CHAPTER 7

LINEAR MOMENTUM

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- define linear momentum and write the mathematical formula for linear momentum from memory.
- distinguish between the unit of force and momentum.
- write Newton's second law of motion in terms of momentum.
- define impulse and write the equation that connects impulse and momentum.
- state the law of conservation of momentum and write, in vector form, the law for a system involving two or more point masses.
- distinguish between a perfectly elastic collision and a completely inelastic collision.
- apply the laws of conservation of momentum and energy to problems involving collisions between two point masses.
- define center of mass and center of gravity and distinguish between the two concepts.

KEY TERMS AND PHRASES

linear momentum ($\vec{\mathbf{p}}$) is a measure of the product of an object's mass (m) and its velocity ($\vec{\mathbf{v}}$).

law of conservation of momentum states that in any collision between two or more objects, the vector sum of the momenta before impact equals the vector sum of the momenta after impact. The total momentum remains constant, i.e., the momentum is conserved.

impulse is defined as the product of the force ($\vec{\mathbf{F}}$) acting on an object and the time (Δt) during which the force acts. The impulse equals the change in the object's momentum.

perfectly elastic collisions occur when the kinetic energy as well as the linear momentum is conserved.

inelastic collisions occur when some (or most) of the kinetic energy of the objects involved in a collision is converted into heat during impact. Problems involving this type of collision can be solved by applying the law of conservation of momentum.

center of gravity (cg) of an object is the point where the entire weight of the object can be considered to be concentrated. At the center of gravity the entire weight of the object can be balanced by a single vertical force equal to the object's weight.

center of mass (CM) is the point where all of the mass of an object can be considered to be concentrated. For almost all objects, the center of mass is at the same point in the object as the center of gravity.

translational motion is where, at any instant, the motion of all points of a moving object has the same velocity and direction of motion. This type of motion, as well as the concept of center of mass, has been implied in solving problems in previous chapters.

SUMMARY OF MATHEMATICAL FORMULAS

momentum	$\vec{p} = m\vec{v}$	Linear momentum ($\vec{p}$) is defined as the product of an object's mass and its velocity.
Newton's second law	$\vec{F} = \Delta\vec{p}/\Delta t$	The average force on an object equals the rate of change of the object's momentum.
conservation of momentum	$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$	In a collision between two or more objects the total momentum is conserved.
impulse	$\vec{F}\Delta t$	Impulse equals the product of the force and the time during which the force acts.
impulse and momentum	$\vec{F}\Delta t = \Delta\vec{p} = m\vec{v}' - m\vec{v}$	An impulse equals the change in an object's momentum.
coordinate positions of the center of mass	$x_{CM} = (\Sigma m_i x_i)/(\Sigma m_i)$ $y_{CM} = (\Sigma m_i y_i)/(\Sigma m_i)$	These equations are used to determine the horizontal and/or vertical position of the center of mass.
translational velocity of the center of mass	$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$	This equation is used to determine the translational velocity of the center of mass.

CONCEPT SUMMARY

Linear Momentum and Force

Linear momentum ($\vec{p}$) is defined as the product of an object's mass (m) and its velocity ($\vec{v}$). Momentum is a vector quantity and the direction of the momentum vector is the same as the velocity vector. The formula for linear momentum is

$\vec{p} = m\vec{v}$ The unit of linear momentum is kg m/s. This is NOT the same as the unit of force, which is $1\ N = 1\ kg\ m/s^2$.

Newton's second law may be written in terms of the average force required to change an object's momentum as follows:

$\vec{F} = \Delta\vec{p}/\Delta t$ where $\Delta\vec{p}$ is the change in momentum and Δt is the time interval during which the change in momentum occurs.

Conservation of Linear Momentum

In any collision between two or more objects, the vector sum of the momenta before impact equals the vector sum of the momenta after impact. The total momentum remains constant, i.e., the momentum is conserved. This principle is known as the **law of conservation of momentum.** For a collision between two objects, the law of conservation of momentum can be written as follows:

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the objects before impact and $\vec{v}_1'$ and $\vec{v}_2'$ are the velocities of the objects after impact.

