CHAPTER 4

DYNAMICS: NEWTON'S LAWS OF MOTION

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- state Newton's three laws of motion and give examples that illustrate each law.
- explain what is meant by the term net force.
- use the methods of vector algebra to determine the net force acting on an object.
- define each of the following terms: mass, inertia, weight, and distinguish between mass and weight.
- identify the SI units for force, mass, and acceleration.
- draw an accurate free body diagram locating each of the forces acting on an object or a system of objects.
- use free body diagrams and Newton's laws of motion to solve word problems.

KEY TERMS AND PHRASES

dynamics is the study of the causes of motion.

net force refers to the vector sum of all of the forces acting on an object.

Newton's first law of motion is also known as Galileo's law of inertia, where **inertia** refers to the tendency of an object to resist any change in its state of motion. Inertia is measured by measuring an object's mass.

Newton's second law of motion refers to an object's motion when a net force does not equal zero. The net force (F) will cause an object to accelerate or decelerate. The rate of acceleration (a) is directly proportional to the magnitude of the net force and inversely proportional to the object's mass (m), i.e., $\vec{\mathbf{a}} = \Sigma\vec{\mathbf{F}}/m$ or $\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.

Newton's third law states that whenever one object exerts a force on a second object, the second object exerts an equal but opposite force on the first.

weight is a measure of the force of gravity on an object.

friction is a contact force that opposes the relative motion of two surfaces as they slide past each other. The frictional force depends on the coefficient of friction (μ) and the normal force (F_N).

coefficient of friction (μ) is a pure number without physical units and varies with the types of surfaces that are in contact. When the object is at rest, the **coefficient of static friction** (μ_s) is used to determine the magnitude of the frictional force just before the object starts to move. When the object is moving, the **coefficient of kinetic friction** (μ_k) is used.

normal force is a contact force that acts perpendicular to the common surface of contact.

SUMMARY OF MATHEMATICAL FORMULAS

Newton's first law	Net $\vec{\mathbf{F}} = 0$ or $\Sigma\vec{\mathbf{F}} = \mathbf{0}$	If the net force on an object equals zero, then the object's motion will not change, i.e., a = 0.
Newton's second law	$\vec{\mathbf{a}} = \Sigma\vec{\mathbf{F}}/m$ or $\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$	An object's acceleration is related to the net force and the object's mass.
Newton's third law	$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$	If object 1 exerts a force on object 2, then object 2 exerts an equal but opposite force on object 1.
weight	$\vec{\mathbf{w}} = m\,\vec{\mathbf{g}}$	An object's weight is directly proportional to its mass and gravitational acceleration.
force of friction	$F_{fr} = \mu\,F_N$	Frictional force depends on the coefficient of friction and the normal force.

CONCEPT SUMMARY

Newton's Laws of Motion

Dynamics is the study of the causes of motion. The basic causes of motion can be explained by using **Newton's three laws of motion**. The **first law** explains the motion of an object on which the **net force** equals zero. The net force refers to the vector sum of all of the forces acting on an object. The magnitude and direction of the net force is determined by using the methods of vector addition described in chapter 3 in the textbook. While the tip to tail and the parallelogram methods are useful, the method used most frequently in determining the resultant is the **vector component method**. If the net force equals zero, the object will tend to remain at rest, or if in motion, will remain in motion in a straight line at a constant speed, i.e., constant velocity.

Newton's first law of motion is also known as Galileo's law of inertia, where **inertia** refers to the tendency of an object to resist any change in its state of motion. Inertia is measured by measuring an object's mass.

Newton's second law of motion refers to an object's motion when a net force does not equal zero. The net force will cause an object to accelerate or decelerate. The rate of acceleration (a) is directly proportional to the magnitude of the net force and inversely proportional to the object's **mass** (m).

$\vec{\mathbf{a}} = \Sigma\vec{\mathbf{F}}/m$ or $\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

The SI unit of force is the **newton** (N) and for mass it is the kilogram (kg). A net force of one newton will cause a 1 kg object to accelerate at 1 m/s^2; thus 1 N = 1 kg m/s^2.

Newton's third law is the "action-reaction" law with which most students are familiar. However, it is necessary to be very careful in interpreting the meaning of this law. The action force and the reaction force are equal and opposite but do not act on the same object. A good way to remember the law is the statement given in the textbook: "Whenever one object exerts a force on a second object, the second object exerts an equal but opposite force on the first."

TEXTBOOK QUESTION 9. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4-36). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.

ANSWER: The key to the answer lies in the concept of inertia. Inertia is the tendency of an object to resist any change in its state of motion. The stone has inertia and this inertia must be overcome in order to move the stone. If a person gives a SHARP pull to the dangling thread, the inertia of the stone resists any change in its motion. The result is that the stone does not move and the force exerted by the person is not transmitted to the section of thread above the stone. The tension in the section below the stone is much greater than above the stone and the lower section snaps.

