10

Energy and Work

Using just a fast run-up and flexible pole, how can a pole vaulter reach an astonishing 6 m (20 ft) off the ground?

Looking Ahead ▸▸

The goal of Chapter 10 is to introduce the concept of energy and learn a new problem-solving strategy based on conservation of energy. In this chapter you will learn to:

- Understand some of the important forms of energy, and how energy can be transformed and transferred.
- Understand what work is, and how to calculate it.
- Understand and use the concepts of kinetic, potential, and thermal energy.
- Solve problems using the law of conservation of energy.
- Apply these ideas to elastic collisions.

Looking Back ◂◂

Part of our introduction to energy will be based on the kinematics of constant acceleration. In addition, we will need ideas from rotational motion. We will also use the before-and-after pictorial representation developed for impulse and momentum problems. Please review

- Section 2.4 Constant-acceleration kinematics.
- Section 7.5 Moment of inertia.
- Sections 9.2–9.3 Before-and-after visual overviews and conservation of momentum.

Energy. It's a word you hear all the time. We use chemical energy to heat our homes and bodies, electrical energy to run our lights and computers, and solar energy to grow our crops and forests. We're told to use energy wisely and not to waste it. Athletes and weary students consume "energy bars" and "energy drinks."

But just what is energy? The concept of energy has grown and changed with time, and it is not easy to define in a general way just what energy is. Rather than starting with a formal definition, we'll let the concept of energy expand slowly over the course of several chapters. In this chapter we introduce several fundamental forms of energy, including kinetic energy, potential energy, and thermal energy. Our goal is to understand the characteristics of energy, how energy is used, and, especially important, how energy is transformed from one form to another. For example, this pole vaulter, after years of training, has become extraordinarily proficient at transforming his energy of motion into energy associated with height from the ground.

We'll also discover a very powerful conservation law for energy. Some scientists consider the law of conservation of energy to be the most important of all the laws of nature. But all that in due time. First we have to start with the basic ideas.

10.1 A "Natural Money" Called Energy

We will start by discussing what seems to be a completely unrelated topic: money. As you will discover, monetary systems have much in common with energy. Let's begin with a short story.

The Parable of the Lost Penny

John was a hard worker. His only source of income was the paycheck he received each month. Even though most of each paycheck had to be spent on basic necessities, John managed to keep a respectable balance in his checking account. He even saved enough to occasionally buy a few savings bonds, his investment in the future.

John never cared much for pennies, so he kept a jar by the door and dropped all his pennies into it at the end of each day. Eventually, he reasoned, his saved pennies would be worth taking to the bank and converting into crisp new dollar bills.

John found it fascinating to keep track of these various forms of money. He noticed, to his dismay, that the amount of money in his checking account did not spontaneously increase overnight. Furthermore, there seemed to be a definite correlation between the size of his paycheck and the amount of money he had in the bank. So John decided to embark on a systematic study of money.

He began, as would any good scientist, by using his initial observations to formulate a hypothesis, which he called a *model* of the monetary system. He found that he could represent his monetary model with the flowchart in Figure 10.1.

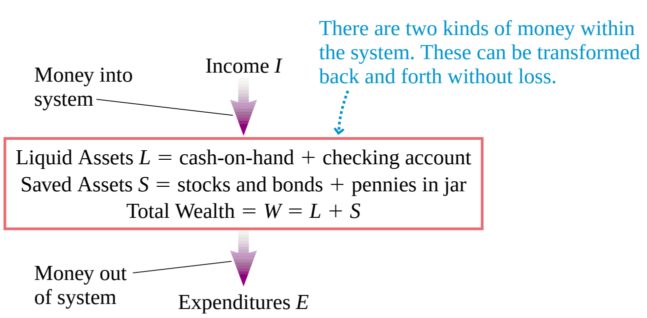


FIGURE 10.1 John's model of the monetary system.

As the chart shows, John divided his money into two basic types, liquid assets and saved assets. The *liquid assets* L, which included his checking account and the cash in his pockets, were moneys available for immediate use. His *saved assets* S, which included his savings bonds as well as the jar of pennies, had the potential to be converted into liquid assets, but they were not available for immediate use.

John decided to call the sum total of assets his *wealth:* $W = L + S$.

John's assets were, more or less, simply definitions. The more interesting question, he thought, was how his wealth depended on his *income* I and *expenditures* E. These represented money transferred *to* him by his employer and money transferred *by* him to stores and bill collectors. After painstakingly collecting and analyzing his data, John finally determined that the relationship between monetary transfers and wealth is

$$\Delta W = I - E$$

John interpreted this equation to mean that the *change* in his wealth, ΔW, was numerically equal to the *net* monetary transfer $I - E$.

During a week-long period when John stayed home sick, isolated from the rest of the world, he had neither income nor expenses. In grand confirmation of his hypothesis, he found that his wealth W_f at the end of the week was identical to his wealth W_i at the week's beginning. That is, $W_f = W_i$. This occurred despite the fact that he had moved pennies from his pocket to the jar and also, by telephone, had sold some bonds and transferred the money to his checking account. In other words, John found that he could make all of the *internal* conversions of assets from one form to another that he wanted, but his total wealth remained constant (W = constant) as long as he was isolated from the world. This seemed such a remarkable rule that John named it the *law of conservation of wealth.*

One day, however, John added up his income and expenditures for the week, and the changes in his various assets, and he was 1¢ off! Inexplicably, some money seemed to have vanished. He was devastated. All those years of careful research, and now it seemed that his monetary hypothesis might not be true. Under some circumstances, yet to be discovered, it looked like $\Delta W \neq I - E$. Off by a measly penny. A wasted scientific life. . . .

But wait! In a flash of inspiration, John realized that perhaps there were other types of assets, yet to be discovered, and that his monetary hypothesis would still be valid if *all* assets were included. Weeks went by as John, in frantic activity, searched fruitlessly for previously *hidden* assets. Then one day, as John lifted the cushion off the sofa to vacuum out the potato chip crumbs—lo and behold, there it was!—the missing penny!

John raced to complete his theory, now including money in the sofa, the washing machine, and behind the radiator as previously unknown forms of assets that were easy to convert from other forms, but often rather difficult to recover. Other researchers soon discovered other types of assets, such as the remarkable find of the "cash in the mattress." To this day, when *all* known assets are included, monetary scientists have never found a violation of John's simple hypothesis that $\Delta W = I - E$. John was last seen sailing for Stockholm to collect the Nobel Prize for his Theory of Wealth.

10.2 The Basic Energy Model

John, despite his diligent efforts, did not discover a law of nature. The monetary system is a human construction that, by design, obeys John's "laws." Monetary system laws, such as that you cannot print money in your basement, are enforced by society, not by nature. But suppose that physical objects possessed a "natural money" that was governed by a theory, or model, similar to John's. An object might have several forms of natural money that could be converted back and forth, but the total amount of an object's natural money would *change* only if natural money were *transferred* to or from the object. Two key words here, as in John's model, are *transfer* and *change*.

One of the greatest and most significant discoveries of science is that there is such a "natural money" called **energy.** You have heard of some of the many forms of energy, such as solar energy or nuclear energy, but others may be new to you. These forms of energy can differ as much as a checking account differs from loose change in the sofa. Much of our study is going to be focused on the *transformation* of energy from one form to another. Much of modern technology is concerned with transforming energy, such as changing the chemical energy of oil molecules to electrical energy or to the kinetic energy of your car.

As we use energy concepts, we will be "accounting" for energy that is transferred in or out of a system or that is transformed from one form to another within a system. Figure 10.2 shows a simple model of energy that is based on John's model of the monetary system. Many details must be added to this model, but it's a good starting point. The fact that nature "balances the books" for energy is one of the most profound discoveries of science.

A major goal of ours is to discover the conditions under which energy is conserved. Surprisingly, the *law of conservation of energy* was not recognized until the mid-nineteenth century, long after Newton. The reason, similar to John's lost penny, was that it took scientists a long time to realize how many types of energy there are and the various ways that energy can be converted from one form to another. As you'll soon learn, energy ideas go well beyond Newtonian mechanics to include new concepts about heat, about chemical energy, and about the energy of the individual atoms and molecules that comprise an object. All of these forms of energy will ultimately have to be included in our accounting scheme for energy.

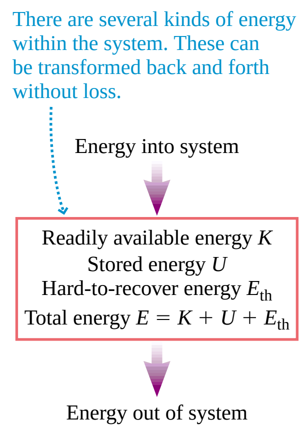


FIGURE 10.2 An initial model of energy.

Systems and Energy

In Chapter 9 we introduced the idea of a *system* of interacting objects. A system can be quite simple, such as a saltshaker sliding across the table, or much more complex, such as a city or a human body. But whether simple or complex, every system in nature has associated with it a quantity we call its **total energy** E. Like John's total wealth, which was made up of assets of many kinds, the total energy of a system is made up of many kinds of energies. In the table below, we give a brief overview of some of the more important forms of energy; in the rest of the chapter we'll look at several of these forms of energy in much greater detail.

Some important forms of energy

Kinetic energy K

Kinetic energy is the energy of *motion.* All moving objects have kinetic energy. The heavier an object, and the faster it moves, the more kinetic energy it has. The wrecking ball in this picture is effective in part because of its large kinetic energy.

Gravitational potential energy U_g

Gravitational potential energy is *stored* energy associated with an object's *height above the ground.* As this roller coaster ascends the track, energy is stored as increased gravitational potential energy. As it descends, this stored energy is converted into kinetic energy.

Elastic or spring potential energy U_s

Elastic potential energy is energy stored when a spring or other elastic object, such as this archer's bow, is *stretched.* This energy can later be transformed into the kinetic energy of the arrow. We'll sometimes use the symbol U to represent potential energy when it is not important to distinguish between U_g and U_s.

Thermal energy E_{th}

Hot objects have more *thermal energy* than cold ones because the molecules in a hot object jiggle around more than those in a cold object. Thermal energy is really just the sum of the microscopic kinetic and potential energies of all the molecules in an object. In boiling water, some molecules have enough energy to escape the water as steam.

Chemical energy E_{chem}

Electric forces cause atoms to bind together to make molecules. Energy can be stored in these bonds, energy that can later be released as the bonds are rearranged during chemical reactions. When we burn fuel to run our car, or eat food to power our bodies, we are using *chemical energy.*

Nuclear energy $E_{nuclear}$

An enormous amount of energy is stored in the *nucleus,* the tiny core of an atom. Certain nuclei can be made to break apart, releasing some of this *nuclear energy,* which is transformed into the kinetic energy of the fragments and then into thermal energy. This is the source of energy of nuclear power plants and nuclear weapons.

A system may have many of these kinds of energy present in it at once. For instance, a moving car has kinetic energy of motion, chemical energy stored in its gasoline, thermal energy in its hot engine, and other forms of energy in its many other parts. The total energy of the system, E, is just the *sum* of the different energies present in the system, so that we have

$$E = K + U_g + U_s + E_{th} + E_{chem} + \cdots \tag{10.1}$$

The energies shown in this sum are the forms of energy in which we'll be most interested in this and the next chapter. The ellipses (. . .) represent other forms of energy, such as nuclear or electric, that also might be present. We'll treat these and others in later chapters.

Energy Transformations

We've seen that all systems contain energy in many different forms. But if the amounts of each form of energy never changed, the world would be a very dull place. What makes the world interesting is that **energy of one kind can *trans-***

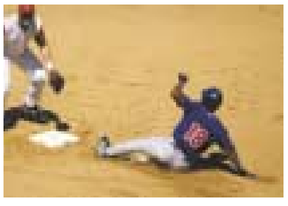
***form* into energy of another kind.** The gravitational potential energy of the roller coaster at the top of the track is rapidly converted into kinetic energy as the coaster descends; the chemical energy of gasoline is converted into the kinetic energy of your moving car. The following table illustrates a few common energy transformations. In this table, we'll use an arrow $\rightarrow$ as a shorthand way of representing an energy transformation.

Some energy transformations

A weightlifter lifts a barbell over her head

The barbell has much more gravitational potential energy when high above her head than when on the floor. To lift the barbell, she is transforming chemical energy in her body into gravitational potential energy of the barbell.

$$E_{\text{chem}} \rightarrow U_{\text{g}}$$



A base runner slides into the base

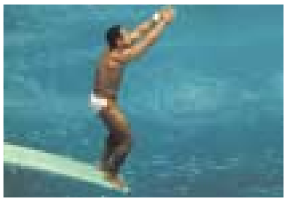
When running, he has lots of kinetic energy. After sliding, he has none. His kinetic energy is transformed mainly into thermal energy: the ground and his legs are slightly warmer.

$$K \rightarrow E_{\text{th}}$$

A burning campfire

The wood contains considerable chemical energy. When the carbon in the wood combines chemically with oxygen in the air, this chemical energy is transformed largely into thermal energy of the hot gases and embers.

$$E_{\text{chem}} \rightarrow E_{\text{th}}$$



A springboard diver

Here's a two-step energy transformation. The picture shows the diver after his first jump onto the board itself. At the instant shown, the board is flexed to its maximum extent. There is a large amount of elastic potential energy stored in the board. Soon this energy will begin to be transformed into kinetic energy; as he rises into the air and slows, this kinetic energy will be transformed into gravitational potential energy.

$$U_{\text{s}} \rightarrow K \rightarrow U_{\text{g}}$$

Figure 10.3 reinforces the idea that **energy transformations are changes of energy *within* the system from one form to another.** Note that it is easy to convert kinetic, potential, or chemical energies into thermal energy. But converting thermal energy back into these other forms is not so easy. How it can be done, and what possible limitations there might be in doing so, will form a large part of the next chapter.

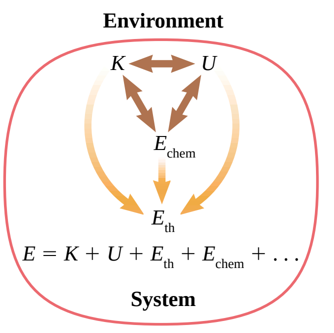


FIGURE 10.3 Energy transformations occur within the system.

Energy Transfers: Work and Heat

We've just seen that energy *transformations* occur between forms of energy *within* a system. In our monetary model, these transformations are like John's shifting of money between his own various assets, such as from his savings

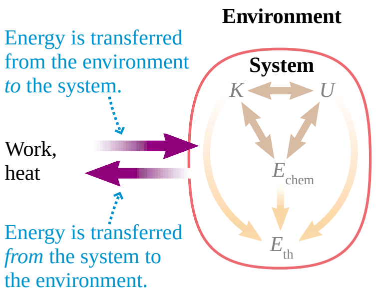


FIGURE 10.4 Work and heat are energy transfers into and out of the system.

One dictionary defines *work* as:

1. Physical or mental effort; labor.
2. The activity by which one makes a living.
3. A task or duty.
4. Something produced as a result of effort, such as a *work of art.*
5. Plural *works:* The essential or operating parts of a mechanism.
6. The transfer of energy to a body by application of a force.

account to stocks. But John also interacted with the greater world around him, receiving money as income and outlaying it as expenditures. Every physical system also interacts with the world around it, that is, with its *environment.* In the course of these interactions, the system can exchange energy with the environment. **An exchange of energy between system and environment is called an energy *transfer*.** There are two primary energy transfer processes: **work,** the *mechanical* transfer of energy to or from a system by pushing or pulling on it, and **heat,** the *nonmechanical* transfer of energy from the environment to the system (or vice versa) *because of a temperature difference between the two.* Figure 10.4 shows how our energy model is modified to include energy transfers. In this chapter we'll focus mainly on work; the concept of heat will be developed much further in Chapters 11 and 12.

Work is a common word in the English language, with many meanings. When you first think of work, you probably think of the first two definitions in this list. After all, we talk about "working out," or we say, "I just got home from work." But that is *not* what work means in physics.

In physics we use *work* in the sense of definition 6: Work is the process of *transferring* energy from the environment to a system, or from a system to the environment, by the application of mechanical forces—pushes and pulls—to the system. Once the energy has been transferred to the system, it can appear in many forms. Exactly what form it takes depends on the details of the system and how the forces are applied. The table below gives a few examples of energy transfers due to work. We use W as the symbol for work.

Energy transfers: work

Putting a shot

The system: The shot.

The environment: The athlete.

As the athlete pushes on the shot to get it moving, he is doing work on the system. That is, he is transferring energy from himself to the ball. The energy transferred to the system appears as kinetic energy.

The transfer: $W \rightarrow K$

Striking a match

The system: The match and matchbox.

The environment: The hand.

As the hand quickly pulls the match across the box, the hand does work on the system, increasing its thermal energy. The matchhead becomes hot enough to ignite.

The transfer: $W \rightarrow E_{th}$

Firing a slingshot

The system: The slingshot.

The environment: The boy.

As the boy pulls back on the elastic bands, he does work on the system, increasing its elastic potential energy.

The transfer: $W \rightarrow U_s$

Notice that in each example above, the environment applies a force while the system undergoes a *displacement.* Energy is transferred as work only when the system *moves* while the force acts. A force applied to a stationary object, such as when you push against a wall, transfers no energy to the object and thus does no work.