TEXTBOOK QUESTION 4. It is said that in ancient times a rich man with a bag of gold coins was frozen to death while stranded on the surface of a frozen lake. Because the ice was frictionless, he could not push himself to shore. What could he have done to save himself had he not been so miserly?

ANSWER: Assuming that the man is at rest on the ice, the initial momentum of the system is zero. If he should throw the bag of coins, he would recoil in the opposite direction. His mass is greater than that of the coins and therefore his recoil speed would be much smaller than the speed of the bag of coins. The final momentum of the coins is equal but opposite the man's momentum and the total final momentum equals zero. Thus, the law of conservation of momentum is upheld. Since the ice is frictionless, his recoil velocity would remain constant until he reached the bank. At that point he could walk around to the opposite bank and retrieve the bag of coins.

An alternate possibility would be to keep the coins in his pocket and throw his shoe or boot. The result would be the same and he would save himself and his gold.

TEXTBOOK QUESTION 10. A light body and a heavy body have the same kinetic energy. Which has the greater momentum?

ANSWER: Assume that the light body has a mass of 1.0 kg and the mass of the heavy body is 4.0 kg. If the speed of the light body is 4.0 m/s then its KE = ½ (1.0 kg)(4.0 m/s)^2 = 8.0 J. The speed of the heavy body can now be determined because it has an equal amount of KE:

$8.0\ \text{J} = \frac{1}{2}(4.0\ \text{kg})\, v^2$ and $v = 2.0$ m/s.

Momentum is the product of the mass and the velocity. The momentum of the light body is (1.0 kg)(4.0 m/s) = 4.0 kg m/s and the momentum of the heavy body is (4.0 kg)(2.0 m/s) = 8.0 kg m/s. Thus, while both objects have the same KE, the heavy object has twice as much momentum.

Impulse

An **impulse** is defined as the product of the force ($\vec{\mathbf{F}}$) acting on an object and the time (Δt) during which the force acts.

impulse = $\vec{\mathbf{F}}\,\Delta t$

The unit of impulse is newton seconds (N s) where 1 N s = 1 kg m/s. Impulse is a vector quantity and the direction of the impulse vector is the same as that of the force vector.

An impulse causes a change in an object's momentum. The mathematical formula that connects impulse and change in momentum is as follows:

$$\vec{\mathbf{F}}\,\Delta t = \Delta\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$$

In a collision between two or more objects, the force each object exerts on the other is usually very large compared to any other force acting and the time interval during which the interaction occurs is usually very short. Usually, both the magnitude of the force and the time interval remain unknown; however, the impulse can be determined if it is possible to determine the change in momentum.

TEXTBOOK QUESTION 7. Cars used to be built as rigid as possible to withstand collisions. Today, though, cars are designed to have "crumple zones" that collapse upon impact. What is the advantage of this new design?

ANSWER: During a collision an outside impulse acts on the car and changes the car's momentum. The change in momentum of the car remains the same whether there are crumple zones or not. However, crumple zones increase the time during which the force exerted by the outside object acts. The increase in time results in a smaller average force acting on the car (and the occupants). Assuming the crumple zones are properly designed, the kinetic energy is dissipated in the form of heat energy and the deformation of the metal.

As an added safety measure, the crumple zones in modern cars are designed to crumple around the passenger compartment. The desired result is that the passenger compartment will remain intact and the passengers are protected from major injury.

EXAMPLE PROBLEM 1. A 0.0600 kg tennis ball is traveling at 30.0 m/s. After being hit by the opponent's racket, the ball's velocity is 20.0 m/s in the opposite direction. Compute the a) change in the ball's momentum and b) average force exerted by the racket if the ball and racket were in contact for 0.0400 s. Hint: Assume that the ball's initial direction of motion is the positive direction.