If the person exerts a slow and steady pull, the inertia of the stone is overcome and the stone will begin to move slowly. The tension in the section of thread is due to the person. The tension in the section above the stone is due to the sum of the pull exerted by the person and the stone's weight. The tension in the section above the stone is greater than below the stone. As a result the section of thread above the stone breaks.

TEXTBOOK QUESTION 10. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?

ANSWER: According to Newton's second law of motion, an object's rate of acceleration is directly proportional to the magnitude of the object's weight but inversely proportional to the object's mass, i.e. $\vec{\mathbf{a}} = \Sigma\vec{\mathbf{F}}/m$. A 2 kg rock has twice as much weight as a 1 kg rock but it also has twice as much mass, i.e., inertia, and is twice as hard to accelerate. The ratio of the force to mass is the same for both objects and as a result the acceleration is the same for each.

TEXTBOOK QUESTION 17. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?

ANSWER: The downward force of your weight is balanced by an equal but opposite force exerted by the ground. The net force on your body is zero, your rate of acceleration is zero, and you remain motionless. As a result, you neither rise nor fall.

TEXTBOOK QUESTION 18. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?

ANSWER: When the car is struck from the rear, the force on the car causes the car to move forward. The seat and seatback are attached to the body of the car and move forward with the car. As the seatback moves forward it pushes the occupant's torso forward. The person's head is above the seatback and the head's inertia causes the head to appear to be "thrown backward" as the car moves forward. In reality, the person's head tends to remain at rest while the car plus victim's torso is pushed forward.

Modern cars are equipped with a headrest which, if properly positioned, will push the head forward along with the torso if the car is struck from the rear.

Weight

The force of gravity on an object, i.e., the **weight,** varies slightly from place to place on the surface of the Earth. The variation is so small that the weight of an object located close to the Earth's surface is usually considered to be constant.

The formula for an object's weight ($\vec{\mathbf{w}}$ or $\vec{\mathbf{F}}_g$) near the Earth's surface can be determined

as follows: $\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$, but $\Sigma\vec{\mathbf{F}} = \vec{\mathbf{F}}_g = \vec{\mathbf{w}}$ and $\vec{\mathbf{a}} = \vec{\mathbf{g}}$; therefore, $\vec{\mathbf{F}}_g = \vec{\mathbf{w}} = m\,\vec{\mathbf{g}}$

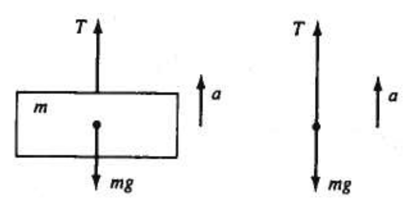
Mass and weight are often confused. Mass depends on an object's inertia and does not vary with location. Weight is the force of gravity acting on an object and varies from place to place. For example, a person standing on the Earth weighs six times as much as he or she would weigh on the Moon; however, his or her mass would remain the same.

EXAMPLE PROBLEM 1. A box of mass 5.0 kg is pulled vertically upward by a force of 68 newtons applied to a rope attached to the box. Determine the a) rate of acceleration of the box and b) vertical velocity of the box after 2.0 s of motion.

Part a. Step 1.

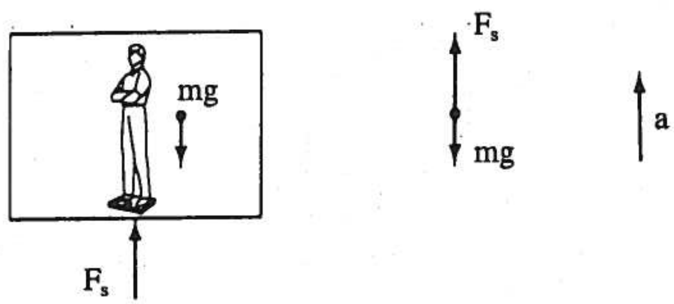
Draw an accurate diagram locating the forces acting on the box, then draw a free body diagram.

Solution: (Sections 4-4 and 4-7)



Part a. Step 2. Determine the net force acting on the box.	The box moves upward; therefore, the tension (T) is greater than the weight, therefore the net force is $T - mg = 68\ N - (5.0\ kg)(9.8\ m/s^2) = 19\ N$
Part a. Step 3. Apply Newton's second law of motion and solve for the rate of acceleration.	$\Sigma \vec{\mathbf{F}} = m\ \vec{\mathbf{a}}$ $68\ N - (5.0\ kg)(9.8\ m/s^2) = (5.0\ kg)\ a$ $19\ N = (5.0\ kg)\ a$ $\vec{\mathbf{a}} = 3.8\ m/s^2$
Part b. Step 1. Determine the velocity the box after 2.0 s.	The acceleration is uniform; therefore, the kinematics equations derived for uniformly accelerated motion can be used to determine the velocity. $v = at + v_o$ $v = (3.8\ m/s^2)(2.0\ s) + 0\ m/s$ $v = 7.6\ m/s$, upward

EXAMPLE PROBLEM 2 A student of mass 50 kg decides to test Newton's laws of motion by standing on a bathroom scale placed on the floor of an elevator. Assume that the scale reads in newtons. Determine the scale reading when the elevator is a) accelerating upward at 0.50 m/s^2, b) traveling upward with a constant speed of 3.0 m/s, and c) traveling upward but decelerating at 1.0 m/s^2.