NOTE ► In the table above, energy is being transferred *from* the athlete *to* the shot by the force of his hand. We say he "does work" on the shot, or "work is done" by the force of his hand. ◄

It is also possible to convert work into gravitational potential, electric, or even chemical energy. We'll have much more to say about work in the next section. But the key points to remember are that **work is the transfer of energy to or from a system by the application of forces,** and that **the system must undergo a displacement for this energy to be transferred.**

There is a second, nonmechanical means of transferring energy between a system and its environment, which we discuss here only briefly. As mentioned before, we'll have much more to say about heat in the next two chapters. When a hot object is placed in contact with a cooler one, energy flows naturally from the hot object to the cool one. **The transfer of energy from a hot to a cold object is called *heat*,** and it is given the symbol Q. It is important to note that heat is not an energy *of* a system, as are kinetic energy and chemical energy. Rather, heat is energy *transferred* between two systems.

As the hand in the photo was held against the wall, heat was transferred from the warm hand to the cool wall, warming up the wall. The warm "handprint" can be imaged using a special camera sensitive to the temperature of objects.

STOP TO THINK 10.1 A child slides down a playground slide at constant speed. The energy transformation is

A. $U_g \to K$ B. $K \to U_g$ C. $W \to K$ D. $U_g \to E_{th}$ E. $K \to E_{th}$

10.3 The Law of Conservation of Energy

Remember that when John was *isolated* from the rest of the world—having neither income nor expenses—his internal wealth could be converted between its many forms, but his *total* wealth remained constant. A similar but much more fundamental law is found for the "natural money" of energy.

Let's start our study of this law by considering an **isolated system** that is separated from its surrounding environment in such a way that no energy can flow into or out of the system. This means that no work is done on the system, nor is any energy transferred as heat. We've already seen that the total energy of a system is made up of many forms of energy that are continually transforming from one kind to another. It is a deep and remarkable fact of nature that during these transformations, the total energy of an isolated system—the *sum* of all of the individual kinds of energy—remains *constant*. Any increase in, say, the system's kinetic energy must be accompanied by a decrease in its potential or thermal energies so that the total energy remains unchanged, as shown in Figure 10.5. We say that **the total energy of an isolated system is *conserved*,** giving us the following *law of conservation of energy.*

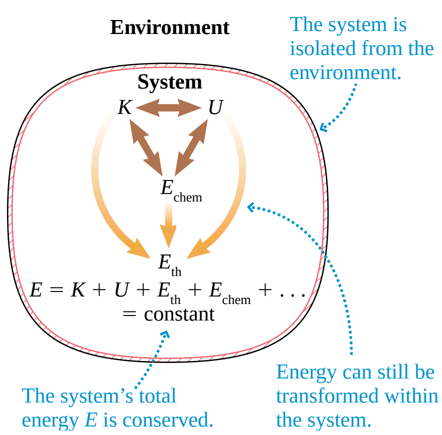


FIGURE 10.5 An isolated system.

Law of conservation of energy for an isolated system The total energy of an isolated system remains constant:

The energies in the system are constantly transforming from one kind to another . . .
. . . but their *sum* is a constant: it doesn't change.

$$K + U_g + U_s + E_{th} + E_{chem} + \ldots = E = \text{constant} \qquad (10.2)$$

Another way to think of this conservation law is in terms of energy *changes*. Recall that we denote the change in a quantity by the symbol Δ, so we write the change in a system's kinetic energy, for instance, as ΔK. Now suppose that an isolated system has its kinetic energy change by ΔK, its gravitational potential energy by ΔU_g, and so on. Then the sum of these changes is the change in the total energy. But since the total energy is constant, its change is *zero*. We can thus write the law of conservation of energy in an alternate form as

Law of conservation of energy for an isolated system (alternate form) The change in the total energy of an isolated system is zero:

$$\Delta E = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = 0 \qquad (10.3)$$

Any increase in one form of energy must be accompanied by a decrease in other forms, so that the total change is zero.

The law of conservation of energy sets a fundamental constraint on those processes that can occur in nature. In any process that occurs within an isolated system, the changes in each form of energy must add up to zero, as required by Equation 10.3.

CONCEPTUAL EXAMPLE 10.1 Energy changes in a bungee ride

A popular fair attraction is the trampoline bungee ride. The rider bounces up and down on large bungee cords. During part of her motion she is found to be moving upward with the cords becoming more stretched. Is she speeding up or slowing down during this interval?

REASON We'll take our system to include the rider, the bungee cords, and the *earth*. We'll see later how gravitational potential energy is stored in the *system* consisting of the earth and an object such as the rider. With this choice of system, to a good approximation the system is isolated, with no energy being transferred into or out of the system. Thus the total energy of the system is constant: $\Delta E = 0$.

Because she's moving upward, her height is increasing—and thus so is her gravitational potential energy. Thus $\Delta U_g > 0$. We also know that the cords are getting more stretched, hence more elastic potential energy is being stored. Thus $\Delta U_s > 0$ as well. Now the law of conservation of energy, Equation 10.3, states that $\Delta E = \Delta K + \Delta U_g + \Delta U_s = 0$, so that $\Delta K = -(\Delta U_g + \Delta U_s)$. Both ΔU_g and ΔU_s are positive, so ΔK must be *negative*. This means that her kinetic energy is *decreasing*. Since kinetic energy is energy of motion, this means that she's slowing down.

ASSESS In that part of her motion where she's moving upward and the cords are stretching, she's approaching the highest point of her motion. It makes sense that she's slowing down here, since at the high point her speed is instantaneously zero.

Systems That Aren't Isolated

When John had income and expenses, his total wealth could change. Indeed, he found that his wealth increased by exactly the amount of his income, and decreased by exactly the amount of his expenditures. Similarly, if a system is *not* isolated, so that it can exchange energy with its environment, the system's energy can change. We have seen that the two primary means of energy exchange are work and heat. If an amount of work W is done on the system, this means that an amount of energy W is transferred from the environment to the system, increasing the system's energy by exactly W. Similarly, if a certain amount of energy is transferred from a hot environment to a cooler system as heat Q, the system's energy will increase by exactly the amount Q. As illustrated in Figure 10.6, **the change in the system's energy is simply the sum of the work done on the system and the heat transferred to the system:**

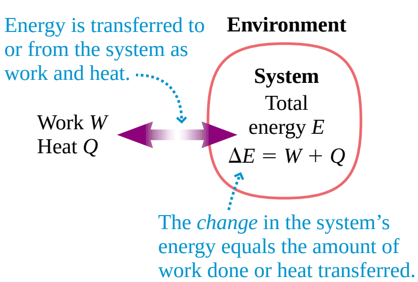


FIGURE 10.6 The law of conservation of energy.

$$\Delta E = W + Q$$

This gives us a more general statement of conservation of energy:

Law of conservation of energy including energy transfers The change in the total energy of a nonisolated system is equal to the energy transferred into or out of the system as work W or heat Q:

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \cdots = W + Q \qquad (10.4)$$

Equation 10.4 is the fullest expression of the law of conservation of energy. It's usually called the **first law of thermodynamics,** but it's really just a restatement of the law of conservation of energy to include the possibility of energy transfers. In this chapter we'll refer to it simply as the law of conservation of energy.

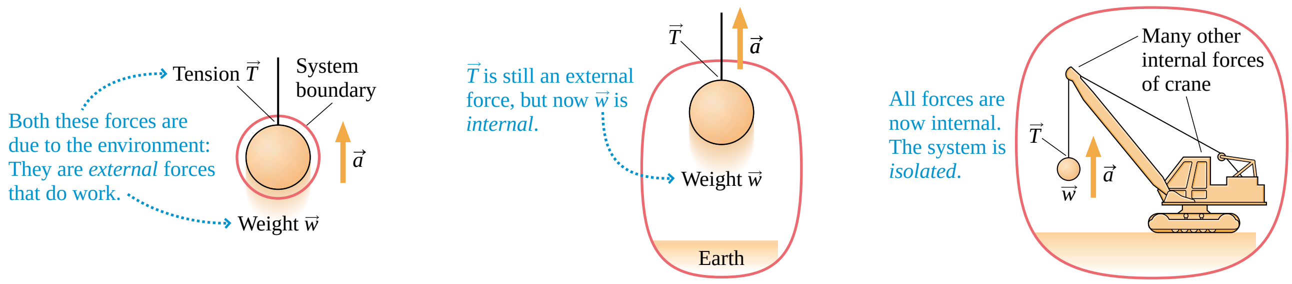
NOTE ► It's important to realize that even when the system is not isolated, energy is conserved overall. The energy transferred to the system as, say, work increases the energy of the system. But this energy is *removed* from the environment, so that the total energy of system *plus* environment is still conserved. ◄

Airplanes are assisted in takeoff from aircraft carriers by a steam-powered catapult under the flight deck. The force of this catapult does work W on the plane, leading to a large increase ΔK in the plane's kinetic energy.

Systems and Conservation of Energy

To apply the law of conservation of energy, you need to carefully define which objects make up the system and which belong to the environment. This choice will affect how we analyze the various energy transfers and tranformations that occur. In doing so, we need to make a distinction between two classes of forces. **Internal forces** are forces between objects *within* the system. If a weightlifter and barbell are both part of the system, the forces $\vec{F}_{\text{weightlifter on barbell}}$ and $\vec{F}_{\text{barbell on weightlifter}}$ are both internal forces. Internal forces are responsible for energy transformations within the system. Because they are internal to the system, however, **internal forces cannot do work on the system** and thereby change its energy. **External forces** act on the system, but their agent is part of the *environment.* **External forces *can* do work on the system,** transferring energy in or out of it. Whether a given force is an internal or external force depends on the choice of what's included in the system. The following table shows some choices for a crane accelerating a heavy ball upward.

Different choices of the system



System:	The ball only	Ball + earth	Ball + earth + crane
Internal forces:	None	$\vec{w}$	$\vec{T}$, $\vec{w}$, many internal forces of crane
External forces:	$\vec{T}$, $\vec{w}$	$\vec{T}$	None
System energies:	K	K, U_g	K, U_g, E_{chem}
Energy analysis:	Tension does positive work and the weight does negative work, but since $T > w$ the net work is positive. This work serves to increase the only energy of the system, its kinetic energy. Notice that since the earth is *not* part of the system, the system has no gravitational potential energy.	The weight force is now an *internal* force. That is, it is an interaction force between two objects—the ball and the earth—that are part of the system. The tension force is still an *external* force that does work on the system. This work increases the gravitational potential energy and the kinetic energy of the system.	Now all the forces are internal, and no work is done on the system: The system is *isolated.* With this choice of system, the increased potential and kinetic energy of the ball come from an energy *transformation* from the chemical energy of the crane's fuel.
Energy equation:	$\Delta K = W$	$\Delta K + \Delta U_g = W$	$\Delta K + \Delta U_g + \Delta E_{\text{chem}} = 0$

There are evidently many possible choices of the system for a given situation. However, certain choices can make problem solving using the law of energy conservation easier. For the crane above, we'd probably choose the second system consisting of the ball and the earth, since it is a good balance between reducing the number of external forces and having only simple system energies such as K

and U_g. The third choice would be hard to work with, since the many complicated internal forces are difficult to calculate. Tactics Box 10.1 gives some suggestions on how to make a good choice for the system.

MP TACTICS BOX 10.1 Choosing the system for conservation-of-energy problems Exercise 6

The system should include all of the objects identified as follows:

1. If the speed of an object or objects is changing, the system should include these moving objects because their kinetic energy is changing.
2. If the height of an object or objects is changing, the system should include the raised object(s) *plus* the earth. This is because potential energy is stored via the gravitational interaction of the earth and object(s).
3. If the compression or extension of a spring is changing, the system should include the spring because elastic potential energy is stored in the spring itself.
4. If kinetic or rolling friction is present, the system should include the moving object and the surface on which it slides or rolls. This is because thermal energy is created in both the moving object and the surface, and we want this thermal energy to all be within the system.

Working with Energy Transformations

The law of conservation of energy applies to every form of energy, from kinetic to chemical to nuclear. For the rest of this chapter, however, we'll narrow our focus a bit and only concern ourselves with the forms of energy typically transformed during the motion of ordinary objects. These energies are the kinetic energy K, the potential energy U (which includes both U_g and U_s), and thermal energy E_{th}. The sum of the kinetic and potential energy, $K + U = K + U_g + U_s$, is called the **mechanical energy** of the system. We'll also limit our analysis to energy transfers in the form of work W. In Chapter 11 we'll expand our scope to include other forms of energy listed in the earlier table, as well as energy transfers as heat Q.

The fact that energy is conserved can be a powerful tool for analyzing the dynamics of moving objects. To see how we can apply the law of conservation of energy to dynamics problems, let's use the fact that the change in any quantity is its final value minus its initial value so that, for example, $\Delta K = K_f - K_i$. Then we can write the law of conservation of energy, Equation 10.4 (with $Q = 0$), as

$$(K_f - K_i) + (U_f - U_i) + \Delta E_{th} = W \tag{10.5}$$

NOTE ► We don't rewrite ΔE_{th} as $(E_{th})_f - (E_{th})_i$ because the initial thermal energy of an object is typically unknown. Only the *change* in E_{th} can be measured. ◄

Rearranging, we have

$$\underbrace{K_i + U_i}_{\text{The initial energy of the system...}} + \underbrace{W}_{\text{...plus the energy transferred to the system as work...}} = \underbrace{K_f + U_f + \Delta E_{th}}_{\text{...equals the final system energy, now possibly including extra thermal energy.}} \tag{10.6}$$

If no external forces do work on the system, $W = 0$ in Equation 10.6 and the system is *isolated*. **If no kinetic friction is present, ΔE_{th} will be zero and mechanical energy will be conserved.** Equation 10.6 then becomes the **law of conservation of mechanical energy:**

$$K_i + U_i = K_f + U_f \tag{10.7}$$

Equations 10.6 and 10.7 summarize what we have learned about the conservation of energy, and they will be the basis of our strategy for solving problems using the law of conservation of energy. Much of the rest of this chapter will be concerned with finding quantitative expressions for the different forms of energy in the system and discussing the important question of what to include in the system. We'll use the following Problem-Solving Strategy as we further develop these ideas.

Spring into action BIO A locust can jump as far as one meter, an impressive distance for such a small animal. To make such a jump, its legs must extend much more rapidly than muscles can ordinarily contract. Thus, instead of using its muscles to make the jump directly, the locust uses them to more slowly stretch an internal "spring" near its knee joint. This stores elastic potential energy in the spring. When the muscles relax, the spring is suddenly released, and its energy is rapidly converted into kinetic energy of the insect.

MP **PROBLEM-SOLVING STRATEGY 10.1** **Conservation of energy problems**

PREPARE Choose what to include in your system (see Tactics Box 10.1). Draw a before-and-after visual overview, as outlined in Tactics Box 9.1. Note known quantities, and determine what quantity you're trying to find. If the system is isolated and if there is no friction, your solution will be based on Equation 10.7, otherwise you should use Equation 10.6.

Identify which mechanical energies in the system are changing:

- If the *speed* of the object is changing, include K_i and K_f in your solution.
- If the *height* of the object is changing, include $(U_g)_i$ and $(U_g)_f$.
- If the *length* of a spring is changing, include $(U_s)_i$ and $(U_s)_f$.
- If kinetic friction is present, ΔE_{th} will be positive. Some kinetic or potential energy will be transformed into thermal energy.

If an external force acts on the system, you'll need to include the work W done by this force in Equation 10.6.

SOLVE Depending on the problem, you'll need to calculate initial and/or final values of these energies and insert them into Equation 10.6 or 10.7. Then you can solve for the unknown energies, and from these any unknown speeds (from K), positions (from U), or displacements or forces (from W).

ASSESS Check the signs of your energies. Kinetic energy, as we'll see, is always positive. In the systems we'll study in this chapter, thermal energy can only increase, so that its change is positive. In Chapters 11 and 12 we'll study systems for which the thermal energy can decrease.

10.4 Work

We've already discussed work as the transfer of energy between a system and its environment by the application of forces on the system. We also noted that in order for energy to be transferred in this way, the system must undergo a displacement—it must *move*—during the time that the force is applied. Let's further investigate the relationship between work, force, and displacement. We'll find that there is a simple expression for work, which we can then use to quantify other kinds of energy as well.

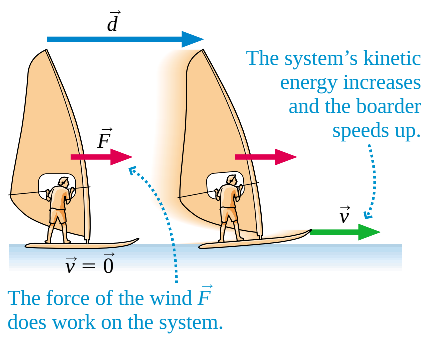


FIGURE 10.7 The force of the wind does work on the system, increasing its kinetic energy *K*.

Consider a system consisting of a windsurfer at rest, as shown on the left in Figure 10.7. Let's assume that there is no friction between his board and the water. Initially the system has no kinetic energy. But if a force from outside the system, such as the force due to the wind, begins to act on the system, the surfer will begin to speed up, and his kinetic energy will increase. In terms of energy transfers, we would say that the energy of the system has increased because of the work done on the system by the force of the wind.