Part a. Step 1. Determine the change in the tennis ball's momentum.	Solution: (Section 7-3) Velocity is a vector quantity. Therefore, if the initial velocity is +30.0 m/s, the final velocity is a -20.0 m/s. $\Delta\vec{\mathbf{p}} = m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$ $= (0.0600\text{ kg})(-20.0\text{ m/s}) - (0.0600\text{ kg})(+30.0\text{ m/s})$

	$\Delta\vec{\mathbf{p}}$ = -1.92 kg m/s
Part b. Step 1. Use the impulse-momentum equation to determine the net force acting on the ball.	$\vec{F}\,\Delta t = m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$ **F** (0.0400 s) = -1.92 kg m/s $\vec{F}$ = -48.0 N Note: the negative sign indicates that the direction of the force is in the direction arbitrarily defined as the negative direction, i.e., in the same direction that the ball is moving after impact.

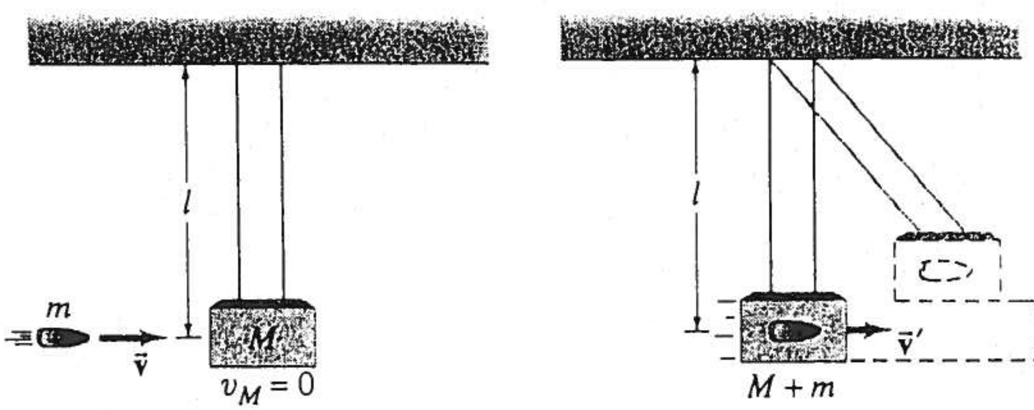
Energy and Momentum Conservation in Collisions

In collisions between two (or more) objects, the total momentum is always conserved. However, some of the kinetic energy may be converted into other forms of energy, usually thermal energy. Problems involving two types of collisions can be solved by using the methods of this chapter: perfectly elastic collisions and completely inelastic collisions.

In **perfectly elastic collisions,** the kinetic energy as well as the linear momentum is conserved. The sum of the kinetic energies of the objects before the collision is equal to the sum of the kinetic energies of the objects after the collision. No heat energy results from the collision. Certain perfectly elastic collision problems can be solved by applying both the law of conservation of energy and the law of conservation of momentum.

In **completely inelastic collisions,** the objects stick together after impact. Most of the kinetic energy is converted into heat. This type of problem can be solved by applying the law of conservation of momentum.

EXAMPLE PROBLEM 2. As shown in the diagram, a bullet of mass 0.0500 kg traveling at 50.0 m/s is fired horizontally into a wooden block suspended from a long rope. The mass of the wooden block is 0.300 kg and it is initially at rest. The collision is completely inelastic and after impact the bullet + wooden block move together until the center of mass of the system rises a vertical distance h above its initial position. Calculate the a) velocity of the bullet + wooden block just after impact and b) vertical distance *h* reached by the bullet + wooden block.



Part a. Step 1.

Use the law of conservation of momentum to determine the velocity of the bullet + pendulum just after impact.

Solution: (Section 7-6)

The initial momentum equals the final momentum. The collision is completely inelastic. The bullet and wooden block stick together after impact; therefore,

$\vec{v}_b' = \vec{v}_w' = \vec{v}'$

$m_b \vec{v}_b + m_w \vec{v}_w = (m_b + m_w)\vec{v}'$

$(0.0500 \text{ kg})(50.0 \text{ m/s}) + (0.300 \text{ kg})(0 \text{ m/s}) = (0.0500 \text{ kg} + 0.300 \text{ kg})\vec{v}'$

$\vec{v}' = (2.50 \text{ kg m/s})/0.350 \text{ kg})$

$\vec{v}' = 7.14 \text{ m/s}$

Part b. Step 1.

