of the y components.
4) Use the Pythagorean theorem and simple trigonometry to solve for the magnitude direction of the resultant.

For problems involving projectile motion:

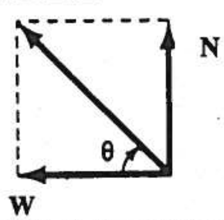
1. Draw an accurate diagram showing the trajectory of the projectile.
2. Use the trigonometric component method to determine v_{xo} and v_{yo}.
3. Complete a data table using information both given and implied in the wording of the problem.
4. As in the free fall problems of chapter 2, use the appropriate sign convention depending on whether the object was initially moving upward or downward.
5. Memorize the formulas for projectile motion. It is also necessary to memorize the meaning of each symbol in each formula. Using the data from the completed data table, determine which formula or combination of formulas must be used to solve the problem.

SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 9. An airplane is traveling 735 km/h in a direction 41.5° west of north (Fig. 3-31). (a) Find the components of the velocity vector in the northerly and westerly directions. (b) How far north and how far west has the plane traveled after 3.00 h?



Part a. Step 1. Determine the angle θ as measured north of west.	Solution: (Section 3-4) θ = 90.0° - 41.5° W of N = 48.5° N of W.
Part a. Step 2. Use trigonometry to determine the north component and the west component	v_{north} = (735 km/h) sin 48.5° = 550 km/h v_{west} = (735 km/h) cos 48.5° = 487 km/h



Part b. Step 1.

Determine the distance traveled in 3.00 h.

d_{north} = (550 km/h)(3.00 h) = 1650 km

d_{west} = (487 km/h)(3.00 h) = 1460 km

TEXTBOOK PROBLEM 10. Three vectors are shown in Fig. 3-32. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (a) components, and (b) magnitude and angle with the x axis.

Part a. Step 1.

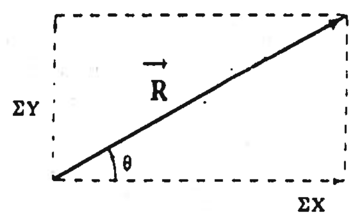
Determine the x and y component of each vector and then deter-rmine ΣX and ΣY. Remember to apply the sign convention.

Solution: (Section 3-4)

A_x = 44.0 cos 28.0° = + 38.8	A_y = 44.0 sin 28.0° = + 20.6
B_x = 26.5 cos 56.0° = - 14.8	B_y = 26.5 sin 56.0° = + 22.0
C_x = 31.0 cos 90.0° = 0.00	C_y = 31.0 sin 90.0° = - 31.0
ΣX = + 24.0	ΣY = + 11.6

Part a. Step 2.

Draw a vector diagram showing ΣX and ΣY. Complete the parallelo-gram and draw in the resultant.



Part a. Step 3.

Use the Pythagorean theorem and trigonometry to determine the magnitude and direction of the resultant.

$R^2 = (\Sigma X)^2 + (\Sigma y)^2 = (+ 24.0)^2 + (+ 11.6)^2 = 711$

R = 26.7

tan θ = (+ 11.6)/(+ 24.0) = 0.488; therefore, θ = 25.8° N of E

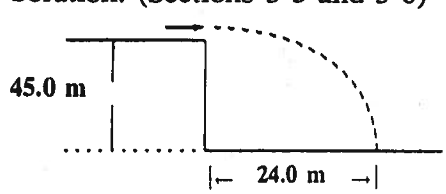
$\vec{\mathbf{R}}$ = 26.7 ∠25.8° N of E

TEXTBOOK PROBLEM 21. A ball is thrown horizontally from the roof of a building 45.0 m tall and lands 24.0 m from the base. What was the ball's initial speed.

Part a. Step 1.

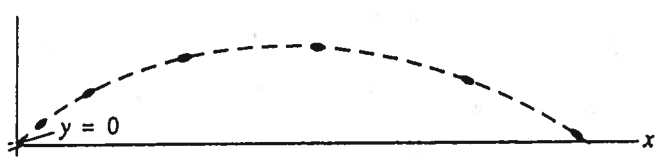
Draw an accurate diagram showing the trajectory of the projectile.