What determines how much work is done by the force of the wind? First, we note that the greater the distance over which the wind pushes the surfer, the faster the surfer goes, and the more his kinetic energy increases. This implies a greater transfer of energy. So **the larger the displacement, the greater the work done.** Second, if the wind pushes with a stronger force, the surfer speeds up more rapidly, and the change in his kinetic energy is greater than with a weaker force. **The stronger the force, the greater the work done.**

This experiment suggests that the amount of energy transferred into a system by a force $\vec{F}$—that is, the amount of work done by $\vec{F}$—depends on both the magnitude F of the force *and* the displacement d of the system. Many experiments of this kind have established that the amount of work done by $\vec{F}$ is *proportional* to both F and d. For the simplest case described above, where the force $\vec{F}$ is constant and points in the direction of the object's displacement, the expression for the work done is found to be

$$W = Fd \tag{10.8}$$

Work done by a constant force $\vec{F}$ in the direction of a displacement $\vec{d}$

LINEAR p.38

The unit of work, that of force multiplied by distance, is N · m. This unit is so important that it has been given its own name, the **joule** (rhymes with *tool*). We define:

$$1 \text{ joule} = 1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

Since work is simply energy being transferred, **the joule is the unit of *all* forms of energy.** Note that work is a *scalar* quantity.

EXAMPLE 10.1 Work done in pushing a crate

Sarah pushes a heavy crate 3.0 m along the floor at a constant speed. She pushes with a constant horizontal force of magnitude 70 N. How much work does Sarah do on the crate?

PREPARE We begin with the visual overview in Figure 10.8. Sarah pushes with a constant force in the direction of the crate's motion, so we can use Equation 10.8 to find the work done.

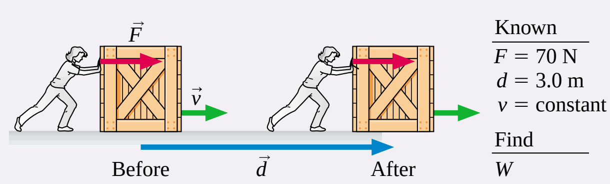


FIGURE 10.8 Sarah pushing a crate.

SOLVE The work done by Sarah is given by

$$W = Fd = (70 \text{ N})(3.0 \text{ m}) = 210 \text{ J}$$

ASSESS Since the crate moves at a constant speed, it must be in dynamic equilibrium with $\vec{F}_{net} = \vec{0}$. This means that a friction force (not shown) must act opposite to Sarah's push. If friction is present, Tactics Box 10.1 suggests taking the crate *and* the floor as the system. The work Sarah does represents energy transferred *into* the system. In this case, the work increases the thermal energy in the crate and the part of the floor along which it slid. Contrast this with the windsurfer, where work increased the windsurfer's kinetic energy. Both situations are consistent with the energy model shown in Figure 10.4, which you should review at this point.

Force at an Angle to the Displacement

Pushing a crate in the same direction as the crate's displacement is the most efficient way to transfer energy into the system, and so the largest possible amount of work is done. Less work is done if the force acts at an angle to the displacement. To see this, consider the kite buggy of Figure 10.9a, pulled along a horizontal path by the angled force of the kite string $\vec{F}$. As shown in Figure 10.9b, we can break $\vec{F}$ into a component $F_\perp$ perpendicular to the motion, and a component $F_\parallel$ parallel to the motion. Only the parallel component acts to accelerate the rider and increase his kinetic energy, so only the parallel component does work on the rider. From Figure 10.9b, we see that if the angle between $\vec{F}$ and the displacement is θ, then the parallel component is $F_\parallel = F\cos\theta$. So when the force acts at an angle θ to the direction of the displacement, we have

$$W = F_\parallel d = Fd\cos\theta \tag{10.9}$$

Work done by a constant force $\vec{F}$ at an angle θ to the displacement $\vec{d}$

Notice that this more general definition of work agrees with Equation 10.8 if $\theta = 0°$.

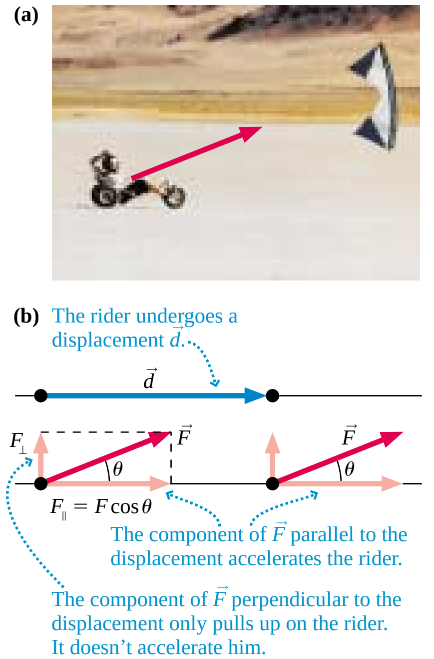


FIGURE 10.9 Finding the work done when the force is at an angle to the displacement.

CONCEPTUAL EXAMPLE 10.2 Work done by a parachute

A drag racer is slowed by a parachute. What is the sign of the work done?

REASON The drag force on the drag racer is shown in Figure 10.10, along with the dragster's displacement as it slows. The force points in the opposite direction to the displacement, so that the angle θ in Equation 10.9 is 180°. Then $\cos\theta = \cos(180°) = -1$. Since F and d in Equation 10.9 are magnitudes, and hence positive, this means that the work $W = Fd\cos\theta = -Fd$ done by the drag force is *negative*.

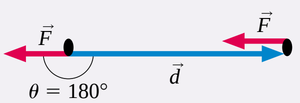


FIGURE 10.10 The force acting on a drag racer.

ASSESS Applying Equation 10.4, the law of conservation of energy, to this situation, we have

$$\Delta K = W$$

because the only system energy that changes is the racer's kinetic energy K. Since the kinetic energy is decreasing, its change ΔK is negative. This agrees with the sign of W. This example illustrates the general principle that **negative work represents a transfer of energy out of the system.**

Tactics Box 10.2 shows how to calculate the work done by a force at any angle to the direction of motion. The system illustrated is a block sliding on a frictionless horizontal surface, so that only the kinetic energy is changing. However, the same relationships hold for any object undergoing a displacement.

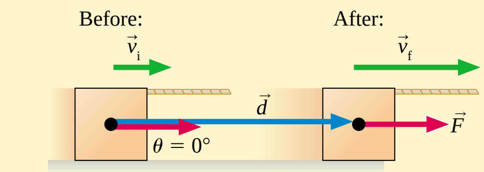
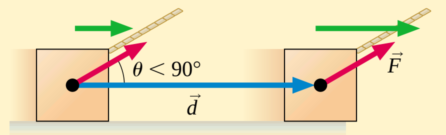
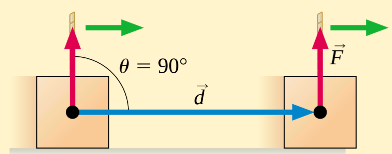
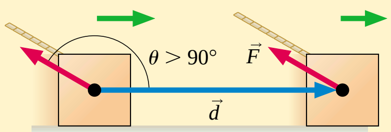
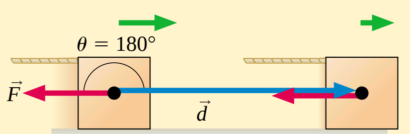
ActivPhysics 5.1

The quantities F and d are always positive, so **the sign of W is determined entirely by the angle θ between the force and the displacement.** Note that

Equation 10.9, $W = Fd\cos\theta$, is valid for any angle θ. In three special cases, $\theta = 0°$, $\theta = 90°$, and $\theta = 180°$, however, there are simple versions of Equation 10.9 that you can use. These are noted in Tactics Box 10.2.

TACTICS BOX 10.2 Calculating the work done by a constant force

Exercises 9,11,12

Direction of force relative to displacement	Angles and work done	Sign of W	Energy transfer
	$\theta = 0°$ $\cos\theta = 1$ $W = Fd$	+	The force is in the direction of motion. The block has its greatest positive acceleration. K increases the most: **Maximum energy transfer to system.**
	$\theta < 90°$ $W = Fd\cos\theta$	+	The component of force parallel to the displacement is less than F. The block has a smaller positive acceleration. K increases less: **Moderate energy transfer to system.**
	$\theta = 90°$ $\cos\theta = 0$ $W = 0$	0	There is no component of force in direction of motion.The block moves at constant speed. No change in K: **No energy transferred.**
	$\theta > 90°$ $W = Fd\cos\theta$	−	The component of force parallel to the displacement is opposite to the motion. The block slows down, and K decreases: **Moderate energy transfer *out* of system.**
	$\theta = 180°$ $\cos\theta = -1$ $W = -Fd$	−	The force is directly opposite to the motion. The block has it greatest deceleration. K decreases the most. **Maximum energy transfer out of system.**

EXAMPLE 10.2 Work done in pulling a suitcase

A strap inclined upward at a 45° angle pulls a suitcase through the airport. The tension in the strap is 20 N. How much work does the tension do if the suitcase is pulled 100 m at a constant speed?

PREPARE Figure 10.11 shows a visual overview. Since the case moves at a constant speed, there must be a rolling friction force acting to the left. Tactics Box 10.1 suggests in this case that we take as our system the suitcase and the floor upon which it rolls.

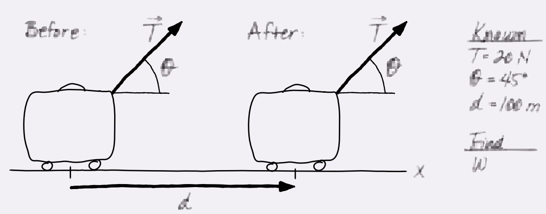


FIGURE 10.11 A suitcase pulled by a strap.

SOLVE We can use Equation 10.9 to find that the tension does work

$$W = Td\cos\theta = (20\ \text{N})(100\ \text{m})\cos 45° = 1400\ \text{J}$$

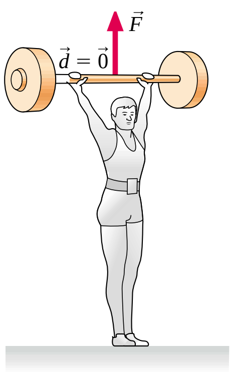
ASSESS Because a person is pulling on the other end of the strap, causing the tension, we would say informally that the person does 1400 J of work on the suitcase. This work represents energy transferred into the suitcase/floor system. Since the suitcase moves at a constant speed, the system's kinetic energy doesn't change. Thus the work goes entirely into increasing the *thermal* energy E_{th} of the suitcase and the floor.

If several forces act on an object that undergoes a displacement, each does work on the object. The **total (or net) work** W_{total} is the sum of the work done by each force. The total work represents the total energy transfer *to* the system from the environment (if $W_{total} > 0$) or *from* the system to the environment (if $W_{total} < 0$).

Forces That Do No Work

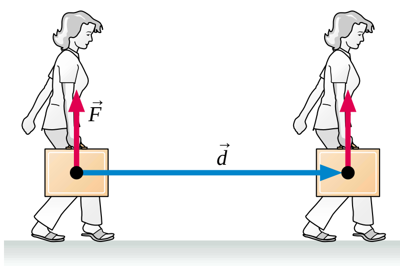
The fact that a force acts on an object doesn't mean that the force will do work on the object. The table below shows three common cases where a force does no work.

Forces that do no work



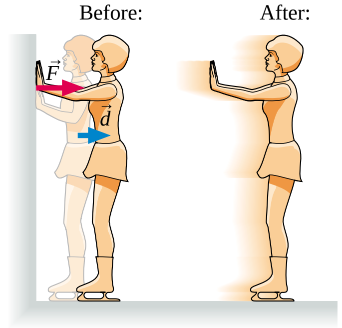
If the object undergoes no displacement while the force acts, no work is done.

This can sometimes seem counterintuitive. The weightlifter struggles mightily to hold the barbell over his head. But during the time the barbell remains stationary, he does no work on it because its displacement is zero. But why then is it so hard for him to hold it there? We'll see in Chapter 11 that it takes a rapid conversion of his internal chemical energy to keep his arms extended under this great load.



A force perpendicular to the displacement does no work.

The woman exerts only a vertical force on the briefcase she's carrying. This force has no component in the direction of the displacement, so the briefcase moves at a constant velocity and its kinetic energy remains constant. Since the energy of the briefcase doesn't change, it must be that no energy is being transferred to it as work. (This is the case where $\theta = 90°$ in Tactics Box 10.2.)

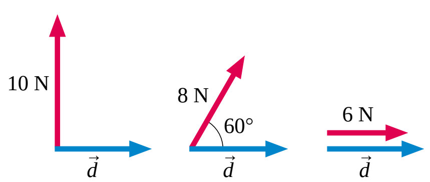


If the part of the object on which the force acts undergoes no displacement, no work is done.

Even though the wall pushes on the skater with a normal force $\vec{n}$ and she undergoes a displacement $\vec{d}$, the wall does no work on her, because the point of her body on which $\vec{n}$ acts—her hands—undergoes no displacement. This makes sense: How could energy be transferred as work from an inert, stationary object? So where does her kinetic energy come from? This will be the subject of much of Chapter 11. Can you guess?

STOP TO THINK 10.2 Which force does the most work?

A. The 10 N force.
B. The 8 N force.
C. The 6 N force.
D. They all do the same amount of work.



10.5 Kinetic Energy

We've already qualitatively discussed kinetic energy, an object's energy of motion. Let's now use what we've learned about work, and some simple kinematics, to find a quantitative expression for kinetic energy. Consider the system consisting of a car being pulled by a tow rope as in Figure 10.12. The rope pulls with a constant force $\vec{F}$ while the car undergoes a displacement $\vec{d}$, so that the force does work $W = Fd$ on the car. If we ignore friction and drag, the work done by $\vec{F}$ will

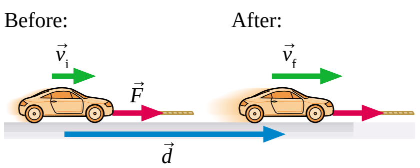


FIGURE 10.12 The work done by the tow rope increases the car's kinetic energy.

be transferred entirely into the car's energy of motion—its kinetic energy. In this case, the law of conservation of energy, Equation 10.6, reads

$$K_\mathrm{i} + W = K_\mathrm{f}$$

or

$$W = K_\mathrm{f} - K_\mathrm{i} \tag{10.10}$$

Using kinematics, we can find another expression for the work done, in terms of the car's initial and final speeds. Recall from Chapter 2 the kinematic equation relating an object's displacement and its change in velocity:

$$v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a\Delta x$$

Applied to the motion of our car, $\Delta x = d$ is the car's displacement and, from Newton's second law, the acceleration is $a = F/m$. Thus we can write

$$v_\mathrm{f}^2 = v_\mathrm{i}^2 + \frac{2Fd}{m} = v_\mathrm{i}^2 + \frac{2W}{m}$$

where we have replaced Fd with the work W. If we now solve for the work, we find

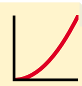
$$W = \frac{1}{2}m(v_\mathrm{f}^2 - v_\mathrm{i}^2) = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_\mathrm{i}^2$$

If we compare this result with Equation 10.10, we see that

$$K_\mathrm{f} = \frac{1}{2}mv_\mathrm{f}^2 \qquad \text{and} \qquad K_\mathrm{i} = \frac{1}{2}mv_\mathrm{i}^2$$

In general, then, an object moving at a speed v has kinetic energy

$$K = \frac{1}{2}mv^2 \tag{10.11}$$



Kinetic energy of an object of mass m moving with speed v

TABLE 10.1 Some approximate kinetic energies

Object	Kinetic energy
Walking ant	1×10^{-8} J
Penny dropped 1 m	2.5×10^{-3} J
Person walking	70 J
100 mph fastball	150 J
Bullet	5000 J
Car, 60 mph	5×10^{5} J
Supertanker	2×10^{10} J

From Equation 10.11, the units of kinetic energy are mass times speed squared, or kg · (m/s)^2. But

$$1\ \mathrm{kg} \cdot (\mathrm{m/s})^2 = \underbrace{1\ \mathrm{kg} \cdot (\mathrm{m/s^2})}_{1\,\mathrm{N}} \cdot \mathrm{m} = 1\,\mathrm{N} \cdot \mathrm{m} = 1\,\mathrm{J}$$

We see that the units of kinetic energy are the same as those of work, as they must be. Table 10.1 gives some approximate kinetic energies. Everyday kinetic energies range from a tiny fraction of a fraction of a joule to nearly a million joules for a speeding car.

CONCEPTUAL EXAMPLE 10.3 Kinetic energy changes for a car

Compare the increase in a 1000 kg car's kinetic energy as it speeds up by 5.0 m/s starting from 5.0 m/s, to its increase in kinetic energy as it speeds up by 5.0 m/s starting from 10 m/s.

REASON The change in the car's kinetic energy in going from 5 m/s to 10 m/s is

$$\Delta K_{5\to10} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_\mathrm{i}^2$$

This gives

$$\Delta K_{5\to10} = \frac{1}{2}(1000\ \mathrm{kg})(10\ \mathrm{m/s})^2 - \frac{1}{2}(1000\ \mathrm{kg})(5.0\ \mathrm{m/s})^2$$
$$= 3.8 \times 10^4\ \mathrm{J}$$

while

$$\Delta K_{10\to15} = \frac{1}{2}(1000\ \mathrm{kg})(15\ \mathrm{m/s})^2 - \frac{1}{2}(1000\ \mathrm{kg})(10\ \mathrm{m/s})^2$$
$$= 6.3 \times 10^4\ \mathrm{J}$$

Even though the increase in the car's *speed* was the same in both cases, the increase in kinetic energy is substantially larger in the second case.