The collision is completely inelastic. The objects stick together after impact. Use the law of conservation of energy to solve for the velocity just after impact.

$KE_{\text{after impact}} = PE_{\text{at top of swing}}$

$\frac{1}{2}(m_b + m_w)v'^2 = (m_b + m_w)\, g\, h$

the mass cancels out, and rearranging gives

$\frac{1}{2} v'^2 = g\, h$

$\frac{1}{2}(7.14 \text{ m/s})^2 = (9.8 \text{ m/s}^2)\, h$ and $h = 2.60 \text{ m}$

EXAMPLE PROBLEM 3. A 2.0 kg object traveling at 1.0 m/s collides head-on with a 1.0 kg object initially at rest. Determine the velocity of each object after the impact if the collision is perfectly elastic.

Part a. Step 1.

Write an equation for the collision using the law of conservation of momentum.

Solution: (Sections 7-4 and 7-5)

$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$

$(2.0 \text{ kg})(1.0 \text{ m/s}) + (1.0 \text{ kg})(0 \text{ m/s}) = (2.0 \text{ kg})\, \vec{v}_1' + (1.0 \text{ kg})\, \vec{v}_2'$

$2 \text{ m/s} = 2\, \vec{v}_1' + 1\, \vec{v}_2'$ (equation 1)

Part a. Step 2.

Write an equation for the collision using the law of conservation of energy.

The collision is perfectly elastic; therefore, kinetic energy is conserved.

$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'^2 + \frac{1}{2} m_2 v'^2$

Since ½ appears in each term, the equation may be simplified by dividing each term by ½. Substituting values gives

$(2.0 \text{ kg})(1.0 \text{ m/s})^2 + (1.0 \text{ kg})(0 \text{ m/s})^2 = (2.0 \text{ kg})\, v_1'^2 + (1.0 \text{ kg})\, v_2'^2$

Upon simplifying, $2 \text{ m}^2/\text{s}^2 = 2\, v_1'^2 + 1\, v_2'^2$ (equation 2)

Part a. Step 3. Application of the conservation laws has resulted in two algebraic equations with the same two unknowns. Solve for the final velocity of each object.	from equation 1: $\vec{v}_2' = 2 - 2\,\vec{v}_1'$ Substituting for $\vec{v}_2'$ in equation 2, $2 = 2\,v_1'^2 + (2 - 2\,v_1')^2$ $2 = 2\,v_1'^2 + 4 - 8\,v_1' + 4\,v_1'^2$ $0 = 6\,v_1'^2 - 8\,v_1' + 2$ and simplifying gives $0 = 3\,v_1'^2 - 4\,v_1' + 1$ and factoring gives $0 = (3\,v_1' - 1)(\,v_1' - 1)$ Either $0 = 3\,v_1' - 1$ or $v_1' - 1 = 0$ and either $\vec{v}_1' = ⅓$ m/s or $\vec{v}_1' = 1$ m/s According to Newton's third law, after impact the velocity of object 1 must change. Therefore, the solution, $v_1' = 1$, while a valid solution, is uninteresting because it means that object 1 missed object 2, so object 2 is still at rest. The interesting answer is $v_1' = ⅓$ m/s. Substitute the value of v_1' into equation 1 and determine v_2'. $\vec{v}_2' = 2 - 2\,v_1' = 2 - 2\,(⅓)$ $\vec{v}_2' = 4/3$ m/s $= 1.3$ m/s

Center of Gravity and Center of Mass

The **center of gravity** (cg) of an object is the point where the entire weight of the object can be considered to be concentrated. At that point the entire weight of the object can be balanced by a single vertical force equal to the object's weight. For example, you can test for the center of gravity of a book by attempting to balance the book with one finger. The center of gravity of a book should be very close to the center of the book. For a baseball bat, the balance point and therefore the center of gravity is displaced from the center toward the fat end of the bat.