Part a. Step 1. Draw a diagram locating the forces acting on the student.	Solution: (Sections 4-4, 4-7) 
Part a. Step 2. Determine the scale reading when the elevator is accelerating upward at 0.50 m/s^2.	Since the acceleration is upward, then F_s is greater than mg and the net force is F_s - mg, directed upward. The student's apparent weight as determined by the scale is greater than his actual weight. Apply Newton's second law and determine the scale reading (F_s). $\Sigma \vec{\mathbf{F}} = m\ \vec{\mathbf{a}}$ and $F_s - m\ g = m\ a$ $F_s - (50\ kg)(9.8\ m/s^2) = (50\ kg)(0.50\ m/s^2)$ $F_s = 515\ N$

Part b. Step 1.

Determine the reading on the scale when the elevator is traveling upward at a constant speed of 3.0 m/s.

The velocity is constant; therefore, the rate of acceleration is zero and the net force must equal zero. The student's apparent weight as read by the scale equals his actual weight.

$\Sigma \vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ and $F_s - mg = m\,a$ but $a = 0\ m/s^2$

$F - (50\ kg)(9.8\ m/s^2) = (50\ kg)(0\ m/s^2)$

$F = 490\ N$

Part c. Step 1.

Determine the reading on on the scale when it is traveling upward but decelerating at $1.0\ m/s^2$.

The elevator is decelerating; therefore, the downward force of his weight is greater than the upward force exerted by the scale. Therefore, the student's apparent weight as determined by the scale is less than his actual weight.

$\Sigma \vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ and $F_s - mg = m\,a$

$F_s - (50\ kg)(9.8\ m/s^2) = (50\ kg)(-\ 1.0\ m/s^2)$

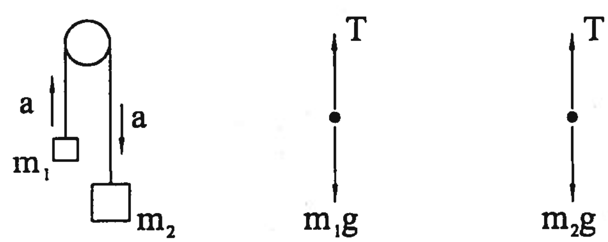
$F_s = 440\ N$

EXAMPLE PROBLEM 3. In a device known as an Atwood machine, a massless, unstretchable rope passes over a frictionless peg. One end of the rope is connected to object $m_1 = 1.0$ kg while the other end is connected to object $m_2 = 2.0$ kg. The system is released from rest and the 2.0 kg object accelerates downward while the 1.0 kg object accelerates upward. Calculate the a) rate of acceleration and b) tension in the rope.

Part a. Step 1.

Draw an accurate diagram locating the forces acting on each object.

Solution: (Section 4-7)



Part a. Step 2.

Use Newton's second law to write a separate equation for the motion of each object.

m_2 accelerates downward; therefore, $m_2 g > T$.

$\Sigma \vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

$m_2\,g - T = m_2\,a$

$(2.0\ kg)(9.8\ m/s^2) - T = (2.0\ kg)\,a$

$19.6\ N - T = (2.0\ kg)\,a$ (equation 2)

m_1 accelerates upward; therefore, $T > m_1\,g$ and

$T - m_1\,g = m_1\,a$

	$T - (1.0\ kg)(9.8\ m/s^2) = (1.0\ kg)\ a$ $T - 9.8\ N = (1.0\ kg)\ a$ (equation 1)
Part a. Step 3. Use algebra to solve for the rate of acceleration of the system.	The above equations have the same two unknowns. To solve for The acceleration, add the two equations together. $19.6\ N - T + T - 9.8\ N = (2.0\ kg + 1.0\ kg)\ a$ $9.8\ N = (3.0\ kg)\ a$ and $a = 3.3\ m/s^2$
Part a. Step 4. Solve for the acceleration using only the external forces acting on the system.	Tension is an internal force in the system. As can be seen in Step 3, the tension algebraically cancels when solving for the acceleration. The external force that causes the motion is the weight of m_2 while the weight of m_1 retards the motion. Applying Newton's second law gives $\Sigma \vec{\mathbf{F}} = m\ \vec{\mathbf{a}}$ $m_2\ g - m_1\ g = (m_2 + m_1)\ a$ $(2.0\ kg)(9.8\ m/s^2) - (1.0\ kg)(9.8\ m/s^2) = (2.0\ kg + 1.0\ kg)\ a$ $19.6\ N - 9.8\ N = (3.0\ kg)\ a$ $a = 3.3\ m/s^2$
Part b. Step 1. Solve for the tension in the rope.	From equation 1: $T - 9.8\ N = (1.0\ kg)\ a$ $T - 9.8\ N = (1.0\ kg)(3.3\ m/s^2)$ $T = 13\ N$