ASSESS Kinetic energy depends on the *square* of the speed v. If we plot the kinetic energy of the car as in Figure 10.13, we see that the energy of the car increases rapidly with speed. We can also see graphically why the change in K for a fixed 5 m/s change in v is greater at high speeds than at low speeds. In part this is why it's harder to accelerate your car at high speeds than at low speeds.

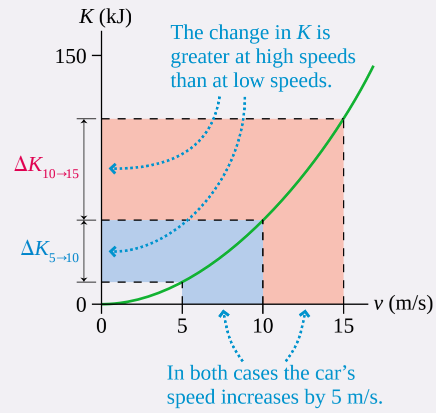
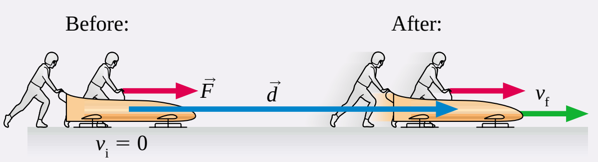


FIGURE 10.13 The kinetic energy increases as the *square* of the speed.

EXAMPLE 10.3 Speed of a bobsled after pushing

A two-man bobsled has a mass of 390 kg. Starting from rest, the two racers push the sled for the first 50 m with a net force of 270 N. Neglecting friction, what is the sled's speed at the end of the 50 m?

PREPARE This is the first example where we fully use Problem-Solving Strategy 10.1. We start by identifying the bobsled as the system; the two racers pushing the sled are part of the environment. The racers do work on the system by pushing it with force $\vec{F}$. Because the speed of the sled changes, we'll need to include kinetic energy. Neither U_g nor U_s changes, so we won't need to consider these energies. Figure 10.14 lists the known quantities and the quantity (v_f) that we want to find.



Known
$m = 390$ kg $\quad F = 270$ N
$d = 50$ m $\quad v_i = 0$ m/s

Find: v_f

FIGURE 10.14 The work done by the pushers increases the sled's kinetic energy.

SOLVE With only kinetic energy changing, the conservation of energy equation, Equation 10.6, is

$$K_i + W = K_f$$

Using our expressions for kinetic energy and work, this becomes

$$\frac{1}{2}mv_i^2 + Fd = \frac{1}{2}mv_f^2$$

Because $v_i = 0$, the energy equation reduces to

$$Fd = \frac{1}{2}mv_f^2$$

We can solve for the final speed to get

$$v_f = \sqrt{\frac{2Fd}{m}} = \sqrt{\frac{2(270\text{ N})(50\text{ m})}{390\text{ kg}}} = 8.3\text{ m/s}$$

ASSESS We solved this problem using the concept of energy conservation. In this case, we could also have solved it using Newton's second law and kinematics. However, we'll soon see that energy conservation can solve problems that would be very difficult for us to solve using Newton's laws alone.

STOP TO THINK 10.3 Rank in order, from greatest to least, the kinetic energies of the sliding pucks.

Rotational Kinetic Energy

We've just found an expression for the kinetic energy of an object moving along a line or some other path. This energy is called **translational kinetic energy.** Consider now an object rotating about a fixed axis, such as the windmill blades in Figure 10.15. Although the blades have no overall translational motion, each

FIGURE 10.15 The large rotating blades of a windmill have a great deal of kinetic energy.

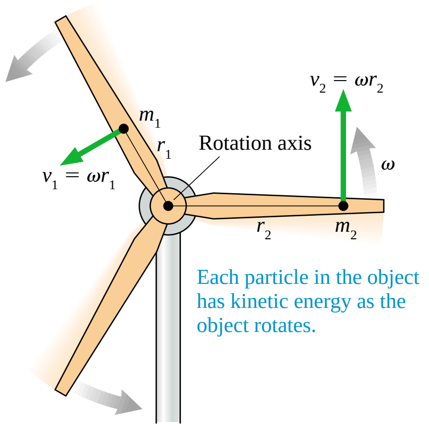


FIGURE 10.16 Rotational kinetic energy is due to the circular motion of the particles.

Rotational recharge The International Space Station (ISS) gets its electrical power from solar panels. But during each 92-min orbit, the ISS is in the earth's shadow for 30 min. The batteries that currently provide power during these blackouts need periodic replacement, which is very expensive in space. A promising new technology would replace the batteries with a *flywheel*—a cylinder rotating at very high angular speed. Energy from the solar cells is used to speed up the flywheel, storing energy as rotational kinetic energy, which can then be converted back into electrical energy when the ISS is in shadow.

particle in the blade is moving and hence has kinetic energy. Adding up the kinetic energy for each particle that makes up the blades, we find that the blades have **rotational kinetic energy,** the kinetic energy due to rotation.

Figure 10.16 shows two of the particles making up a windmill blade that rotates with angular velocity ω. Recall from Section 7.2 that a particle moving with angular velocity ω in a circle of radius r has a speed $v = \omega r$. Thus particle 1, which rotates in a circle of radius r_1, moves with speed $v_1 = r_1\omega$. Particle 2, which rotates in a circle with a larger radius r_2, moves with a larger speed $v_2 = r_2\omega$. The object's rotational kinetic energy is the sum of the kinetic energies of *all* of the particles:

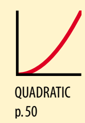
$$K_{\text{rot}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \cdots$$

$$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \cdots = \frac{1}{2}\left(\sum mr^2\right)\omega^2$$

You will recognize the term in parentheses as our old friend, the moment of inertia I. Thus the rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \qquad (10.12)$$

Rotational kinetic energy of object with moment of inertia I and angular velocity ω



NOTE ▶ Rotational kinetic energy is *not* a new form of energy. This is the familiar kinetic energy of motion, only now expressed in a form that is especially convenient for rotational motion. Comparison with the familiar $\frac{1}{2}mv^2$ shows again that the moment of inertia I is the rotational equivalent of mass. ◀

A rolling object, such as a wheel, is undergoing both rotational *and* translational motions. Consequently, its total kinetic energy is the sum of its rotational and translational kinetic energies:

$$K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Recall from Section 6.3 that v and ω of a rolling object of radius R are related by $\omega = v/R$. Thus we can write the kinetic energy of a rolling object as

$$K_{\text{rolling}} = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2 \qquad (10.13)$$

This illustrates the important fact that **the kinetic energy of a rolling object is always greater than that of a nonrotating object moving at the same speed.**

EXAMPLE 10.4 **Kinetic energy of a bicycle**

Bike 1 has a 10.0 kg frame and 1.00 kg wheels, while bike 2 has a 9.00 kg frame and 1.50 kg wheels. Both bikes thus have the same 12.0 kg total mass. What is the kinetic energy of each bike when they are ridden at 12.0 m/s? Model each wheel as a hoop of radius 35.0 cm.

PREPARE Each bike's frame has only translational kinetic energy $K_{\text{frame}} = \frac{1}{2}Mv^2$, where M is the mass of the frame. The kinetic energy of each rolling wheel is given by Equation 10.13. From Table 7.2, we find that I for a hoop is mR^2, where m is the mass of one wheel.

SOLVE From Equation 10.13 the kinetic energy of each rolling wheel is

$$K_{\text{wheel}} = \frac{1}{2}\left(m + \frac{mR^2}{R^2}\right)v^2 = \frac{1}{2}(2m)v^2 = mv^2$$

Then the total kinetic energy of a bike is

$$K = K_{\text{frame}} + 2K_{\text{wheel}} = \frac{1}{2}Mv^2 + 2mv^2$$

The factor of 2 in the second term occurs because each bike has two wheels. Thus the kinetic energies of the two bikes are

$$K_1 = \frac{1}{2}(10.0\text{ kg})(12.0\text{ m/s})^2 + 2(1.00\text{ kg})(12.0\text{ m/s})^2 = 1010\text{ J}$$

$$K_2 = \frac{1}{2}(9.00\text{ kg})(12.0\text{ m/s})^2 + 2(1.50\text{ kg})(12.0\text{ m/s})^2 = 1080\text{ J}$$

The kinetic energy of bike 2 is about 7% higher than that of bike 1. Note that the radius of the wheels was not needed in this calculation.

ASSESS As the cyclists on these bikes accelerate from rest to 12 m/s, they must convert some of their internal chemical energy into the kinetic energy of the bikes. Racing cyclists want to use as little of their own energy as possible. Although both bikes have the same total mass, the one with the lighter wheels will take less energy to get it moving. Shaving a little extra weight off your wheels is more useful than taking that same weight off your frame.

It's important that racing bike wheels are as light as possible.

10.6 Potential Energy

When two or more objects in a system interact, it is sometimes possible to *store* energy in that system in a way that the energy can be easily recovered. For instance, the earth and a ball interact by the gravitational force between them. If the ball is lifted up into the air, energy is stored in the ball-earth system, energy that can later be recovered as kinetic energy when the ball is released and falls. Similarly, a spring is a system made up of countless atoms that interact via their atomic "springs." If we push a box against a spring, energy is stored that can be recovered when the spring later pushes the box across the table. This sort of stored energy is called **potential energy,** since it has the *potential* to be converted into other forms of energy such as kinetic or thermal energy.

The forces due to gravity and springs are special in that they allow for the storage of energy. Other interaction forces do not. When a crate is pushed across the floor, the crate and the floor interact via the force of friction, and the work done on the system is converted into thermal energy. But this energy is *not* stored up for later recovery—it slowly diffuses into the environment and cannot be recovered.

Interaction forces that can store useful energy are called **conservative forces.** The name comes from the important fact, which we'll soon look at in detail, that when only conservative forces act, the mechanical energy of a system is *conserved.* Gravity and elastic forces are conservative forces, and later we'll see that the electric force is a conservative force as well. Friction, on the other hand, is a **nonconservative force.** When two objects interact via a friction force, energy is not stored. It is usually transformed into thermal energy.

Let's look more closely at the potential energies associated with the two conservative forces—gravity and springs—that we'll study in this chapter.

Gravitational Potential Energy

To find an expression for gravitational potential energy, let's consider the system of the book and the earth shown in Figure 10.17a on the next page. The book is lifted at a constant speed from its initial position at y_i to a final height y_f.

We can analyze this situation using the approach of Problem-Solving Strategy 10.1. The lifting force of the hand is external to the system and so does work W on the system, increasing its energy. The book is lifted at a constant speed, so its kinetic energy doesn't change. Because there's no friction, the book's thermal energy doesn't change either. Thus the work done goes entirely into increasing the gravitational potential energy of the system. The law of conservation of energy, Equation 10.6, then reads

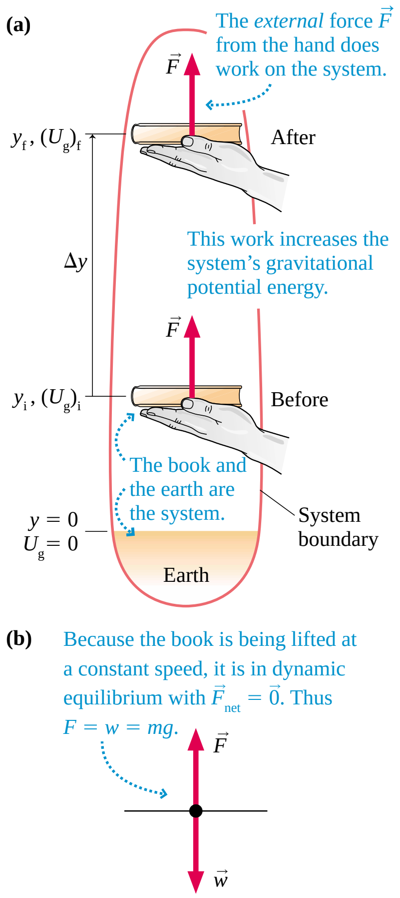


FIGURE 10.17 Lifting a book increases its gravitational potential energy.

The initial gravitational potential energy. . .   . . .plus the energy put into the system as work. . .

$$(U_g)_i + W = (U_g)_f \tag{10.14}$$

. . .equals the final gravitational potential energy.

The work done is $W = F\Delta y$, where $\Delta y = y_f - y_i$ is the vertical distance that the book is lifted. From the free-body diagram of Figure 10.17b, we see that $F = mg$. This gives $W = mg\Delta y$, so that

$$(U_g)_i + mg\Delta y = (U_g)_f$$

or

$$(U_g)_f = (U_g)_i + mg\Delta y \tag{10.15}$$

Since our final height was greater than our initial height, Δy is positive and $(U_g)_f > (U_g)_i$. **The higher the object is lifted, the greater the gravitational potential energy in the object/earth system.**

Equation 10.15 gives the final gravitational potential energy $(U_g)_f$ in terms of its initial value $(U_g)_i$. But what is the value of $(U_g)_i$? We can gain some insight by writing Equation 10.15 in terms of energy *changes*. We have

$$(U_g)_f - (U_g)_i = mg\Delta y$$

or

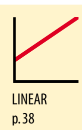
$$\Delta U_g = mg\Delta y$$

For example, if we lift a 1.5 kg book up by 2.0 m, we increase its gravitational potential energy by $\Delta U_g = (1.5\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m}) = 29.4\text{ J}$. This increase is *independent* of the book's starting height: We would get the *same* increase whether we lifted the book 2.0 m starting at sea level or starting at the top of Mount Everest. If we then dropped the book 2.0 m, we would recover the same 29.4 J as kinetic energy, whether in Miami or on Everest. This illustrates an important general fact about *every* form of potential energy: **Only *changes* in potential energy are significant.**

Because of this fact, we are free to choose a *reference level* where we define U_g to be zero. Our expression for U_g is particularly simple if we choose this reference level to be at $y = 0$. We then have

$$U_g = mgy \tag{10.16}$$

Gravitational potential energy of an object of mass m at a height y (assuming $U_g = 0$ when the object is at $y = 0$)



NOTE ▶ We've emphasized that gravitational potential energy is an energy of the earth-object *system*. In solving problems using the law of conservation of energy, you'll need to include the earth as part of your system. For simplicity, we'll usually speak of "the gravitational potential energy of the ball," but what we really mean is the potential energy of the earth-ball system. ◀

EXAMPLE 10.5 Hitting the bell

At the county fair, Katie tries her hand at the ring-the-bell attraction, as shown in Figure 10.18. She swings the mallet hard enough to give the ball an initial upward speed of 8.0 m/s. Will the ball ring the bell, 3.0 m from the bottom?

PREPARE As discussed above and in Tactics Box 10.1, we'll choose the ball *and* the earth as the system. Figure 10.18 shows the visual overview. If we assume that the track along which the ball moves is frictionless, then only the *mechanical* energy of the system changes. The only force on the ball after it leaves the

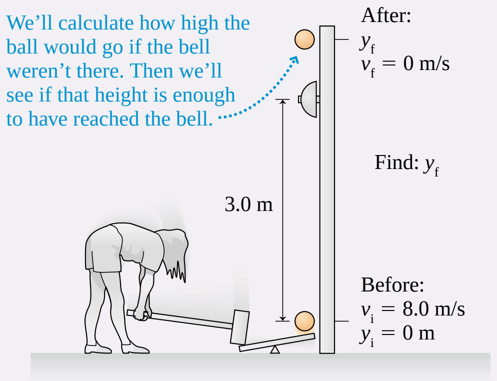


FIGURE 10.18 Before-and-after visual overview of the ring-the-bell attraction.

bottom lever is gravity, but gravity is an *internal* force due to our choice of the ball *plus* the earth as the system. This means that the gravitational interaction is included as gravitational potential energy rather than as external work. Since no *external* forces do work on the earth-ball system, the system is isolated. We can then use the law of conservation of mechanical energy, Equation 10.7.

SOLVE Equation 10.7 tells us that $K_i + (U_g)_i = K_f + (U_g)_f$. We can use our expressions for kinetic and potential energy to write this as

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Let's ignore the bell for the moment and figure out how far the ball would rise if there were nothing in its way. We know that the ball starts at $y_i = 0$ m and that its speed v_f at the highest point is zero. Thus the energy equation simplifies to

$$mgy_f = \frac{1}{2}mv_i^2$$

This is easily solved for the height y_f:

$$y_f = \frac{v_i^2}{2g} = \frac{(8.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 3.3 \text{ m}$$

This is higher than the point where the bell sits, so the ball would actually hit it on the way up.

ASSESS Notice that the mass canceled and wasn't needed, a fact about free fall that you should remember from Chapter 2.

An important conclusion from Equation 10.16 is that **gravitational potential energy depends only on the height of the object above the reference level $y = 0$, not on the object's horizontal position.** Consider carrying a briefcase while walking on level ground at a constant speed. As shown in the table on page 15, the force of your hand on the briefcase is *vertical* and hence *perpendicular* to the displacement. No work is done on the briefcase and consequently its gravitational potential energy remains constant as long as its height above the ground doesn't change as you walk.