The **center of mass,** which is abbreviated CM, is the point where all of the mass of an object can be considered to be concentrated. For almost all objects, the center of mass is at the same point in the object as the center of gravity. An extreme example to show the difference is a thin, uniform rod that extends from the surface of the Earth vertically upward. The center of mass of the rod would be at its center. However, since the value of g decreases with altitude and the rod is extremely long, the lower half of the rod would weigh more than the upper half. In this case the center of gravity would fall below the center of mass.

The following formula can be used to determine the distance from one end to the center of mass of a massless rod along which a number of point masses have been attached.

$$x_{CM} = (m_1\,x_1 + m_2\,x_2 + \ldots + m_n\,x_n)/(m_1 + m_2 + \ldots + m_n)$$

EXAMPLE PROBLEM 4. The mass of the Sun is 2.0×10^{30} kg while the mass of the Earth is 6.0×10^{24} kg. The center-to-center distance between the Earth and the Sun is 1.50×10^{11} m. Determine the distance from the center of the Sun to the center of mass of the Earth-Sun system.

Part a. Step 1.	Solution: (Section 7-8)
The center of mass is to be measured from the center of the Sun. Therefore, $x_{Sun} = 0$ m. Complete a data table listing the mass and position of each object.	$m_{Sun} = 2.0 \times 10^{30}$ kg, $x_{Sun} = 0$ m $m_{Earth} = 6.0 \times 10^{24}$ kg, $x_{Earth} = 1.50 \times 10^{11}$ m
Part a. Step 2. Use the formula for determining the position of the center of mass of objects arranged along a horizontal line.	$x_{CM} = (m_{Sun}\, x_{Sun} + m_{Earth}\, x_{Earth})/(m_{Sun} + m_{Earth})$ $m_{Sun}\, x_{Sun} = (2.0 \times 10^{30}\ \text{kg})(0.0\ \text{m}) = 0$ kg m $m_{Earth}\, x_{Earth} = (6.0 \times 10^{24}\ \text{kg})(1.50 \times 10^{11}\ \text{m}) = 9.0 \times 10^{35}$ kg m $m_{Sun} + m_{Earth} = 2.0 \times 10^{30}\ \text{kg} + 6.0 \times 10^{24}\ \text{kg} \approx 2.0 \times 10^{30}$ kg $x_{CM} = (0\ \text{kg m} + 9.0 \times 10^{35}\ \text{kg m})/(2.0 \times 10^{30}\ \text{kg})$ $x_{CM} = 4.5 \times 10^{5}\ \text{m} \approx 280$ miles

Note: the Earth-Sun system revolves around the center of mass. The inside cover of the textbook lists the radius of the Sun as 6.96×10^{8} m (approximately 433,000 miles). Therefore, the center of mass of the Earth-Sun system is very close (approximately 280 miles) from the center of the Sun. For all practical purposes, the Earth revolves about the center of the Sun.

Center of Mass and Translational Motion

The term **translational motion** is used in situations where the motion of all points of a moving object have at any instant the same velocity and direction of motion. This type of motion, as well as the concept of center of mass, has been implied in solving problems in previous chapters of the textbook. For example, the free-body diagrams used in chapter 3 assumed that the mass of an object could be considered to be concentrated at a point with its weight acting at that point.

The concept of center of mass is also useful when discussing the motion of a group of particles. For example, as stated in the textbook, "The total linear momentum of a system of particles is equal to the product of the total mass M and the velocity of the center of mass of the system. Or, the linear momentum of an extended body is the product of the body's mass and the velocity of its CM." Therefore,

$$M\,\vec{v}_{cm} = m_1\,\vec{v}_1 + m_2\,\vec{v}_2 + m_3\,\vec{v}_3 + \ldots + m_n\,\vec{v}_n$$

Also, Newton's second law can be written

$$\Sigma \mathbf{F} = M\,\vec{a}_{CM}$$

And as stated in the textbook, "The sum of all of the forces acting on the system is equal to the total mass of the system times the acceleration of the center of mass."