Solution: (Sections 3-5 and 3-6)



Part a. Step 2.. Complete a data table using information both given and implied in the problem.	Using the technique discussed in the textbook, the upward direction will be designated as positive. The top of the building is the zero point of position, i.e. $y_o = 0$. Therefore, the street below is at $y = -45.0$ m. $g = 9.8\ m/s^2$ $v_{yo} = 0$ $t = ?$ $y = -45.0$ m $v_y = ?$ $x = 24.0$ m $x_o = 0$ $v_{xo} = ?$
Part a. Step 3. Determine the total time that the stone is in the air.	$y = v_{yo}\, t - ½\, g\, t^2$ $-45.0\ m = (0\ m/s)\ t - ½\ (9.8\ m/s^2)\ t^2$ $t^2 = 2(-45.0\ m)/(-9.8\ m/s^2) = 9.18\ s^2$ Either t = 3.03 s or - 3.03 s. Since the time of flight cannot be negative, the answer is t = 3.03 s.
Part a. Step 4. Determine the ball's initial speed.	The horizontal velocity remains constant, i.e., $v_{xo} = v_x$. $x = v_{xo}\, t$ $24.0\ m = v_{xo}\ (3.03\ s)$ $v_{xo} = 7.92$ m/s

TEXTBOOK PROBLEM 24. An athlete executing a long jump leaves the ground at a 28.0° angle and travels 7.80 m. (a) What was the takeoff speed? (b) If this speed was increased by just 5.0 percent, how much longer would the jump be?

Part a. Step 1. Draw an accurate diagram showing the trajectory of the athlete.	Solution: (Sections 3-5 and 3-6) 
Part a. Step 2. Complete a data table using information both given and implied.	Since the initial motion is upward, the upward direction will be designated as the positive direction. Also, the athlete starts at ground level and returns to ground level; therefore, $y = y_o = 0$ m. $g = 9.80\ m/s^2$ $t = ?$ $y = 0$ m $y_o = 0$ m $x = 7.80$ m $x_o = 0$ $v_y = ?$ $v_{yo} = v_o \sin 28.0° = (0.469)\ v_o$ $v_{xo} = v_o \cos 28.0° = (0.883)\ v_o$

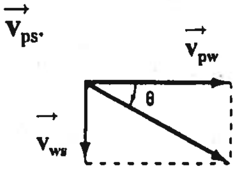
Part a. Step 3. Write an equation for the horizontal distance in terms of the horizontal speed and time.	$x = v_{xo}\ t$ $7.80\ m = (0.883)\ v_o\ t$ $v_o\ t = 8.83\ m$
Part a. Step 4. Determine the time in air.	$y = v_{yo}\ t\ - ½\ g\ t^2$ $0 = (0.469)\ v_o\ t - (4.90\ m/s^2)\ t^2$ but $v_o\ t = 8.83\ m$ $0 = (0.469)(8.83\ m) - (4.90\ m/s^2)\ t^2$ $t^2 = 0.846\ s^2$ and $t = 0.920\ s$
Part a. Step 5. Combine the results of steps 2 and 3 to determine the initial speed.	$v_o\ t = 8.83\ m$ but $t = 0.920\ s$ $v_o\ (0.920\ s) = 8.83\ m$ $v_o = 9.60\ m/s$
Part b. Step 1. Determine the value of the velocity.	As a result of the 5 percent increase, the new velocity would be 1.05 greater than in part a. $v_o = (1.05)(9.60\ m/s) = 10.1\ m/s$
Part b. Step 2. Complete a data table using the new information.	$g = 9.80\ m/s^2$ $t = ?$ $y = 0\ m$ $y_o = 0\ m$ $x = ?\ m$ $x_o = 0$ $v_y = ?$ $v_{yo} = (10.1\ m/s)\ \sin 28.0° = 4.74\ m/s$ $v_{xo} = (10.1\ m/s)\ \cos 28.0° = 8.92\ m/s$
Part b. Step 3. Determine the time in air.	$y = v_{yo}\ t\ - ½\ g\ t^2$ $0 = (4.74\ m/s)\ t - (4.9\ m/s^2)\ t^2$ Using algebra gives either $t = 0.967\ s$ or $t = 0\ s$
Part b. Step 4. Determine the horizontal distance.	$x = v_{xo}\ t$ $x = (8.92\ m/s)(0.967\ s)$ $x = 8.60\ m$

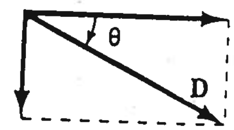
Part b. Step 5. Determine the increased distance.	$8.60\ \text{m} - 7.80\ \text{m} = 0.80\ \text{m}$

Alternate Method. This problem is readily solved by using the range equation derived in the textbook. This special equation can only be used because the athlete returns to his original height above the ground.