This idea can be applied to more complicated cases, such as the 51 kg hiker in Figure 10.19. His gravitational potential energy depends *only* on his height y above the reference level, so it's the same value $U_g = mgy = 50$ kJ at any point on path A where he is at a height $y = 100$ m above the reference level. If he had instead taken path B, his gravitational potential energy at 100 m would be the same 50 kJ. It doesn't matter *how* he gets to 100 m, his potential energy at that height will be the same. This demonstrates an important aspect of all potential energies: **The potential energy depends only on the *position* of the object and not on the path the object took to get to that position.** This fact will allow us to use the law of conservation of energy to easily solve a variety of problems that would be very difficult to solve using Newton's laws alone, because we won't need to know the details of the path of the object—just its starting and ending points.

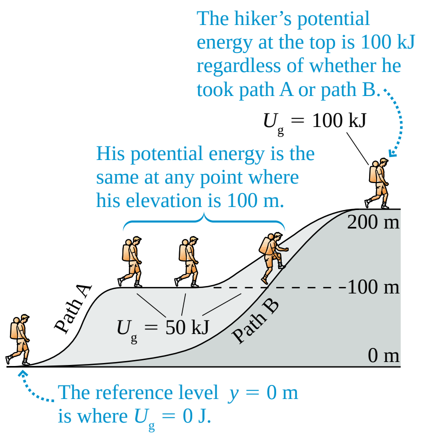


FIGURE 10.19 The hiker's gravitational potential energy depends only on his height above the $y = 0$ reference level.

EXAMPLE 10.6 Speed at the bottom of a water slide

Still at the county fair, Katie tries the water slide, whose shape is shown in Figure 10.20. The starting point is 9.0 m above the ground. She pushes off with an initial speed of 2.0 m/s. If the slide is frictionless, how fast will Katie be traveling at the bottom?

PREPARE Figure 10.20 on the next page shows a visual overview of the slide. Because there is no friction, Tactics Box 10.1 suggests that we take as our system Katie (the moving object) and the earth. With this choice of system, the only energies in the system are kinetic and gravitational potential energy. Note that the slope of the slide is not constant, so Katie's

Continued

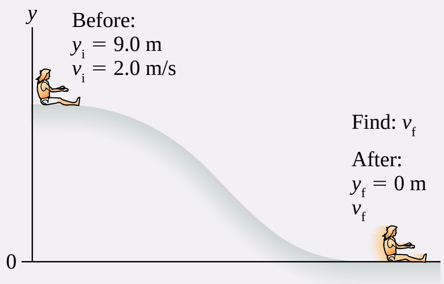


FIGURE 10.20 Before-and-after visual overview of Katie on the water slide.

acceleration will not be constant either. Thus we can't use constant-acceleration kinematics to find her speed. But we *can* use the law of conservation of energy to easily solve for her speed. Because there is no friction, the mechanical energy is conserved.

SOLVE Conservation of mechanical energy gives

$$K_i + (U_g)_i = K_f + (U_g)_f$$

or

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Taking $y_f = 0$ m we have

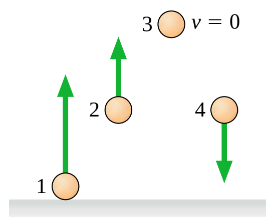
$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2$$

which we can solve to get

$$v_f = \sqrt{v_i^2 + 2gy_i}$$
$$= \sqrt{(2.0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(9.0\text{ m})} = 13\text{ m/s}$$

ASSESS It is important to realize that the *shape* of the slide does not matter because gravitational potential energy depends only on the *height* above a reference level. **In sliding down any (frictionless) slide of the same height, your speed at the bottom would be the same.**

STOP TO THINK 10.4 Rank in order, from largest to smallest, the gravitational potential energies of identical balls 1 to 4.



Elastic Potential Energy

Energy can also be stored in a compressed or extended spring as **elastic** (or **spring**) **potential energy** U_s. We can find out how much energy is stored in a spring by using an external force to slowly compress the spring. This external force does work on the spring, transferring energy to the spring. Since only the elastic potential energy of the spring is changing, the law of conservation of energy reads

$$W = \Delta U_s \tag{10.17}$$

That is, we can find out how much elastic potential energy is stored in the spring by calculating the amount of work needed to compress the spring.

Figure 10.21 shows a spring being compressed by a hand. In Section 8.4 we found that the force that the spring will exert on the hand is equal to $-kx$, where x is the displacement of the end of the spring from its equilibrium position at $x = 0$ and k is the spring constant. By Newton's third law, this means that the force that the hand exerts on the spring is equal to $+kx$.

As we compress the end of the spring from its equilibrium position to a final displacement x, the force we apply increases from zero to kx. This is not a constant force, so we can't use Equation 10.8, $W = Fd$, to find the work done, because this equation is valid only for a constant force. However, it seems reasonable that we could calculate the work by using the *average* force in Equation 10.8. Because the force varies from $F_i = 0$ to $F_f = kx$, the average force used to compress the spring is

$$F_{avg} = \frac{1}{2}(F_f + F_i) = \frac{1}{2}(kx + 0) = \frac{1}{2}kx$$

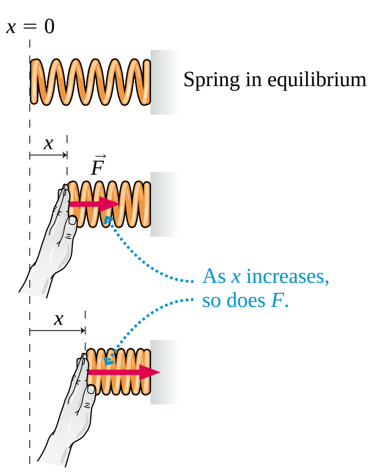


FIGURE 10.21 The force required to compress a spring is not constant.

Thus the work done by the hand is

$$W = F_{\text{avg}}d = F_{\text{avg}}x = \left(\frac{1}{2}kx\right)x = \frac{1}{2}kx^2$$

This work is stored as potential energy in the spring, so we can use Equation 10.17 to find that the elastic potential energy increases by

$$\Delta U_s = \frac{1}{2}kx^2$$

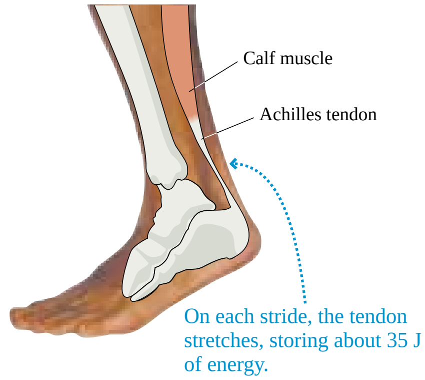
Just as in the case of gravitational potential energy, we have found an expression for the *change* in U_s, not U_s itself. Again, we are free to set $U_s = 0$ at any convenient spring extension. An obvious choice is to set $U_s = 0$ at the point where the spring is in equilibrium, neither compressed nor stretched; that is, at $x = 0$. With this choice we have

$$U_s = \frac{1}{2}kx^2 \qquad (10.18)$$

QUADRATIC p.50

Elastic potential energy of a spring displaced a distance x from equilibrium (assuming $U_s = 0$ when the end of the spring is at $x = 0$)

NOTE ► Since U_s depends on the *square* of the displacement x, U_s is the same whether x is positive (the spring is compressed as in Figure 10.21) or negative (the spring is stretched). ◄



Spring in your step BIO As you run, you lose some of your mechanical energy each time your foot strikes the ground; this energy is transformed into unrecoverable thermal energy. Luckily, about 35% of the decrease of your mechanical energy when your foot lands is stored as elastic potential energy in the stretchable Achilles tendon of the lower leg. On each plant of the foot the tendon is stretched, storing some energy. The tendon springs back as you push off the ground again, helping to propel you forward. This recovered energy reduces the amount of internal chemical energy you use, increasing your efficiency.

EXAMPLE 10.7 Speed of a spring-launched ball

A spring-loaded toy gun is used to launch a 10 g plastic ball. The spring, which has a spring constant of 10 N/m, is compressed by 10 cm as the ball is pushed into the barrel. When the trigger is pulled, the spring is released and shoots the ball back out. What is the ball's speed as it leaves the barrel? Assume that friction is negligible.

PREPARE Assume the spring obeys Hooke's law $F = -kx$, and is massless so that it has no kinetic energy of its own. Using Tactics Box 10.1 we choose the system to be the spring and the ball. There's no friction, hence the system's mechanical energy $K + U_s$ is conserved.

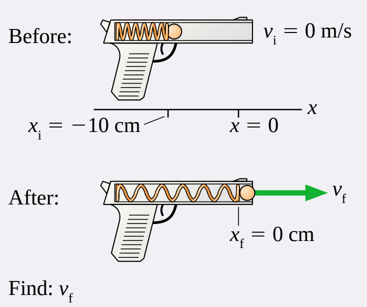


FIGURE 10.22 The before-and-after visual overview of a ball being shot out of a spring-loaded toy gun.

Figure 10.22 shows a before-and-after visual overview. The compressed spring will push on the ball until the spring has returned to its equilibrium length. We have chosen the origin of the coordinate system at the equilibrium position of the free end of the spring, making $x_i = -10$ cm and $x_f = 0$ cm.

SOLVE The energy conservation equation is $K_i + (U_s)_i = K_f + (U_s)_f$. We can use the elastic potential energy of the spring, Equation 10.18, to write this as

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$$

We know that $x_f = 0$ m and $v_i = 0$ m/s, so this simplifies to

$$\frac{1}{2}mv_f^2 = \frac{1}{2}kx_i^2$$

It is now straightforward to solve for the ball's speed:

$$v_f = \sqrt{\frac{kx_i^2}{m}} = \sqrt{\frac{(10\text{ N/m})(-0.10\text{ m})^2}{0.010\text{ kg}}} = 3.2\text{ m/s}$$

ASSESS This is *not* a problem that we could have easily solved with Newton's laws. The acceleration is not constant, and we have not learned how to handle the kinematics of nonconstant acceleration. But with conservation of energy—it's easy!

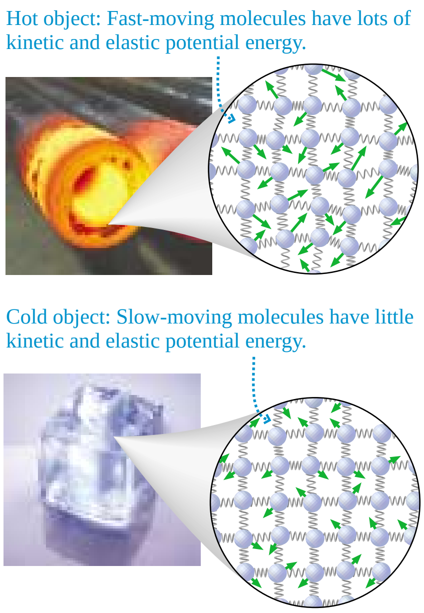


FIGURE 10.23 A molecular view of thermal energy.

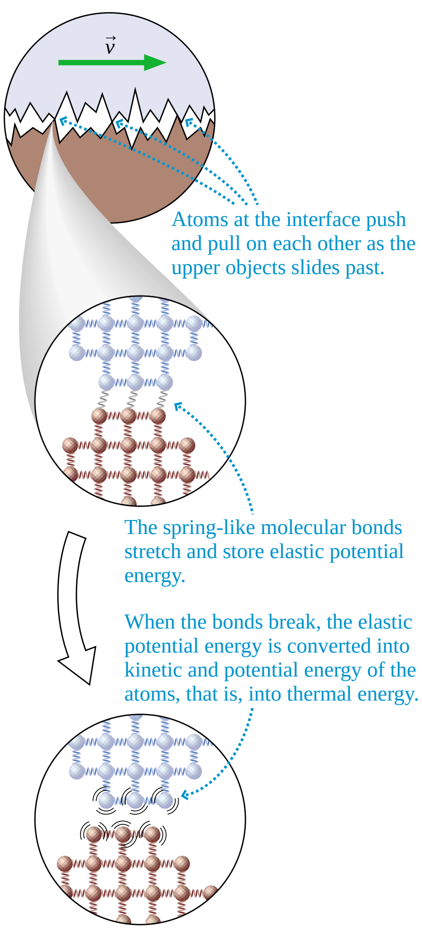


FIGURE 10.24 How friction causes an increase in thermal energy.

STOP TO THINK 10.5 A spring-loaded gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

A. 2 m/s. B. 4 m/s. C. 8 m/s. D. 16 m/s.

10.7 Thermal Energy

We noted earlier that thermal energy is related to the microscopic motion of the molecules of an object. As Figure 10.23 shows, the molecules in a hot object jiggle around their average positions more than the molecules in a cold object. This has two consequences. First, each atom is on average moving faster in the hot object. This means that each atom has a higher *kinetic energy.* Second, each atom in the hot object tends to stray further from its equilibrium position, leading to a greater stretching or compressing of the spring-like molecular bonds. This means that each atom has on average a higher *potential energy.* The potential energy stored in any one bond and the kinetic energy of any one atom are both exceedingly small, but there are incredibly many bonds and atoms. The sum of all these microscopic potential and kinetic energies is what we call **thermal energy.**

Is this microscopic energy worth worrying about? To see, consider a 500 g ($\approx$1 lb) iron ball moving at the respectable speed of $v_{\text{ball}} = 20$ m/s ($\approx$45 mph). Its kinetic energy is $K = \frac{1}{2}mv_{\text{ball}}^2 = 100$ J.

How fast do the atoms jiggle about their equilibrium positions? This speed depends on the temperature, but at room temperature it's very high—roughly 500 m/s. So each atom is on average traveling in a straight line at 20 m/s, but jiggling about this average motion at a speed of 500 m/s! This a factor of 25 times faster. And since kinetic energy is proportional to the *square* of the speed, the kinetic energy due to the microscopic motion is about 625 times greater than that due to the overall motion. And it turns out that the microscopic potential energy is just as large. Thus the ball that has an ordinary kinetic energy of 100 J has an internal thermal energy of $2 \times 625 \times 100\text{ J} = 125{,}000$ J!

Transforming Mechanical Energy into Thermal Energy

Consider a snowboarder sliding on level snow. After a while, he will glide to a stop because of the friction force of the snow on his board. We can analyze this using the law of conservation of energy. Following Tactics Box 10.1 we'll take the system to be the boarder *plus* the snow. Then there are no forces external to the system that do work on it and, since he's moving horizontally, his potential energy doesn't change. Then the law of conservation of energy is $K_i = K_f + \Delta E_{\text{th}}$, or $\Delta E_{\text{th}} = K_i - K_f$. He's slowing to a stop, so $K_i > K_f$ and ΔE_{th} is positive. The system's thermal energy *increases* as kinetic energy is transformed into thermal energy.

This increase in thermal energy is a general feature of any system where friction between sliding objects is present: **When two objects slide against each other with friction present, mechanical energy is always transformed into thermal energy.** An atomic-level explanation is shown in Figure 10.24.

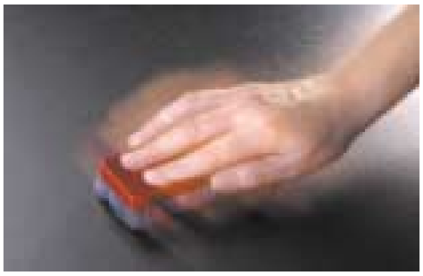
The presence of friction has two important consequences for our conservation of energy Problem-Solving Strategy 10.1:

1. As stated in Tactics Box 10.1, we must include in the system not only the moving object but also the surface against which it slides. This is because the thermal energy generated by friction resides in *both* object and surface (as in Figure 10.24), and it is usually impossible to tell what fraction resides

in each. By choosing both object and surface to be in the system, we know that *all* the thermal energy ends up in the system.

2. In addition to the mechanical energy $K + U$ we now must include ΔE_{th} in the conservation of energy equation.

TRY IT YOURSELF



Agitating atoms Vigorously rub a somewhat soft object such as a blackboard eraser on your desktop for about 10 seconds. If you then pass your fingers over the spot where you rubbed, you'll feel a distinct warm area. Congratulations: you've just set some 100,000,000,000,000,000,000,000 atoms into motion!

EXAMPLE 10.8 Thermal energy created sledding down a hill

George jumps onto his sled and starts from rest at the top of a 5.0-m-high hill. His speed at the bottom is 8.0 m/s. The mass of George and the sled is 55 kg. How much thermal energy was produced in this process?

PREPARE Figure 10.25 shows the before-and-after visual overview. The statement of the problem implies that thermal energy will be generated, so following Tactics Box 10.1, we'll take the system to include both George and the sled *and* the slope. Because his height is changing, his gravitational potential energy is changing and we'll need to include the earth in the system as well. No forces act from outside this system, so the work W is zero.

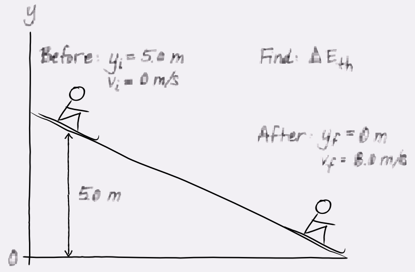


FIGURE 10.25 Visual overview of George sliding down the hill.