PROBLEM SOLVING SKILLS

For problems involving impulse-change of momentum:

1. Draw an accurate diagram locating all of the forces acting on the system.
2. If a net external force acts on the object(s), then the momentum of the system will change. Determine the magnitude and direction of the net force.
3. Apply the impulse-momentum equation taking note that force and velocity are vectors and that direction of the vector plays an important part in the solution.

For problems involving no external force acting on the system:

1. Use the law of conservation of momentum to solve the problem. Take note that momentum is a vector quantity and must be considered in the solution.

For problems involving graphical integration:

1. Determine the sum of the partial and complete blocks that lie under the curve.
2. Determine the magnitude of the impulse represented by one block.
3. Determine the magnitude of the total impulse by multiplying the impulse represented by one block by the total number of blocks found in Step 2.
4. Use the impulse-momentum equation and solve the problem.

For problems involving perfectly elastic and completely inelastic collisions:

1. Determine which type of collision is described in the problem.
2. Use the law of conservation of momentum to solve problems where the collision is inelastic.
3. If the collision is perfectly elastic use both conservation of momentum and conservation of mechanical energy. Each law produces an algebraic equation with two unknowns. The final velocity of each object can be determined by applying standard algebraic techniques.

SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 4. A child in a boat throws a 6.40 kg package out horizontally with a speed of 10.0 m/s, Fig. 7-31. Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 26.0 kg and that of the boat is 45.0 kg. Ignore water resistance.

Part a. Step 1.	Solution: (Section 7-3)
Complete a data table including information both given and implied.	$m_1 = 6.40$ kg, $v_1 = 0$ m/s, $v_1' = +10.0$ m/s $m_2 = 26.0$ kg, $v_2 = 0$ m/s, $v_2' = ?$ $m_3 = 45.0$ kg, $v_3 = 0$ m/s, $v_3' = ?$ but the child and the boat recoil together; therefore, $\vec{v}_2' = \vec{v}_3' = \vec{v}' = ?$
Part a. Step 2.	$m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 = m_1 \vec{v}_1' + m_2 \vec{v}_2' + m_3 \vec{v}_3'$
Apply the law of conservation of momentum.	$(6.40\text{ kg})(0\text{ m/s}) + (26.0\text{ kg})(0\text{ m/s}) + (45.0\text{ kg})(0\text{ m/s}) =$ $(6.40\text{ kg})(10.0\text{ m/s}) + [(26.0\text{ kg}) + (45.0\text{ kg})]\,\vec{v}'$

	$0 = 64.0\ \text{kg m/s} + (71.0\ \text{kg})\ \vec{v}'$ $\vec{v}' = -\ 0.901\ \text{m/s}$ Note: the negative sign indicates that the child + boat recoil in the direction opposite from the package.

TEXTBOOK PROBLEM 10. A 3800 kg open railroad car coasts along with a constant speed of 8.60 m/s along a level track. Snow begins to fall vertically and fills the car at rate of 3.50 kg/min. Ignoring friction with the tracks, what is the speed of the car after 90 min?

Part a. Step 1. Complete a data table listing information both given and implied.	Solution: (Section 7-2) $m_1 = 3800\ \text{kg}$ $m_{2\text{initial}} = 0\ \text{kg}$ $v_1 = 8.60\ \text{m/s}$ $m_{2\text{final}} = (3.50\ \text{kg/min})(90\ \text{min}) = 315\ \text{kg}$ $v' = ?$ Note: the falling snow has no horizontal component of motion; therefore, $v_2 = 0\ \text{m/s}$.
Part a. Step 2. Use the law of conservation of momentum to solve for the speed of the car after 90 minutes.	$m_1\ \vec{v}_1 + m_2\ \vec{v}_2 = (m_1 + m'_2)\ \vec{v}'$ $(3800\ \text{kg})(8.60\ \text{m/s}) + (0.0\ \text{kg})(0\ \text{m/s}) = [(3800\ \text{kg}) + (3.50\ \text{kg/min})(90\ \text{min})]\ \vec{v}'$ $32680\ \text{kg m/s} = (4115\ \text{kg})\ \vec{v}'$ $\vec{v}' = 7.94\ \text{m/s}$

TEXTBOOK PROBLEM 12. A 23 g bullet traveling at 230 m/s penetrates a 2.0 kg block of wood and emerges cleanly at 170 m/s. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?