Friction

Friction is a contact force that opposes the relative motion of two surfaces as they slide past each other. The maximum value of the frictional force (F_{fr}) is proportional to the **coefficient of friction** (μ) and also to the **normal force** (F_N) acting on the object.

$$F_{fr} = \mu\ F_N$$

The coefficient of friction is a pure number without physical units and varies with the types of surfaces that are in contact with each other. The force to overcome friction when the object is initially at rest is greater than when the object is moving. When the object is at rest, the **coefficient of static friction** (μ_s) is used to determine the magnitude of the frictional force just before the object starts to move. When the object is moving, the **coefficient of kinetic friction** (μ_k) is used.

The normal force is a contact force that acts perpendicular to the common surface of contact. For example, a book at rest on a horizontal table is acted upon by two forces. The book's weight is downward but is opposed by the normal force which is equal in magnitude but directed upward, perpendicular to the surface of the table.

TEXTBOOK QUESTION 19. A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it too accelerates. What force causes the crate to accelerate.

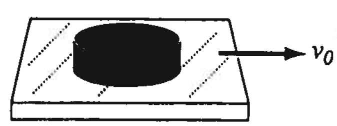
ANSWER: The net force acting on the crate is due to static friction. In order for the crate to remain at rest relative to the surface of the bed of the truck, the frictional force must be sufficient to cause the crate to accelerate at the same rate as the truck. The maximum value of the frictional force is related to the coefficient of static friction as well as the normal force. If the rate of acceleration of the truck exceeds a certain value, the frictional force will be insufficient to accelerate the crate at the same rate as the truck and the crate will slide toward the back of the truck.

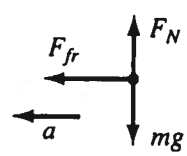
EXAMPLE PROBLEM 4. A hockey puck of mass 0.50 kg traveling at 10 m/s on a horizontal surface slows to 2.0 m/s over a distance of 80 m. Determine the a) magnitude of the frictional force acting on the puck and b) coefficient of kinetic friction between the puck and the surface.

Part a. Step 1.

Draw an accurate diagram locating the forces acting on the puck, then draw a free body diagram.

Solution: (Sections 4-4 and 4-7)





Part a. Step 2.

Complete a data table.

$m = 0.50$ kg, $x = 80$ m

$v_o = 10$ m/s, $x_o = 0$ m, $v = 2.0$ m/s

Part a. Step 3.

Use the kinematics equations to determine the acceleration.

$v^2 = v_o^2 + 2\,a\,(x - x_o)$

$(2.0\text{ m/s})^2 = (10\text{ m/s})^2 + 2\,a(80\text{ m} - 0)$

$a = -\,0.60\text{ m/s}^2$

Part a. Step 4.

Use Newton's second law to determine the magnitude of the frictional force.

$\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

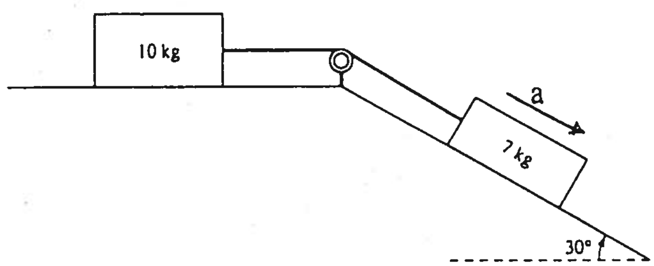
$F_{fr} = (0.50\text{ kg})(-\,0.60\text{ m/s}^2)$

$F_{fr} = -\,0.30$ newtons

Note: the frictional force is negative because it is in the direction opposite from the direction of motion.

Part b. Step 1. Determine the magnitude and direction of the normal force.	Since the puck is on a level surface, the normal force is equal in magnitude but in the opposite direction from the puck's weight. $F_N = mg = (0.50\text{ kg})(9.8\text{ m/s}^2)$ $F_N = 4.9\text{ N}$
Part b. Step 2. Determine the coefficient of kinetic friction between the puck and the ice.	$F_{fr} = \mu_k F_N = \mu_k mg$ $0.30\text{ N} = \mu_k (0.50\text{ kg})(9.8\text{ m/s}^2)$ $\mu_k = 0.061$

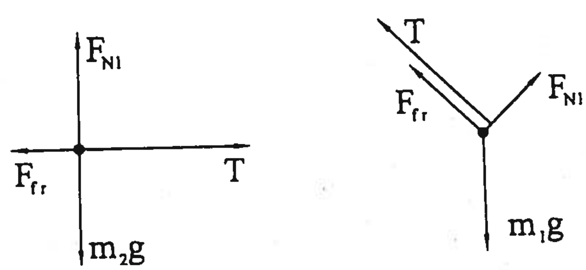
EXAMPLE PROBLEM 5. As shown in the diagram, a 10.0-kg box on a level surface is attached by a weightless, unstretchable rope to a 7.00-kg box that rests on a 30.0° incline. The coefficient of kinetic friction between each box and the surface is 0.100. The system is released from rest and both objects accelerate toward the right. Determine the a) rate of acceleration of the system and b) tension in the rope that connects the boxes.