SOLVE Here the law of conservation of energy reads

$$K_i + (U_g)_i = K_f + (U_g)_f + \Delta E_{th}$$

In terms of positions and speeds,

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f + \Delta E_{th}$$

Because $v_i = 0$ m/s and $y_f = 0$ m, this simplifies to

$$mgy_i = \frac{1}{2}mv_f^2 + \Delta E_{th}$$

from which we have

$$\begin{aligned}\Delta E_{th} &= mgy_i - \frac{1}{2}mv_f^2 \\ &= (55\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m}) - \frac{1}{2}(55\text{ kg})(8.0\text{ m/s})^2 \\ &= 940\text{ J}\end{aligned}$$

ASSESS The change in E_{th} is positive, as it must be. This extra thermal energy resides in the sled and all along the slope where George slid. You should be able to show that about 35% of George's original gravitational potential energy was transformed into thermal energy as he slid down the hill.

10.8 Further Examples of Conservation of Energy

In this section, we'll tie together what we've learned about using the law of conservation of energy to solve dynamics problems. In each, we use the key idea of setting the "before" energy equal to the "after" energy.

ActivPhysics 5.2–5.7, 7.11–7.13

EXAMPLE 10.9 Where will the sled stop?

A sledder, starting from rest, slides down a 10-m-high hill. At the bottom of the hill is a long horizontal patch of rough snow. The hill is nearly frictionless, but the coefficient of friction between the sled and the rough snow at the bottom is $\mu_k = 0.30$. How far will the sled slide along the rough patch?

PREPARE A picture of the sledder is shown in Figure 10.26. We'll break this problem into Part A, his motion down the hill; and Part B, his motion along the ice. We know how to use conservation of energy to find his speed at the bottom of the hill. Along the rough patch, however, we'll use kinematics and Newton's laws, as studied in Chapters 2 and 5, to find how far he slides.

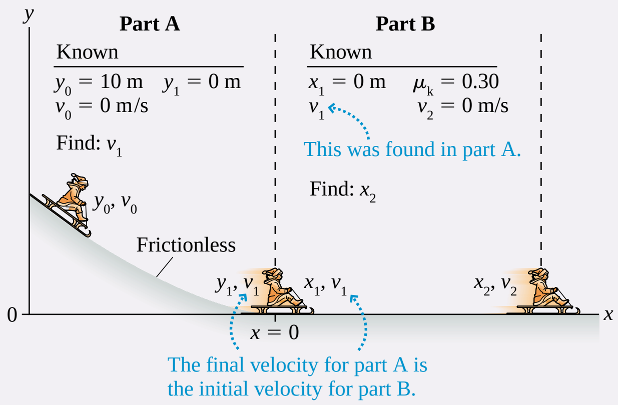


FIGURE 10.26 Visual overview of a sledder sliding downhill.

SOLVE We'll solve Part A first and find the sled's speed v_f at the bottom. The hill is frictionless, so mechanical energy is conserved and we have

$$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$$

Since $v_i = 0$ m/s and $y_f = 0$ m, this reduces to

$$mgy_i = \frac{1}{2}mv_f^2$$

so that

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.8\text{ m/s}^2)(10\text{ m})} = 14.0\text{ m/s}$$

On the rough patch in Part B, where the only horizontal force is the kinetic friction force f_k pointing to the left, the sled's acceleration is

$$a = -\frac{f_k}{m} = -\frac{\mu_k n}{m} = -\frac{\mu_k mg}{m}$$

$$= -\mu_k g = -(0.30)(9.8\text{ m/s}^2) = -2.94\text{ m/s}^2$$

The negative acceleration indicates that the sled is slowing down, as expected.

We now use kinematics to find how far the sled slides. We know the acceleration a, as well as the initial and final velocities along the horizontal patch, and we want to know the final position x_2. This suggests using the kinematic equation

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

to find the final position. For the motion of Part B, the final velocity $v_f = v_2 = 0$ m/s, the initial velocity is $v_1 = 14.0$ m/s, and the initial position is $x_i = x_1 = 0$ m. We can then solve for the final position in the kinematic equation to get

$$x_2 = -\frac{v_1^2}{2a} = -\frac{(14.0\text{ m/s})^2}{-5.88\text{ m/s}^2} = 33\text{ m}$$

ASSESS When friction is present, mechanical energy is *not* conserved: Some of the mechanical energy of the system is inevitably transformed into thermal energy. Thus we cannot use the law of conservation of mechanical energy for such problems. Instead, we'll need to use Newton's laws and kinematics to find how far objects slide. As in this example, however, there will often be a part of the problem with no friction that we *can* solve using the law of conservation of mechanical energy.

EXAMPLE 10.10 Who wins the great downhill race?

Figure 10.27 shows a contest in which a sphere, a cylinder, and a circular hoop, each with mass M and radius R, are placed at height h on a slope of angle θ. All three are simultaneously released from rest and roll down the ramp without slipping. Which one will win the race to the bottom of the hill?

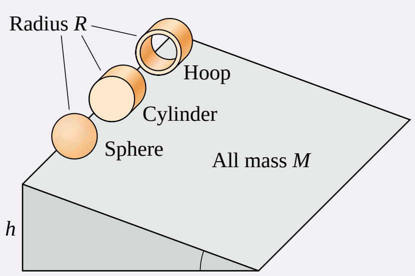


FIGURE 10.27 Which will win the downhill race?

PREPARE With no sliding friction, the total mechanical energy is conserved. However, the kinetic energy of each object must include a contribution from its rotational kinetic energy.

SOLVE Conservation of energy tells us that the gravitational potential energy $(U_g)_i = Mgh$ at the top will be transformed into an equal amount of kinetic energy K_f at the bottom. Thus

$$(U_g)_i = Mgh = K_f = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2$$

where we used Equation 10.13 for the total (translational plus rotational) kinetic energy. The speed at the bottom is then

$$v = \sqrt{\frac{2Mgh}{M + \dfrac{I}{R^2}}}$$

Table 7.2 gives the moment of inertia for each of the three shapes. We have

Shape	Moment of Inertia I	$v = \sqrt{2Mgh/(M + I/R^2)}$
Sphere	$\frac{2}{5}MR^2$	$v = \sqrt{\frac{10}{7}gh} = 1.19\sqrt{gh}$
Cylinder	$\frac{1}{2}MR^2$	$v = \sqrt{\frac{4}{3}gh} = 1.15\sqrt{gh}$
Hoop	MR^2	$v = \sqrt{gh}$

The sphere has the largest speed at the bottom, a full 19% faster than the hoop. Because the sphere always travels faster than the hoop or the cylinder, it will win the race, followed by the cylinder and the hoop.

ASSESS All the objects have the same kinetic energy at the bottom, because they all started with the same energy, Mgh, at the top. But the object with the smallest I will have the smallest *rotational* kinetic energy at the bottom, and hence the largest *translational* kinetic energy and the largest v. An ordinary sliding object (no rotation) reaches the bottom with speed $v = \sqrt{2gh} = 1.41\sqrt{gh}$. This is significantly faster than any of the rolling objects. The sliding object is faster because *all* its kinetic energy is translational—and it's the translational motion that gets you down the hill.

10.9 Energy in Collisions

In Chapter 9 we studied collisions between two objects. We found that if no external forces are acting on the objects, the total *momentum* of the objects will be conserved. Now we wish to study what happens to *energy* in collisions. The energetics of collisions are important in many applications in biokinetics, such as designing safer automobiles and bicycle helmets.

Let's first re-examine a perfectly inelastic collision. We studied just such a collision in Example 9.8. Recall that in such a collision the two objects stick together and then move with a common final velocity. What happens to the energy?

EXAMPLE 10.11 Energy transformations in a perfectly inelastic collision

Figure 10.28 shows two air track gliders that are pushed toward each other, collide, and stick together. In Example 9.8, we used conservation of momentum to find the final velocity shown in Figure 10.28 from given initial velocities. Compare the initial and final *mechanical energies* of the system.

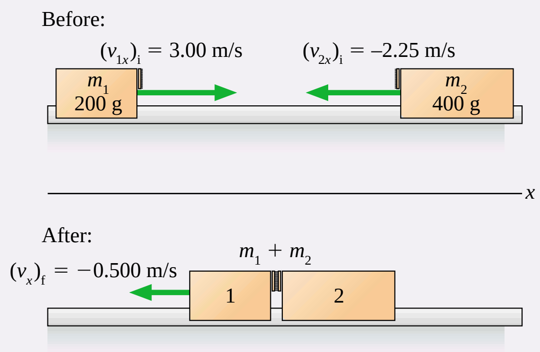


FIGURE 10.28 Initial and final velocities in a completely inelastic collision.

PREPARE We'll choose our system to be the two gliders. Because the tracks are horizontal, there is no change in potential energy. Thus the law of conservation of energy, Equation 10.6, reads $K_i = K_f + \Delta E_{th}$. The total energy before the collision must equal the total energy after, but the total *mechanical* energies need not be equal.

SOLVE The initial kinetic energy is

$$K_i = \frac{1}{2}m_1(v_{1x})_i^2 + \frac{1}{2}m_2(v_{2x})_i^2$$

$$= \frac{1}{2}(0.200\text{ kg})(3.00\text{ m/s})^2 + \frac{1}{2}(0.400\text{ kg})(-2.25\text{ m/s})^2$$

$$= 1.91\text{ J}$$

Because the gliders stick together and move as a single object with mass $m_1 + m_2$, the final kinetic energy is

$$K_f = \frac{1}{2}(m_1 + m_2)(v_x)_f^2$$

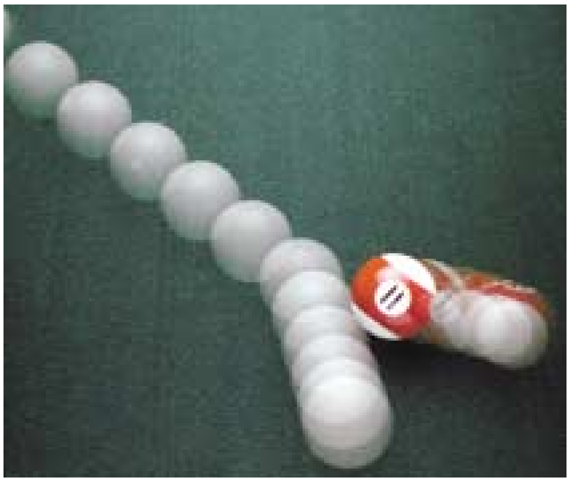
$$= \frac{1}{2}(0.600\text{ kg})(-0.500\text{ m/s})^2 = 0.0750\text{ J}$$

From the conservation of energy equation above, we find that the thermal energy increases by

$$\Delta E_{th} = K_i - K_f = 1.91\text{ J} - 0.075\text{ J} = 1.84\text{ J}$$

This amount of the initial kinetic energy is transformed into thermal energy during the impact of the collision.

ASSESS About 96% of the initial kinetic energy is transformed into thermal energy. This is typical of many real-world collisions.



In a collision between a cue ball and a stationary ball, the mechanical energy of the balls is almost perfectly conserved.

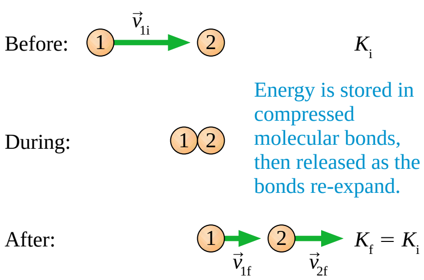


FIGURE 10.29 A perfectly elastic collision.

Elastic Collisions

Figure 9.1 showed a collision of a tennis ball with a racket. The ball is compressed and the racket strings stretch as the two collide, then the ball expands and the strings relax as the two are pushed apart. In the language of energy, the kinetic energy of the objects is transformed into the elastic potential energy of the ball and strings, then back into kinetic energy as the two objects spring apart. If *all* of the kinetic energy is stored as elastic potential energy, and then *all* of the elastic potential energy is transformed back into the post-collision kinetic energy of the objects, then mechanical energy is conserved. A collision for which mechanical energy is conserved is called a **perfectly elastic collision.**

Needless to say, most real collisions fall somewhere between perfectly elastic and perfectly inelastic. A rubber ball bouncing on the floor might "lose" 20% of its kinetic energy on each bounce and return to only 80% of the height of the previous bounce. But collisions between two very hard objects, such as two pool balls or two steel balls, come close to being perfectly elastic. And collisions between microscopic particles, such as atoms or electrons, can be perfectly elastic.

Figure 10.29 shows a head-on, perfectly elastic collision of a ball of mass m_1, having initial velocity $(v_{1x})_i$, with a ball of mass m_2 that is initially at rest. The balls' velocities after the collision are $(v_{1x})_f$ and $(v_{2x})_f$. These are velocities, not speeds, and have signs. Ball 1, in particular, might bounce backward and have a negative value for $(v_{1x})_f$.

The collision must obey two conservation laws: conservation of momentum (obeyed in any collision) and conservation of mechanical energy (because the collision is perfectly elastic). Although the energy is transformed into potential energy during the collision, the mechanical energy before and after the collision is purely kinetic energy. Thus

$$\text{momentum conservation:}\qquad m_1(v_{1x})_i = m_1(v_{1x})_f + m_2(v_{2x})_f$$

$$\text{energy conservation:}\qquad \frac{1}{2}m_1(v_{1x})_i^2 = \frac{1}{2}m_1(v_{1x})_f^2 + \frac{1}{2}m_2(v_{2x})_f^2$$

Momentum conservation alone is not sufficient to analyze the collision because there are two unknowns: the two final velocities. That is why we did not consider perfectly elastic collisions in Chapter 9. Energy conservation gives us another condition. The complete solution of these two equations involves straightforward but rather lengthy algebra. We'll just give the solution here, which is:

$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i \qquad (v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i \qquad (10.19)$$

Perfectly elastic collision with object 2 initially at rest

Equations 10.19 allow us to compute the final velocity of each object. Let's look at a common and important example: a perfectly elastic collision between two objects of equal mass.

EXAMPLE 10.12 Velocities in an air hockey collision

On an air hockey table, a moving puck, traveling at 2.3 m/s, makes a head-on collision with a stationary puck. What are the final velocities of each of the pucks?

PREPARE The before-and-after visual overview is shown Figure 10.30. We've sketched in final velocities in the picture, but we don't really know yet which way the pucks will move. Because one puck was initially at rest, we can use Equation 10.19 to find the final velocities of the pucks. The pucks are identical, so we have $m_1 = m_2 = m$.

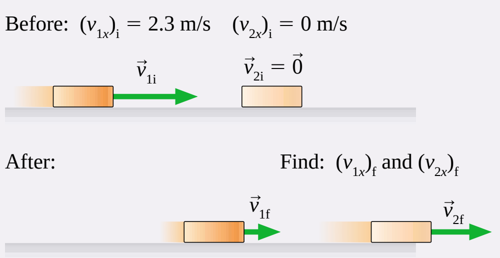


FIGURE 10.30 A moving puck collides with a stationary puck.

SOLVE We use Equation 10.19 with $m_1 = m_2 = m$ to get

$$(v_{1x})_f = \frac{m - m}{m + m}(v_{1x})_i = 0 \text{ m/s}$$

$$(v_{2x})_f = \frac{2m}{m + m}(v_{1x})_i = (v_{1x})_i = 2.3 \text{ m/s}$$

The incoming puck stops dead, and the initially stationary puck goes off with the same velocity that the incoming one had.

ASSESS You can see that momentum and energy are conserved: the incoming puck's momentum and energy are completely transferred to the outgoing puck. If you've ever played pool, you've probably seen this sort of collision when you hit a ball head-on with the cue ball: the cue ball stops and the other ball picks up the cue ball's velocity.

Other cases where the colliding objects are of unequal mass will be treated in the end-of-chapter problems.

Forces in Collisions

The collision between two pool balls occurs very quickly, and the forces are typically very large and difficult to calculate. Fortunately, by using the concepts of momentum and energy conservation we can often calculate the final velocities of the balls without having to know the forces between them. There are collisions, however, where knowing the forces involved is of critical importance. The following example shows how a helmet helps protect the head from the large forces involved in a bicycle accident.

EXAMPLE 10.13 Protecting your head

A bike helmet is basically a shell of hard, crushable foam 3.0 cm thick. In testing, the helmet is strapped onto a 5.0 kg headform that is dropped from a height of 2.0 m onto a hard anvil. What force is encountered by the head in such a fall?

The foam inside a bike helmet is designed to crush upon impact.

PREPARE A visual overview of the test is shown in Figure 10.31. We can use the law of conservation of energy, Equation 10.6, to estimate the force on the headform. We'll choose the headform and the earth to be the system; the foam in the helmet will be part of the environment. We make this choice so that the force on the headform due to the foam is an *external* force that does work W on the headform.