Part a. Step 1. Complete a data table listing information both given and implied.	Solution: (Section 7-2) $m_1 = 23\ \text{g} = 0.023\ \text{kg}$ $m_2 = 2.0\ \text{kg}$ $\vec{v}_1 = 230\ \text{m/s}$ $\vec{v}_2 = 0\ \text{m/s}$ $\vec{v}_1' = 170\ \text{m/s}$ $\vec{v}_2' = ?$
Part a. Step 2. Use the law of conservation of momentum to solve for the final speed of the block of wood.	$m_1\ \vec{v}_1 + m_2\ \vec{v}_2 = m_1\ \vec{v}_1' + m_2\ \vec{v}_2'$ $(0.023\ \text{kg})(230\ \text{m/s}) + (2.0\ \text{kg})(0\ \text{m/s}) = (0.023\ \text{kg})(170\ \text{m/s}) + (2.0\ \text{kg})\ \vec{v}_2'$ $5.29\ \text{kg m/s} = (3.91\text{kg m/s}) + (2.0\ \text{kg})\ \vec{v}_2'$ $\vec{v}_2' = 0.69\ \text{m/s}$

TEXTBOOK PROBLEM 15. A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 3.5×10^{-3} s. Find (a) the impulse imparted to the golf ball, and (b) the average force exerted on the ball by the golf club.

Part a. Step 1. Complete a data table listing information both given and implied.	Solution: (Section 7-3) $F = ?$ $\quad \Delta t = 3.5 \times 10^{-3}$ s $\quad m = 0.045$ kg $v = 0$ m/s $\quad v' = 45$ m/s
Part a. Step 2. Use the impulse-momentum equation to determine the impulse imparted to the golf ball.	impulse = $\vec{\mathbf{F}}\,\Delta t$ but $\vec{\mathbf{F}}\,\Delta t = m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$ impulse = $m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$ impulse = (0.045 kg)(45 m/s) - (0.0450 kg)(0.0 m/s) impulse = 2.0 N s
Part b. Step 1. Use the impulse-momentum equation to determine the average force on the ball.	$\vec{\mathbf{F}}\,\Delta t = m\,\vec{\mathbf{v}}' - m\,\vec{\mathbf{v}}$ $\vec{\mathbf{F}}\,(3.5 \times 10^{-3}$ s) = (0.045 kg)(45 m/s) - (0.0450 kg)(0.0 m/s) $\vec{\mathbf{F}} = 5.8 \times 10^{2}$ N

TEXTBOOK PROBLEM 23. A 0.450 kg ice puck, moving east with a speed of 3.00 m/s, has a head-on collision with a 0.900 kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

Part a. Step 1. Complete a data table listing information both given and implied.	Solution: (Section 7-4 and 7-5) $m_1 = 0.450$ kg $\quad m_2 = 0.900$ kg $\quad \vec{\mathbf{v}}_1 = 3.00$ m/s $\vec{\mathbf{v}}_2 = 0$ m/s $\quad \vec{\mathbf{v}}_1' = ?$ $\quad \vec{\mathbf{v}}_2' = ?$
Part a. Step 2. Write an equation for the collision using the law of conservation of momentum.	$m_1\,\vec{\mathbf{v}}_1 + m_2\,\vec{\mathbf{v}}_2 = m_1\,\vec{\mathbf{v}}_1' + m_2\,\vec{\mathbf{v}}_2'$ (0.450 kg)(3.00 m/s) + (0.900 kg)(0 m/s) = (0.450 kg) $\vec{\mathbf{v}}_1'$ + (0.900 kg) $\vec{\mathbf{v}}_2'$ Dividing each term by 0.450 kg, and then simplifying gives 3 m/s = 1 $\vec{\mathbf{v}}_1'$ + 2 $\vec{\mathbf{v}}_2'$ (equation 1)
Part a. Step 3. Write an equation for the collision using the law of conservation of energy.	The collision is perfectly elastic; therefore, kinetic energy is conserved. $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v'^2 + \frac{1}{2} m_2 v'^2$ Since ½ appears in each term, the equation may be simplified by dividing each term by ½. Substituting values and again dividing each term by 0.450 kg, and then simplifying gives