Part a. Step 1.

Draw a free body diagram locating the forces acting on each object. Note: these forces include tension in the rope, weight, normal force, and frictional force acting on each.

Solution: (Sections 4-4 and 4-7)



(10.0-kg box) (7.00-kg box)

Part a. Step 2.

Determine the components of the 7.00 kg object's weight that are perpendicular and parallel to the incline.

$F_{\parallel} = m_1 g \sin 30.0°$

$F_{\parallel} = (7.00\text{ kg})(9.80\text{ m/s}^2)(0.500) = 34.3\text{ N}$

$F_{\perp} = m_1 g \cos 30.0°$

$F_{\perp} = (7.00\text{ kg})(9.80\text{ m/s}^2)(0.866) = 59.4\text{ N}$

Part a. Step 3. Determine the magnitude of the frictional force acting on the box.	The frictional force is given by $F_{fr} = \mu_k F_N$, where F_N is equal in magnitude but opposite in direction to $F_\perp$. For the box on the horizontal surface, the force of friction $F_{fr} = \mu_k F_N = \mu_k m_2 g \cos 0°$ $F_{fr} = (0.100)(10.0 \text{ kg})(9.80 \text{ m/s}^2)(1.00) = 9.80 \text{ N}$ For the box on the 30.0° incline, the frictional force $F_{fr} = \mu_k m_1 g \cos 30.0° = (0.100)(59.4 \text{ N})$ $F_{fr} = 5.94 \text{ N}$
Part a. Step 4. Use Newton's second law to write a separate equation for the motion of each object.	The motion of object 1 is parallel to the incline, i.e., $F_\parallel$ causes the motion while F_{fr} and the tension in the string retard the motion. object 1: (7.00 kg box) $F_\parallel - F_{fr} - T = m_1 a$ $34.3 \text{ N} - 5.94 \text{ N} - T = (7.00 \text{ kg}) a$ $28.4 \text{ N} - T = (7.00 \text{ kg}) a$ The motion of the 10.0 kg box, i.e., object 2, is along the level surface. The tension in rope causes object 2 to accelerate while friction retards its motion. object 2: (10.0 kg box) $T - F_{fr} = m_2 a$ $T - 9.80 \text{ N} = (10.0 \text{ kg}) a$
Part a. Step 5. Use algebra to solve for the rate of acceleration of the system.	The above equations have the same two unknowns. To solve for the acceleration, add the two equations together. $28.4 \text{ N} - T + T - 9.80 \text{ N} = (7.00 \text{ kg} + 10.0 \text{ kg}) a$ $18.6 \text{ N} = (17.0 \text{ kg}) a$ and $a = 1.09 \text{ m/s}^2$
Part a. Step 6. Solve for the acceleration considering only the external forces acting on the system.	Tension is an internal force and the problem can be solved by considering only the external forces acting on the system. The forces acting on the system are $F_\parallel$ and the retarding force of by friction acting on each box. Apply the second law and solve for the acceleration. Net F = m a $F_\parallel - F_{fr1} - F_{fr2} = (m_1 + m_2) a$ $34.4 \text{ N} - 5.97 \text{ N} - 9.80 \text{ N} = (7.0 \text{ kg} + 10.0 \text{ kg}) a$ $18.5 \text{ N} = (17.0 \text{ kg}) a$ and $a = 1.09 \text{ m/s}^2$

Part b. Step 1. Solve for the tension in the rope.	$T - 9.80\ N = (10.0\ kg)(1.09\ m/s^2)$ $T = 20.8\ N$

PROBLEM SOLVING SKILLS

For problems related to Newton's second law of motion:

1. Draw an accurate diagram locating each of the forces acting on the object or system of objects.
2. Draw a free body diagram locating the forces acting on the object(s) in question.
3. If the object is on an incline, the weight is replaced with components acting parallel and perpendicular to the incline.
4. If a frictional force is involved, the magnitude of the force is related to the coefficient of friction and the normal force.
5. Determine the magnitude of the net force acting on the object.
6. Use Newton's second law to write an equation for the motion of each object in the system. Solve for the rate of acceleration of each object in the system.
7. If the problem involves uniform acceleration, then the kinematics equations developed for uniformly accelerated motion can be used to determine the velocity, displacement, time, etc.

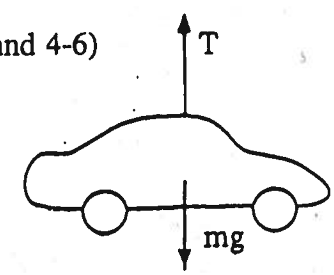
SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 10. How much tension must a rope withstand if it is used to accelerate a 1200 kg car vertically upward at 0.80 m/s^2?

Part a. Step 1.

Draw a free body diagram locating the forces acting on the car.