SOLVE The headform starts at initial height $y_i = 2.0$ m above the anvil and ends at rest with the foam fully crushed. Then the law of conservation of energy is

$$K_i + (U_g)_i + W = K_f + (U_g)_f$$

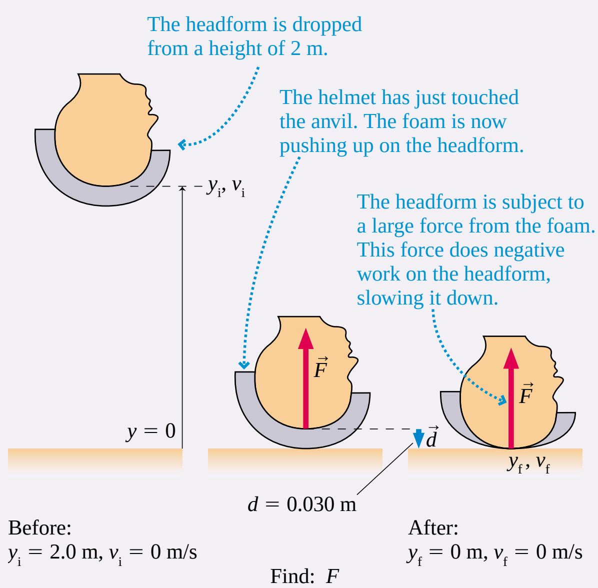


FIGURE 10.31 The foam in the helmet does negative work on the headform.

Continued

In words, this states that the initial energy of the system, plus the energy transferred to the system as work, equals the final energy of the system.

Since the headform starts and ends at rest, both K_i and K_f are zero. Taking our reference height $y = 0$ at the anvil, $(U_g)_f$ is zero as well. Since $(U_g)_i = mgy_i$, conservation of energy gives simply $mgy_i + W = 0$, or

$$mgy_i = -W$$

As the foam is crushed, it pushes up on the headform with force $\vec{F}$, doing work on it. This force is directed *opposite* to the displacement $\vec{d}$ of the headform, so the work done is *negative*—kinetic energy is being *removed* from the headform, slowing it down. The work done is $-Fd$, if we assume the force is relatively constant, so we have

$$mgy_i = -(-Fd)$$

or

$$F = \frac{mgy_i}{d} = \frac{(5.0 \text{ kg})(9.8 \text{ m/s})(2.0 \text{ m})}{0.030 \text{ m}} = 3300 \text{ N}$$

This is the force that acts on the head to bring it to a halt in only 3 cm. More important from the perspective of possible brain injury is the head's *acceleration*

$$a = \frac{F}{m} = \frac{3300 \text{ N}}{5.0 \text{ kg}} = 660 \text{ m/s}^2 = 67g$$

where g is the acceleration due to gravity.

ASSESS The accepted threshold for serious brain injury is around $300g$, so this helmet would protect the rider in all but the most serious accidents. Without the helmet, the rider's head would come to a stop in a much smaller distance and thus be subjected to a much larger acceleration.

It's also interesting to ask where the original energy of the headform went. The work on it was negative, indicating a transfer of energy from the headform to the environment—the foam. As the foam crushes, there is a great deal of internal friction and rubbing between parts of the foam. This causes the foam to get warmer, increasing its thermal energy. This increase must be exactly equal to the energy lost by the headform.

10.10 Power

Both these cars take about the same energy to reach 60 mph, but the race car gets there in a much shorter time, so its *power* is much greater.

We've now studied how energy can be transformed from one kind to another and how it can be transferred between the environment and the system as work. In many situations we would like to know *how quickly* the energy is transformed or transferred. Is a transfer of energy very rapid, or does it take place over a long time? In passing a truck, your car needs to transform a certain amount of the chemical energy in its fuel into kinetic energy. It makes a *big* difference whether your engine can do this in 20 s or 60 s!

The question "How quickly?" implies that we are talking about a *rate*. For example, the velocity of an object—how fast it is going—is the *rate of change* of position. So when we raise the issue of how fast the energy is transformed, we are talking about the *rate of transformation* of energy. Suppose in a time interval Δt an amount of energy ΔE is transformed from one form to another. The rate at which this energy is transformed is called the **power** P, and it is defined as

$$P = \frac{\Delta E}{\Delta t} \tag{10.20}$$

Power when amount of energy ΔE is transformed in time interval Δt

The unit of power is the **watt,** which is defined as 1 watt $= 1$ W $= 1$ J/s.

Power also measures the rate at which energy is transferred into or out of a system as work W. If work W is done in time interval Δt, the rate of energy *transfer* is

$$P = \frac{W}{\Delta t} \tag{10.21}$$

Power when amount of work W is done in time interval Δt

A force that is doing work (i.e., transferring energy) at a rate of 3 J/s has an "output power" of 3 W. A system gaining energy at the rate of 3 J/s is said to "consume" 3 W of power. Common prefixes used with power are mW (milliwatts), kW (kilowatts), and MW (megawatts).

We can express Equation 10.21 in a different form. If in the time interval Δt an object undergoes a displacement Δx, the work done by a force acting on the object is $W = F\Delta x$. Then Equation 10.21 can be written

$$P = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\frac{\Delta x}{\Delta t} = Fv$$

The rate at which energy is transferred to an object as work—the power—is the product of the force that does the work, and the velocity of the object:

$$P = Fv \qquad (10.22)$$

Rate of energy transfer due to a force F acting on an object moving at velocity v

LINEAR p. 38

The English unit of power is the *horsepower*. The conversion factor to watts is

$$1 \text{ horsepower} = 1 \text{ hp} = 746 \text{ W}$$

Many common appliances, such as motors, are rated in hp.

EXAMPLE 10.14 Power to pass a truck

You are behind a 1500 kg truck traveling at 60 mph (27 m/s). To pass it, you speed up to 75 mph (34 m/s) in 6.0 s. What power is required to do this?

PREPARE Your car is undergoing an energy transformation from the chemical energy of your fuel to the kinetic energy of the car. We can calculate the amount of energy transformed by finding the change ΔK in the kinetic energy.

SOLVE We have

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1500 \text{ kg})(27 \text{ m/s})^2 = 5.47 \times 10^5 \text{ J}$$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(1500 \text{ kg})(34 \text{ m/s})^2 = 8.67 \times 10^5 \text{ J}$$

so that

$$\begin{aligned}\Delta K &= K_f - K_i \\ &= 8.67 \times 10^5 \text{ J} - 5.47 \times 10^5 \text{ J} = 3.20 \times 10^5 \text{ J}\end{aligned}$$

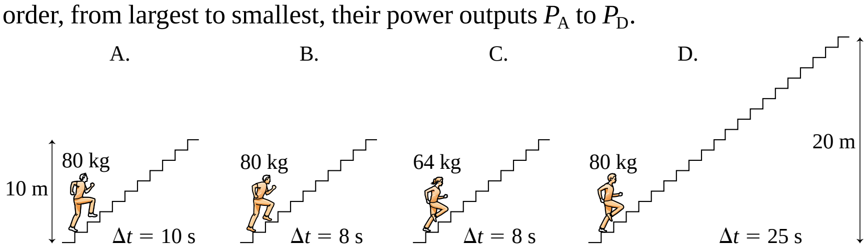
To transform this amount of energy in 6 s, the power required is

$$P = \frac{\Delta K}{\Delta t} = \frac{3.20 \times 10^5 \text{ J}}{6.0 \text{ s}} = 53{,}000 \text{ W} = 53 \text{ kW}$$

This is about 71 hp. This power is in addition to the power needed to overcome drag and friction and cruise at 60 mph, so the total power required from the engine will be even greater than this.

ASSESS You use a large amount of energy to perform a simple driving maneuver such as this. 3.20×10^5 J is enough energy to lift an 80 kg person 410 m in the air—the height of a tall skyscraper. And 53 kW would lift him there in only 6 s!

STOP TO THINK 10.6 Four students run up the stairs in the time shown. Rank in order, from largest to smallest, their power outputs P_A to P_D.



SUMMARY

The goal of Chapter 10 has been to learn about energy and how to solve problems using the law of conservation of energy.

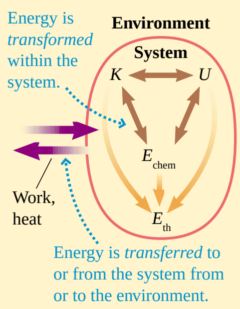
GENERAL PRINCIPLES

General Energy Model

Within a system, energy can be **transformed** between various forms.

Energy can be **transferred** into or out of a system in two basic ways:

- **Work:** The transfer of energy by mechanical forces.
- **Heat:** The nonmechanical transfer of energy from a hotter to a colder object.



Solving Energy Conservation Problems

PREPARE Choose your system (Tactics Box 10.1). Decide what forms of energy are changing. If there is friction, then thermal energy will be created. Check for external forces that will do work on your system.

SOLVE Use Equation 10.6 to relate the initial energy of your system, plus the work done, to the final energy of the system:

$$K_i + U_i + W = K_f + U_f + \Delta E_{th}$$

ASSESS Kinetic energy is always positive. The *change* in thermal energy should be positive.

Law of Conservation of Energy

Isolated system: No energy is transferred into or out of the system. Each form of energy within the system can change, but the total change in energy is zero. Energy of the system is *conserved:*

The change in the system's energy is zero.

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \ldots = 0$$

Nonisolated system: Energy can be exchanged with the environment as work or heat. The energy of the system changes by the amount of work done or heat transferred:

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} + \Delta E_{chem} + \ldots = W + Q$$

The system's energy changes by the amount of work done and heat transferred.

Systems with mechanical and thermal energy only: The initial mechanical energy, plus the work done, equals the final mechanical energy plus additional thermal energy:

$$K_i + U_i + W = K_f + U_f + \Delta E_{th}$$

In terms of energy changes, this can be written

$$\Delta K + \Delta U_g + \Delta U_s + \Delta E_{th} = W$$

IMPORTANT CONCEPTS

Mechanical energy is the sum of a system's kinetic and potential energies:

$$\text{Mechanical energy} = K + U = K + U_g + U_s$$

Kinetic energy is an energy of motion

$$K = \underbrace{\tfrac{1}{2}mv^2}_{\text{Translational}} + \underbrace{\tfrac{1}{2}I\omega^2}_{\text{Rotational}}$$

Potential energy is energy stored in a system of interacting objects

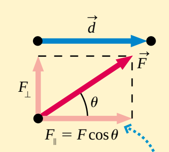
- **Gravitational potential energy:** $U_g = mgy$
- **Elastic potential energy:** $U_s = \frac{1}{2}kx^2$

Thermal energy is the sum of the microscopic kinetic and potential energy of all the molecules in an object. The hotter an object, the more thermal energy it has. When kinetic (sliding) friction is present, mechanical energy will be transformed into thermal energy.

Work is the process by which energy is transferred to or from a system by the application of mechanical forces.

If a particle moves through a displacement $\vec{d}$ while acted upon by a constant force $\vec{F}$, the force does work

$$W = F_{\parallel}d = Fd\cos\theta$$

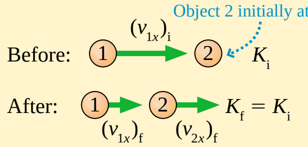


Only the component of the force parallel to the

APPLICATIONS

Perfectly elastic collisions

Both mechanical energy and momentum are conserved.



$$(v_{1x})_f = \frac{m_1 - m_2}{m_1 + m_2}(v_{1x})_i$$

$$(v_{2x})_f = \frac{2m_1}{m_1 + m_2}(v_{1x})_i$$

Power is the rate at which energy is transformed . . .

$$P = \frac{\Delta E}{\Delta t}$$

ΔE: Amount of energy transformed
Δt: Time required to transform it

. . . or at which work is done.

$$P = \frac{W}{\Delta t}$$

W: Amount of work done
Δt: Time required to do work

For instructor-assigned homework, go to www.masteringphysics.com

National difficulty data for each problem is shown on a scale of | to ||||.

Problems labeled: can be done on a Workbook Energy Worksheet; INT integrate significant material from earlier chapters; BIO are of biological or medical interest.

QUESTIONS

Conceptual Questions

1. The brake shoes of your car are made of a material that can tolerate very high temperatures without being damaged. Why is this so?
2. When you pound a nail with a hammer, the nail gets quite warm. Describe the energy transformations that lead to the addition of thermal energy in the nail.

For Questions 3 through 8, give a specific example of a system with the energy transformation shown. In these questions, W is the work done on the system by the environment, and K and U are the kinetic and potential energies of the system, respectively.

3. $W \rightarrow K$ with $\Delta U = 0$.
4. $W \rightarrow U$ with $\Delta K = 0$.
5. $K \rightarrow U$ with $W = 0$.
6. $K \rightarrow W$ with $\Delta U = 0$.
7. $U \rightarrow K$ with $W = 0$.
8. $U \rightarrow W$ with $\Delta K = 0$.
9. A ball of putty is dropped from a height of 2 m onto a hard floor, where it sticks. What object or objects need to be included within the system if the system is to be isolated during this process?
10. A 0.5 kg mass on a 1-m-long string swings in a circle on a horizontal, frictionless table at a steady speed of 2 m/s. How much work does the tension in the string do on the mass during one revolution? Explain.
11. INT Particle A has less mass than particle B. Both are pushed forward across a frictionless surface by equal forces for 1 s. Both start from rest.
    a. Compare the amount of work done on each particle. That is, is the work done on A greater than, less than, or equal to the work done on B? Explain.
    b. Compare the impulses delivered to particles A and B. Explain.
    c. Compare the final speeds of particles A and B. Explain
12. The meaning of the word "work" is quite different in physics from its everyday usage. Give an example of an action a person could do that "feels like work" but that does not involve any work as we've defined it in this chapter.
13. To change a tire, you need to use a jack to raise one corner of your car. While doing so, you happen to notice that pushing the jack handle down 20 cm raises the car only 0.2 cm. Use energy concepts to explain why the handle must be moved so far to raise the car by such a small amount.

Questions 14 through 17 refer to a weightlifter raising a barbell from the floor to above his head. Describe the energy transformations that occur if the system is chosen as specified in the question. Use the notation of Section 10.2 for the various forms of energy and energy transfer.

14. The system is the barbell alone.
15. The system is the weightlifter alone.
16. The system is the barbell plus the earth.
17. The system is the barbell plus the earth plus the weightlifter.

In Questions 18 through 20, imagine yourself doing a chin-up. You start from rest with your arms extended above your head, and end at rest with your elbows bent and your hands still gripping the bar. Describe the energy transformations that occur if the system is chosen as specified in the question. Use the notation of Section 10.2 for the various forms of energy and energy transfer.

18. The system is you alone.
19. The system is you plus the chin-up bar.
20. The system is you plus the chin-up bar plus the earth.
21. One kilogram of matter contains approximately 10^{17} J of nuclear energy. Why don't we need to include this energy when we study ordinary energy transformations?
22. A roller coaster car rolls down a frictionless track, reaching speed v_f at the bottom.
    a. If you want the car to go twice as fast at the bottom, by what factor must you increase the height of the track?
    b. Does your answer to part a depend on whether the track is straight or not? Explain.
23. A spring gun shoots out a plastic ball at speed v_i. The spring is then compressed twice the distance it was on the first shot.
    a. By what factor is the spring's potential energy increased?
    b. By what factor is the ball's speed increased? Explain.
24. Sandy and Chris stand on the edge of a cliff and throw identical mass rocks at the same speed. Sandy throws her rock horizontally while Chris throws his upward at an angle of 45° to the horizontal. Are the rocks moving at the same speed when they hit the ground, or is one moving faster than the other? If one is moving faster, which one? Explain.
25. If you allow a can of chicken broth to join the rolling-object race discussed in Example 10.10, it wins handily. A can of tomato paste, on the other hand, ties with the cylinder. Why? **Hint:** Try to picture how the stuff inside each can moves as the can rolls.
26. A solid cylinder and a cylindrical shell have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has light-weight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground as shown in Figure Q10.26. Both blocks are released simultaneously. The ropes do not slip. Which block hits the ground first? Or is it a tie? Explain.

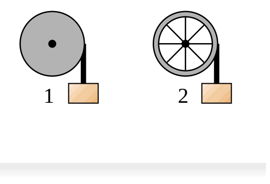


FIGURE Q10.26

27. You are much more likely to be injured if you fall on a concrete sidewalk than if you fall on a grassy field. Use energy and work concepts to explain why this is so. BIO

Multiple-Choice Questions

28. If you walk up a flight of stairs at constant speed, gaining vertical height h, the work done on you (the system, of mass m) is
    A. $+mgh$, by the normal force of the stairs.
    B. $-mgh$, by the normal force of the stairs.
    C. $+mgh$, by the gravitational force of the earth.
    D. $-mgh$, by the gravitational force of the earth.
29. You and a friend each carry a 15 kg suitcase up two flights of stairs, walking at a constant speed. Take each suitcase to be the system. Suppose you carry your suitcase up the stairs in 30 s while your friend takes 60 s. Which of the following is true?
    A. You did more work, but both of you expended the same power.
    B. You did more work and expended more power.
    C. Both of you did the same work, but you expended more power.
    D. Both of you did the same work, but you expended less power.
30. A woman uses a pulley and a rope to raise a 20 kg weight to a height of 2 m. If it takes 4 s to do this, about how much power is she supplying?
    A. 100 W B. 200 W C. 300 W D. 400 W
31. A hockey puck sliding along frictionless ice with speed v to the right collides with a horizontal spring and compresses it by 2.0 cm before coming to a momentary stop. What will be the spring's maximum compression if the same puck hits it at a speed of $2v$?
    A. 2.0 cm B. 2.8 cm C. 4.0 cm
    D. 5.6 cm E. 8.0 cm
32. A block slides down a smooth ramp and moves onto a level, rough surface at a speed of 2.0 m/s. It comes to rest after traveling 1.0 m. At what distance from the base of the ramp was the block moving at 1.0 m/s?
    A. 0.12 m B. 0.25 m C. 0.50 m D. 0.75 m
33. A wrecking ball is suspended from a 5.0-m-long cable that makes a 30° angle with the vertical. The ball is released and swings down. What is the ball's speed at the lowest point?
    A. 7.7 m/s B. 4.4 m/s C. 3.6 m/s D. 3.1 m/s

PROBLEMS

Section 10.4 Work

1. During an etiquette class, you walk slowly and steadily at 0.20 m/s for 2.5 m with a 0.75 kg book balanced on top of your head. How much work does your head do on the book?
2. A 2.0 kg book is lying on a 0.75-m-high table. You pick it up and place it on a bookshelf 2.3 m above the floor.
    a. How much work does gravity do on the book?
    b. How much work does your hand do on the book?
3. The two ropes seen in Figure P10.3 are used to lower a 255 kg piano exactly 5 m from a second-story window to the ground. How much work is done by each of the three forces?