	$9\ m^2/s^2 = 1\ v_1'^2 + 2\ v_2'^2$ (equation 2)
Part a. Step 4. Application of the conservation laws has resulted in two algebraic equations with the same two unknowns. Solve for the final velocity of each object.	From equation 1: $\vec{v}_1' = 3 - 2\ \vec{v}_2'$ Substituting for $\vec{v}_1'$ in equation 2, $9 = (3 - 2\ v_2')^2 + 2\ v_2'^2$ $9 = 4\ v_2'^2 - 12\ v_2' + 9 + 2\ v_2'^2$ $0 = 6\ v_2'^2 - 12\ v_2'$ and dividing each term by 6 gives $0 = v_2'^2 - 2\ v_2'$ factor the equation and solve for v_2' $0 = (v_2')(v_2' - 2)$ Either $0 = v_2'$ or $0 = v_2' - 2$ and either $\vec{v}_2' = 0$ m/s or $\vec{v}_2' = 2.00$ m/s (east) $v_2' = 0$ indicates that object 1 missed object 2, so object 2 is still at rest. According to Newton's third law, after impact the velocity of object 2 must change. Since object 1 struck object 2, then the correct answer is $v_2' = 2.00$ m/s (east). Substitute the value of $\vec{v}_2'$ into equation 1 and determine $\vec{v}_1'$. $3\ m/s = 1\ \vec{v}_1' + 2\ \vec{v}_2'$ (equation 1) $3 = 1\ \vec{v}_1' + 2\ (2\ m/s)$ $\vec{v}_1' = -1.00$ m/s Note: the negative sign indicates that object 1 rebounds and travels west after impact.

TEXTBOOK PROBLEM 48. The CM of an empty 1050 kg car is 2.50 m behind the front of the car. How far from the front of the car will the CM be when two people sit in the front seat 2.80 m from the front of the car, and three people sit in the back seat 3.90 m from the front? Assume that each person has a mass of 70.0 kg.

Part a. Step 1. Complete a data table listing the mass and distance of each object from the front of the car.	Solution: (Section 7-8) Note: let the center of mass (CM) be measured from the front of the car. $m_{car} = 1050$ kg $\quad x_{car} = 2.50$ m $m_{people\ in\ front} = (2)(70.0\ kg) = 140$ kg $\quad x_{people\ in\ front} = 2.80$ m $m_{people\ in\ back} = (3)(70.0\ kg) = 210$ kg $\quad x_{people\ in\ back} = 3.90$ m

Part a. Step 2. Use the formula for determining the position of the center of mass of objects placed along a horizontal line.	$x_{CM} = [m_{car}\, x_{car} + m_{people\ in\ front}\, x_{people\ in\ front} + m_{people\ in\ back}\, x_{people\ in\ back}]/[m_{total}]$ $m_{car}\, x_{car} = (1050\ kg)(2.50\ m) = 2625\ kg\ m$ $m_{people\ in\ front}\, x_{people\ in\ front} = (140\ kg)(2.80\ m) = 392\ kg\ m$ $m_{people\ in\ back}\, x_{people\ in\ back} = (210\ kg)(3.90\ m) = 819\ kg\ m$ $m_{total} = 1050\ kg + 140\ kg + 210\ kg = 1400\ kg$ $x_{CM} = [2625\ kg\ m + 392\ kg\ m + 819\ kg\ m]/[1400\ kg]$ $x_{CM} = 2.74\ m$