Solution: (Sections 4-4 and 4-6)



Part a. Step 2.

Apply Newton's second law and solve for the tension.

The car accelerates upward; therefore, the tension (T) is greater than the weight, therefore the $\Sigma F = T - mg$.

$\Sigma F = ma$

$T - (1200\ kg)(9.8\ m/s^2) = (1200\ kg)(0.80\ m/s^2)$

$T - 11760\ N = 960\ N$

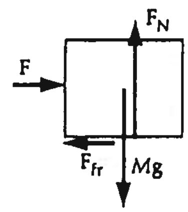
$T = 1.3 \times 10^4\ N$

TEXTBOOK PROBLEM 37. A force of 48.0 N is required to start a 5.0-kg box moving across a horizontal concrete floor. (a) What is the coefficient of static friction between the box and the floor? (b) If the 48.0 N force continues, the box accelerates at 0.70 m/s^2. What is the coefficient of kinetic friction?

Part a. Step 1.

Draw a free body diagram locating the forces acting on the box.

Solution: (Section 4-8)



Part a. Step 2.

Use Newton's second law to solve for the coefficient of static friction.

Note: the force of static friction $F_{fr} \leq \mu_s F_N$. When the box is just about to move $F_{fr} = \mu_s F_N$ where $F_N = m$ g. At that point the rate of acceleration is zero.

$\Sigma \vec{\mathbf{F}} = m \vec{\mathbf{a}}$

$F - F_{fr} = ma$ but $a = 0$ m/s^2 and $F_{fr} = \mu_s m_1 g$

48.0 N $- \mu_s (5.0$ kg$)(9.8$ m/s$^2) = 0$ m/s^2

$- \mu_s = (- 48.0$ N$)/(49$ N$)$

$\mu_s = 0.98$

Part b. Step 1.

Use Newton's second law to solve for the coefficient of kinetic friction (μ_k).

When the box begins to move, kinetic friction opposes the motion.

$\Sigma \vec{\mathbf{F}} = m \vec{\mathbf{a}}$

$F - F_{fr} = m\,a$ where $F_{fr} = \mu_k m_1 g$

48.0 N $- \mu_k (5.0$ kg$)(9.8$ m/s$^2) = (5.0$ kg$)(0.70$ m/s$^2)$

48.0 N $- \mu_k (49$ N$) = 3.5$ N

$- \mu_k = (- 48.0$ N $+ 3.5$ N$)/(49$ N$)$

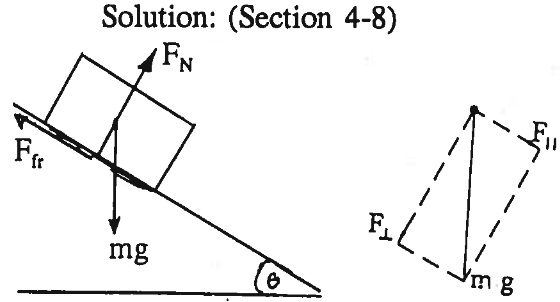
$\mu_k = 0.91$

TEXTBOOK PROBLEM 52. A carton shown in Fig. 4-55 lies on a plane tilted at an angle $\theta = 22.0°$ to the horizontal, with $\mu_k = 0.12$. (a) Determine the rate of acceleration of the carton as it slides down the plane. (b) If the carton starts from rest 9.30 m up the plane from the base, what will be the carton's speed when it reaches the bottom of the incline?

Part a. Step 1.

Draw a free body diagram locating all of the forces acting on the carton.

Solution: (Section 4-8)



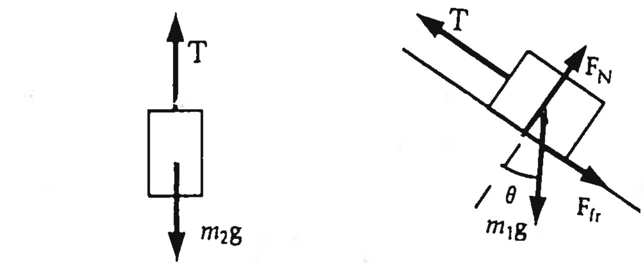
Part a. Step 2. Determine the components of the object's weight perpendicular and parallel to the incline.	$F_{\parallel}/mg = \sin 22.0°$ and $F_{\parallel} = mg \sin 22.0° = 0.375\ mg$ $F_{\perp}/mg = \cos 22.0°$ and $F_{\perp} = mg \cos 22.0° = 0.927\ mg$
Part a. Step 3. Determine the magnitude and direction of the carton's acceleration.	There is no motion perpendicular to the incline; the magnitude of the normal force equals the perpendicular component of the weight. The frictional force is related to the perpendicular component of the weight by the formula $F_{fr} = \mu_k F_N$, where F_N is equal in magnitude but opposite in direction to $F_{\perp}$. The component of the weight parallel to the incline causes the motion. Since $F_{\parallel}$ is greater than F_{fr}, the $\Sigma F = F_{\parallel} - F_{fr}$. $\Sigma \vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ $F_{\parallel} - F_{fr} = m\,a$ where $F_{fr} = \mu_k F_N = \mu_k\, mg \cos\theta$ $(0.375)\ mg - (0.12)(0.927)\ mg = m\,a$ Note that the mass of the object appears in each term. Dividing each term by m gives $(9.8\ m/s^2)(0.375) - (0.12)(9.8\ m/s^2)(0.927) = a$ $3.68\ m/s^2 - 1.09\ m/s^2 = a$ $a = 2.6\ m/s^2$
Part b. Step 1. Determine the object's speed at the bottom of the incline. Note: the initial speed at the top $v_o \approx 0$.	$2\,a\,(x - x_o) = v^2 - v_o^2$ $2\,(2.6\ m/s^2)(9.30\ m) = v^2 - (0\ m/s)^2$ $v^2 = 48.4\ m^2/s^2$ $v = 7.0\ m/s$