Part a. Step 1. Use the range equation to determine the take-off speed.	$R = v_o^2 \sin 2\theta/g$ and rearranging gives $v_o^2 = R\ g/\sin 2\theta$ where $\sin 2\theta = \sin 2(28°) = \sin 56° = 0.829$ $v_o^2 = (7.80\ \text{m})(9.8\ \text{m/s}^2)/(0.829) = 92.2\ \text{m}^2/\text{s}^2$ $v_o = 9.60\ \text{m/s}$
Part b. Step 2. Use the range equation to determine the increased distance.	$R = v_o^2 \sin 2\theta/g$ $R = [(1.05)(9.60\ \text{m/s})]^2(0.829)/(9.8\ \text{m/s}^2) = 8.60\ \text{m}$ Increased distance: $8.60\ \text{m} - 7.80\ \text{m} = 0.80\ \text{m}$

TEXTBOOK PROBLEM 47. A swimmer is capable of swimming 0.45 m/s in still water. (a) If she aims her body directly across a 75 m wide river whose current is 0.40 m/s, how far downstream (from a point opposite her starting point) will she land? (b) How long will it take her to reach the other side?

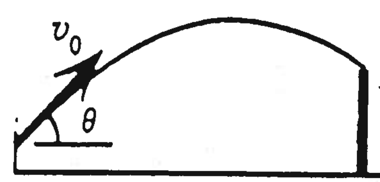
Part a. Step 1. Determine the woman's velocity relative to the riverbank ($\vec{v}_{ps}$).	Solution: (Section 3-8) The woman's velocity relative to the water ($\vec{v}_{pw}$) is at right angles to the current ($\vec{v}_{ws}$). The Pythagorean theorem can be used to solve for the woman's velocity relative to the river bank $\vec{v}_{ps}$.  $v_{ps} = [(v_{pw})^2 + (v_{ws})^2]^{1/2}$ $= [(0.45\ \text{m/s})^2 + (0.40\ \text{m/s})^2]^{1/2}$ $v_{ps} = 0.60\ \text{m/s}$, $\tan\theta = (0.40\ \text{m/s})/(0.45\ \text{m/s}) = 0.89$, $\theta = 41.6°$ $\vec{v}_{ps} = 0.60\ \text{m/s}\ \angle 41.6°$
Part a. Step 2. Determine the distance downstream where she reaches the opposite bank of the river.	$\tan\theta = (D_{downstream})/(D_{across\ river})$ $\tan 41.6° = (D_{downstream})/(75\ \text{m})$ $D_{downstream} = (75\ \text{m})(\tan 41.6°) = (75\ \text{m})(0.89) = 67\ \text{m}$

Part b. Step 1.	time = displacement/resultant velocity
Determine the time required to cross the river.	$t = D/v_{ps}$  but as shown in the diagram, D is the actual distance traveled crossing the river. But cos 41.6° = (75 m)/D D = (75 m)/(cos 41.6°) = (75 m)/(0.74.8) = 100 m $t = D/v_{ps}$ = (100 m)/(0.60 m/s) = 170 s

Alternate Method: Since the woman swims perpendicular to the current, the current does not affect the time for the trip across. An analogous situation is that of an airplane passenger walking from a window on the left side of the plane to a window on the right side of the plane. The time is the same if the plane is at rest or if it is moving at 450 miles per hour. Thus, time to cross river equals the distance across the river divided by the woman's velocity perpendicular to the river current.

$t = D/v_{pw}$ = (75 m)/(0.45 m/s) = 170 s

TEXTBOOK PROBLEM 62. When Babe Ruth hit a homer over a 7.5 m high right-field fence 95 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume that the ball was hit 1.0 m above the ground and its path initially made a 38° angle with the ground.

Part a. Step 1.	Solution: (Sections 3-5 and 3-6)
Draw an accurate diagram showing the path of the ball.	
Part a. Step 2. Complete a data table using the information both given and implied.	$g = 9.8\ m/s^2$ $t = ?$ $y = 7.5\ m$ $y_0 = 1.0\ m$ $x = 95\ m$ $x_o = 0$ $v_y = ?$ $v_{yo} = v_o \sin 38° = 0.62\ v_o$ $v_{xo} = v_o \cos 38° = 0.79\ v_o$

Part a. Step 3. Determine the horizontal distance in terms of the initial velocity and time of flight.	$x = v_{xo}\, t$ $95\ m = 0 + 0.79\ v_o\, t$ $v_o\, t = 121\ m$
Part a. Step 4. Determine the time required for the ball to reach the right-field fence.	$y = y_o + v_{yo}\, t - \frac{1}{2} g\, t^2$ $7.5\ m = 1.0\ m + 0.62\ v_o\, t - (4.9\ m/s^2)\, t^2$ but $v_o\, t = 121\ m$ $6.5\ m = (0.62)(121\ m) - (4.9\ m/s^2)\, t^2$ $t^2 = (-68.5\ m)/(-4.9\ m/s^2) = 13.9\ s^2$ $t = 3.74\ s$
Part a. Step 5. Determine the ball's initial speed.	From Part a. Step 3. $v_o\, t = 121\ m$ but $t = 3.74\ s$ $v_o\,(3.74\ s) = 121\ m$ $v_o \approx 32\ m/s$