TEXTBOOK PROBLEM 64. (a) Suppose the coefficient of kinetic friction between the m_1 and the plane shown in Fig. 4-57 is $\mu_k = 0.150$ and that $m_1 = m_2 = 2.70$ kg. As m_2 moves down, determine the magnitude of the acceleration of m_1 and m_2 given $\theta = 25.0°$. (b) What smallest value of μ_k will keep this system from accelerating?

Part a. Step 1.

Draw a free body diagram locating the forces acting on each object. Note: these forces include tension in the rope, the weight, the normal force, and frictional force acting on each object.

Solution: (Section 4-8)



object m_2 object m_1

Part a. Step 2.

Determine the components of m_1's weight that are perpendicular and parallel to the incline.

$F_{\parallel}/m_1g = \sin 25.0°$

$F_{\parallel} = m_1g \sin 25.0° = (2.70\text{ kg})(9.80\text{ m/s}^2)(0.423) = 11.2\text{ N}$

$F_{\perp}/m_1g = \cos 25.0°$

$F_{\perp} = m_1g \cos 25.0° = (2.70\text{ kg})(9.80\text{ m/s}^2)(0.906) = 24.0\text{ N}$

Part a. Step 3.

Determine the magnitude of the frictional force (F_{fr}) acting on m_1.

For the box on the 25.0° incline, $F_{fr} = \mu_k F_N$, where F_N is equal in magnitude but opposite in direction to $F_{\perp}$.

$F_{fr} = \mu_k m_1g \cos 25.0° = (0.150)(24.0\text{ N}) = 3.60\text{ N}$

Part a. Step 4.

Use Newton's second law to write a separate equation for the motion of each object.

The motion of object 1 is parallel to the incline, i.e., the tension (T) in the string causes the motion while $F_{\parallel}$ and F_{fr} retard the motion.

object 1: $T - F_{\parallel} - F_{fr} = m_1 a$

$T - 11.2\text{ N} - 3.60\text{ N} = (2.70\text{ kg})\, a$

(equation 1) $T - 14.8\text{ N} = (2.70\text{ kg})\, a$

The motion of m_2 is directed downward. The weight of m_2 causes the motion while the tension in the string opposes the motion.

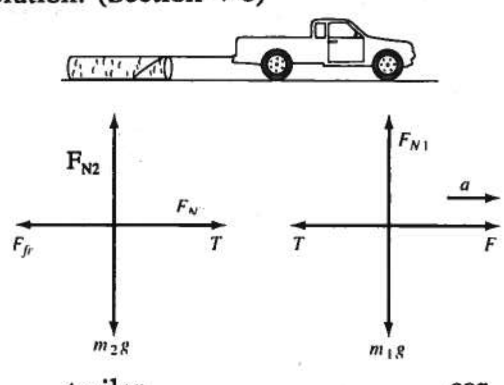
object 2: $m_2 g - T = m_2 a$

$(2.70\text{ kg})(9.80\text{ m/s}^2) - T = (2.70\text{ kg})\, a$

(equation 2) $26.5\text{ N} - T = (2.70\text{ kg})\, a$

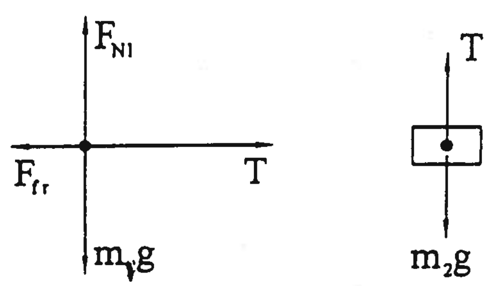
Part a. Step 5. Use algebra to solve for the rate of acceleration of the system.	The above equations have the same two unknowns. To solve for the acceleration, add the two equations together. 26.5 N - T + T - 14.8 N = (2.70 kg) a + (2.70 kg) a 11.7 N = (5.40 kg) a and a = 2.17 m/s^2
Part b. Step 1. Determine the smallest value of μ_k which will keep this system from accelerating.	If the system does not accelerate, then $\Sigma F = 0$. $F_{\parallel} - F_{fr} = 0$ $m_1 g \sin 25.0° - \mu_k m_1 g \cos 25.0° = 0$ note that $m_1 g$ cancels $\sin 25.0° - \mu_k \cos 25.0° = 0$ solving for μ_k gives $\mu_k = (\sin 25.0°)/(\cos 25.0°) = \tan 25.0°$ $\mu_k = 0.47$

TEXTBOOK PROBLEM 69. A 1150 kg car pulls a 450 kg trailer. The car exerts a horizontal force of 3.8 x 10^3 N against the ground in order to accelerate. What force does the car exert on the trailer. Assume an effective friction coefficient of 0.15 for the trailer.

Part a. Step 1. Draw a free body diagram locating the forces acting on each object. Note: these forces include tension in the connection between the car and the trailer, weight, normal force, and frictional force acting on the trailer.	Solution: (Section 4-8) 
Part a. Step 2. Use Newton's second law to write a separate equation for the motion of each object.	$\Sigma F = m a$ $F_{car} - T = m_{car}\, a$ (car) 3.8 x 10^3 N - T = (1150 kg) a $T - F_{fr} = m_{trailer}\, a$ (trailer)

	$T - \mu_k m_{trailer} g \cos 0° = m_{trailer} a$ $T - (0.15)(450 \text{ kg})(9.80 \text{ m/s}^2)(1.00) = (450 \text{ kg}) a$ $T - 660 \text{ N} = (450 \text{ kg}) a$
Part a. Step 3. Use algebra to solve for the rate of acceleration of the system.	The above equations have the same two unknowns. To solve for the acceleration, add the two equations together. $3.8 \times 10^3 \text{ N} - T + T - 660 \text{ N} = (1150 \text{ kg} + 450 \text{ kg}) a$ $3.1 \times 10^3 \text{ N} = (1600 \text{ kg}) a$ and $a = 2.0 \text{ m/s}^2$
Part b. Step 1. Solve for the tension in the connection between the car and the trailer.	$T - 660 \text{ N} = (450 \text{ kg}) a$ $T - 660 \text{ N} = (450 \text{ kg})(2.0 \text{ m/s}^2)$ $T = 1.54 \times 10^3 \text{ N}$

TEXTBOOK PROBLEM 76. A 28.0-kg block is connected to an empty 1.35-kg bucket by a cord running over a frictionless pulley (Fig. 4-59). The coefficient of static friction between the table and the block is 0.450 and the coefficient of kinetic friction between the table and the block is 0.320. Sand is gradually added to the bucket until the system just begins to move. (a) Calculate the mass of sand added to the bucket. (b) Calculate the acceleration of the system.

Part a. Step 1. Draw a free body diagram locating the forces acting on each object. Note: let m_2 represent the mass of the bucket + sand while m_1 represents the mass of the block.	Solution: (Section 4-7) 
Part a. Step 2. Use Newton's second law to write a separate equation for the motion for each object.	Once enough sand is added, the bucket + sand accelerate downward, therefore, $m_2g > T$ Net F = m a $m_2 g - T = m_2 a$ (equation 1) The normal force F_N that the table exerts upward balances the weight of object 1. However, a frictional force F_{fr} opposes the motion of m_1. Since m_1 accelerates horizontally across the table, $T > F_{fr}$ and

	$T - F_{fr} = m_1\,a$ where $F_{fr} = \mu_s\,F_N = \mu_s\,m_1\,g$ (equation 2)
Part a. Step 3. Use algebra to solve for the mass m_2 required to just cause the system to move.	At the point where motion is *about* to begin, the force of static friction will be a maximum but the rate of acceleration equals zero. The above equations have the same two unknowns. To solve for m_2, add the two equations together. $m_2\,g - T + T - \mu_s\,m_1\,g = (m_2 + m_1)\,a$ $m_2\,g - \mu_s\,m_1\,g = (m_2 + m_1)\,a$ (equation 3) but $a = 0\ m/s^2$ $m_2\,(9.8\ m/s^2) - (0.450)(28.0\ kg)(9.8\ m/s^2) = 0$ $m_2 = (123\ N)/(9.8\ m/s^2) = 12.6\ kg$ m_2 equals the mass of the bucket + sand. Since the mass of the bucket is 1.35 kg, the mass of sand is 11.3 kg.
Part b. Step 1. Use equation 3 from Part a. Step 3 to solve for the rate of acceleration.	Once the system begins to move, the motion is opposed by kinetic friction instead of static friction; i.e., $F_{fr} = \mu_k\,F_N = \mu_k\,m_1\,g$. $m_2\,g - \mu_k\,m_1\,g = (m_2 + m_1)\,a$ $(12.6\ kg)(9.8\ m/s^2) - (0.320)(28.0\ kg)(9.8\ m/s^2) = (12.6\ kg + 28.0\ kg)\,a$ $123\ N - 87.8\ N = (40.6\ kg)\,a$ $a = 0.879\ m/s^2$