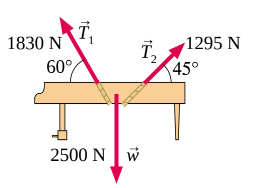


FIGURE P10.3

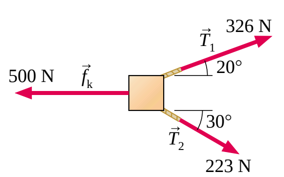


FIGURE P10.4

4. The two ropes shown in the bird's-eye view of Figure P10.4 are used to drag a crate exactly 3 m across the floor. How much work is done by each of the ropes on the crate?
5. a. At the airport, you ride a "moving sidewalk" that carries you horizontally for 25 m at 0.70 m/s. Assuming that you were moving at 0.70 m/s before stepping onto the moving sidewalk and continue at 0.70 m/s afterward, how much work does the moving sidewalk do on you? Your mass is 60 kg.
    b. An escalator carries you from one level to the next in the airport terminal. The upper level is 4.5 m above the lower level, and the length of the escalator is 7.0 m. How much work does the up escalator do on you when you ride it from the lower level to the upper level?
    c. How much work does the down escalator do on you when you ride it from the upper level to the lower level?
6. A boy flies a kite with the string at a 30° angle to the horizontal. The tension in the string is 4.5 N. How much work does the string do on the boy if the boy
    a. stands still?
    b. walks a horizontal distance of 11 m away from the kite?
    c. walks a horizontal distance of 11 m toward the kite?

Section 10.5 Kinetic Energy

7. Which has the larger kinetic energy, a 10 g bullet fired at 500 m/s or a 10 kg bowling ball sliding at 10 m/s?
8. At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/hr?
9. An oxygen atom is four times as massive as a helium atom. In an experiment, a helium atom and an oxygen atom have the same kinetic energy. What is the ratio v_{He}/v_O of their speeds?
10. Sam's job at the amusement park is to slow down and bring to a stop the boats in the log ride. If a boat and its riders have a mass of 1200 kg and the boat drifts in at 1.2 m/s, how much work does Sam do to stop it?
11. A 20 g plastic ball is moving to the left at 30 m/s. How much work must be done on the ball to cause it to move to the right at 30 m/s?
12. The turntable in a microwave oven has a moment of inertia of 0.040 kg · m^2 and is rotating once every 4.0 s. What is its kinetic energy?
13. An energy storage system based on a flywheel (a rotating disk) can store a maximum of 4.0 MJ when the flywheel is rotating at 20,000 revolutions per minute. What is the moment of inertia of the flywheel?

Section 10.6 Potential Energy

14. | The lowest point in Death Valley is 85.0 m below sea level. The summit of nearby Mt. Whitney has an elevation of 4420 m. What is the change in gravitational potential energy of an energetic 65.0 kg hiker who makes it from the floor of Death Valley to the top of Mt. Whitney?

15. | a. What is the kinetic energy of a 1500 kg car traveling at a speed of 30 m/s ($\approx$65 mph)?
b. From what height should the car be dropped to have this same amount of kinetic energy just before impact?
c. Does your answer to part b depend on the car's mass?

16. || A boy reaches out of a window and tosses a ball straight up with a speed of 10 m/s. The ball is 20 m above the ground as he releases it. Use conservation of energy to find
a. The ball's maximum height above the ground.
b. The ball's speed as it passes the window on its way down.
c. The speed of impact on the ground.

17. || a. With what minimum speed must you toss a 100 g ball straight up to just barely hit the 10-m-high ceiling of the gymnasium if you release the ball 1.5 m above the floor? Solve this problem using energy.
b. With what speed does the ball hit the floor?

18. || What minimum speed does a 100 g puck need to make it to the top of a frictionless ramp that is 3.0 m long and inclined at 20°?

19. || A car is parked at the top of a 50-m-high hill. It slips out of gear and rolls down the hill. How fast will it be going at the bottom? (Ignore friction.)

20. || A pendulum is made by tying a 500 g ball to a 75-cm-long string. The pendulum is pulled 30° to one side, then released.
a. What is the ball's speed at the lowest point of its trajectory?
b. To what angle does the pendulum swing on the other side?

21. ||| A 1500 kg car is approaching the hill shown in Figure P10.21 at 10 m/s when it suddenly runs out of gas.
a. Can the car make it to the top of the hill by coasting?
b. If your answer to (a) is yes, what is the car's speed after coasting down the other side?

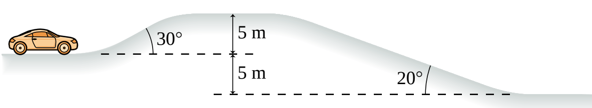


FIGURE P10.21

22. | How much energy can be stored in a spring with a spring constant of 500 N/m if its maximum possible stretch is 20 cm?

23. | How far must you stretch a spring with $k = 1000$ N/m to store 200 J of energy?

24. | A student places her 500 g physics book on a frictionless table. She pushes the book against a spring, compressing the spring by 4.00 cm, then releases the book. What is the book's speed as it slides away? The spring constant is 1250 N/m.

25. | A 10 kg runaway grocery cart runs into a spring with spring constant 250 N/m and compresses it by 60 cm. What was the speed of the cart just before it hit the spring?

26. | As a 15,000 kg jet lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

27. |||| BIO The elastic energy stored in your tendons can contribute up to 35% of your energy needs when running. Sports scientists have studied the change in length of the knee extensor tendon in sprinters and nonathletes. They find (on average) that the sprinters' tendons stretch 41 mm, while nonathletes' stretch only 33 mm. The spring constant for the tendon is the same for both groups, 33 N/mm. What is the difference in maximum stored energy between the sprinters and the nonathletes?

28. || You're driving at 35 km/hr when the road suddenly descends 15 m into a valley. You take your foot off the accelerator and coast down the hill. Just as you reach the bottom you see the police officer hiding behind the speed limit sign that reads "70 km/hr." Are you going to get a speeding ticket?

29. || Your friend's Frisbee has become stuck 16 m above the ground in a tree. You want to dislodge the Frisbee by throwing a rock at it. The Frisbee is stuck pretty tight, so you figure the rock needs to be traveling at least 5.0 m/s when it hits the Frisbee. If you release the rock 2.0 m above the ground, with what minimum speed must you throw it?

Section 10.7 Thermal Energy

30. | A 1500 kg car traveling at 20 m/s skids to a halt.
a. What energy transfers and transformations occur during the skid?
b. What is the change in the thermal energy of the car and the road surface?

31. || A 20 kg child slides down a 3.0-m-high playground slide. She starts from rest, and her speed at the bottom is 2.0 m/s.
a. What energy transfers and transformations occur during the slide?
b. What is the change in the thermal energy of the slide and the seat of her pants?

32. ||| A fireman of mass 80 kg slides down a pole. When he reaches the bottom, 4.2 m below his starting point, his speed is 2.2 m/s. By how much has thermal energy increased during his slide?

Section 10.9 Energy in Collisions

33. | A 50 g marble moving at 2.0 m/s strikes a 20 g marble at rest. What is the speed of each marble immediately after the collision? Assume the collision is perfectly elastic.

34. | Ball 1, with a mass of 100 g and traveling at 10 m/s, collides head-on with ball 2, which has a mass of 300 g and is initially at rest. What are the final velocities of each ball if the collision is (a) perfectly elastic? (b) perfectly inelastic?

35. || A proton is traveling to the right at 2.0×10^7 m/s. It has a head-on, perfectly elastic collision with a stationary carbon atom. The mass of the carbon atom is 12 times the mass of the proton. What are the speed and direction of each after the collision?

36. | Two balls undergo a perfectly elastic head-on collision, with one ball initially at rest. If the incoming ball has a speed of 200 m/s, what are the final speed and direction of each ball if
a. the incoming ball is *much* more massive than the stationary ball?
b. the stationary ball is *much* more massive than the incoming ball?

37. |||| Derive Equations 10.19 for the final speeds of two objects undergoing a perfectly elastic collision, with one object initially stationary.

Section 10.10 Power

38. | a. How much work does an elevator motor do to lift a 1000 kg elevator a height of 100 m?
b. How much power must the motor supply to do this in 50 s at constant speed?

39. || a. How much work must you do to push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s? The coefficient of kinetic friction for steel on steel is 0.60.
b. What is your power output while doing so?

40. | Which consumes more energy, a 1.2 kW hair dryer used for 10 min or a 10 W night light left on for 24 hr?

41. ||| A 1000 kg sports car accelerates from 0 to 30 m/s in 10 s. What is the average power of the engine?

42. ||| In just 0.30 s, you compress a spring (spring constant 5000 N/m), which is initially at its equilibrium length, by 4.0 cm. What is your average power output?

43. |||| In the winter sport of curling, players give a 20 kg stone a push across a sheet of ice. A curler accelerates a stone to a speed of 3.0 m/s over a time of 2.0 s.
a. How much force does the curler exert on the stone?
b. What average power does the curler use to bring the stone up to speed?

44. |||| A 710 kg car drives at a constant speed of 23 m/s. It is subject to a drag force of 500 N. What power is required from the car's engine to drive the car
a. on level ground?
b. up a hill with a slope of 2.0°?

45. || An elevator weighing 2500 N ascends at a constant speed of 8.0 m/s. How much power must the motor supply to do this?

General Problems

46. ||| A 2.3 kg box, starting from rest, is pushed up a ramp by a 10 N force parallel to the ramp. The ramp is 2.0 m long and tilted at 17°. The speed of the box at the top of the ramp is 0.80 m/s. Consider the system to be the box + ramp + earth.
a. How much work W does the force do on the system?
b. What is the change ΔK in the kinetic energy of the system?
c. What is the change ΔU_g in the gravitational potential energy of the system?
d. What is the change ΔE_{th} in the thermal energy of the system?

47. || A 55 kg skateboarder wants to just make it to the upper edge of a "half-pipe" with a radius of 3.0 m, as shown in Figure P10.47. What speed v_i does he need at the bottom if he is to coast all the way up?

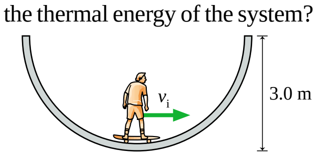


FIGURE P10.47

a. First do the calculation treating the skateboarder and board as a point particle, with the entire mass nearly in contact with the half-pipe.
b. More realistically, the mass of the skateboarder in a deep crouch might be thought of as concentrated 0.75 m from the half-pipe. Assuming he remains in that position all the way up, what v_i is needed to reach the upper edge ?

48. || Fleas have remarkable jumping ability. If a 0.50 mg flea jumps straight up, it will reach a height of 40 cm if there is no air resistance. In reality, air resistance limits the height to 20 cm.
BIO
a. What is the flea's kinetic energy as it leaves the ground?
b. At its highest point, what fraction of the initial kinetic energy has been converted to potential energy?

49. ||| A marble slides without friction in a *vertical* plane around the inside of a smooth, 20-cm-diameter horizontal pipe. The marble's speed at the bottom is 3.0 m/s; this is fast enough so that the marble makes a complete loop, never losing contact with the pipe. What is its speed at the top?
INT

50. ||| A 20 kg child is on a swing that hangs from 3.0-m-long chains, as shown in Figure P10.50. What is her speed v_i at the bottom of the arc if she swings out to a 45° angle before reversing direction?

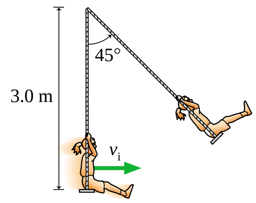


FIGURE P10.50

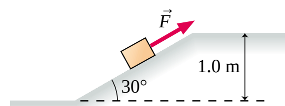


FIGURE P10.51

51. | Suppose you lift a 20 kg box by a height of 1.0 m.
a. How much work do you do in lifting the box?
Instead of lifting the box straight up, suppose you push it up a 1.0-m-high ramp that makes a 30° degree angle with the horizontal, as shown in Figure P10.51. Being clever, you choose a ramp with no friction.
b. How much force F is required to push the box straight up the slope at a constant speed?
c. How long is the ramp?
d. Use your force and distance results to calculate the work you do in pushing the box up the ramp. How does this compare to your answer to part a?

52. || A cannon tilted up at a 30° angle fires a cannon ball at 80 m/s from atop a 10-m-high fortress wall. What is the ball's impact speed on the ground below? Ignore air resistance.

53. ||| The sledder shown in Figure P10.53 starts from the top of a frictionless hill and slides down into the valley. What initial speed v_i does the sledder need to just make it over the next hill?

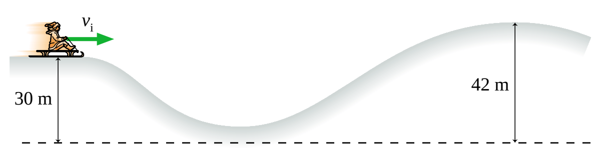


FIGURE P10.53

54. ||| A 100 g granite cube slides down a frictionless 40° incline. At the bottom, just after it exits onto a horizontal table, the granite collides with a 200 g steel cube at rest. How high above the table should the granite cube be released to give the steel cube a speed of 150 cm/s?

55. || A 50 g ice cube can slide without friction up and down a 30° slope. The ice cube is pressed against a spring at the bottom of the slope, compressing the spring 10 cm. The spring constant is 25 N/m. When the ice cube is released, what distance will it travel up the slope before reversing direction?

56. |||| In a physics lab experiment, a spring clamped to the table shoots a 20 g ball horizontally. When the spring is compressed 20 cm, the ball travels horizontally 5.0 m and lands on the floor 1.5 m below the point at which it left the spring. What is the spring constant?
INT

57. The desperate contestants on a TV survival show are very hungry. The only food they can see is some fruit hanging on a branch high in a tree. Fortunately, they have a spring they can use to launch a rock. The spring constant is 1000 N/m, and they can compress the spring a maximum of 30 cm. All the rocks on the island seem to have a mass of 400 g.
   a. With what speed does the rock leave the spring?
   b. If the fruit hangs 15 m above the ground, will they feast or go hungry?

58. The maximum energy a bone can absorb without breaking is surprisingly small. For a healthy human of mass 60 kg, experimental data show that the leg bones can absorb about 200 J.
   a. From what maximum height could a person jump and land rigidly upright on both feet without breaking his legs? Assume that all the energy is absorbed in the leg bones in a rigid landing.
   b. People jump from much greater heights than this; explain how this is possible.

   Hint: Think about how people land when they jump from greater heights.

59. In an amusement park water slide, people slide down an essentially frictionless tube. They drop 3.0 m and exit the slide, moving horizontally, 1.2 m above a swimming pool. What horizontal distance do they travel from the exit point before hitting the water? Does the mass of the person make any difference?

60. The 5.0-m-long rope in Figure P10.60 hangs vertically from a tree right at the edge of a ravine. A woman wants to use the rope to swing to the other side of the ravine. She runs as fast as she can, grabs the rope, and swings out over the ravine.

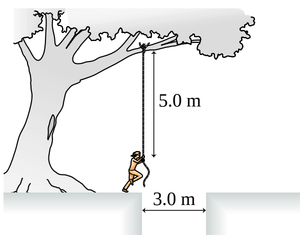


FIGURE P10.60

   a. As she swings, what energy conversion is taking place?
   b. When she's directly over the far edge of the ravine, how much higher is she than when she started?
   c. Given your answers to parts a and b, how fast must she be running when she grabs the rope in order to swing all the way across the ravine?

61. You have been asked to design a "ballistic spring system" to measure the speed of bullets. A bullet of mass m is fired into a block of mass M. The block, with the embedded bullet, then slides across a frictionless table and collides with a horizontal spring whose spring constant is k. The opposite end of the spring is anchored to a wall. The spring's maximum compression d is measured.
   a. Find an expression for the bullet's initial speed v_B in terms of m, M, k, and d.

   Hint: This is a two-part problem. The bullet's collision with the block is an inelastic collision. What quantity is conserved in an inelastic collision? Subsequently the block hits a spring on a frictionless surface. What quantity is conserved in this collision?

   b. What was the speed of a 5.0 g bullet if the block's mass is 2.0 kg and if the spring, with $k = 50$ N/m, was compressed by 10 cm?
   c. What fraction of the bullet's initial kinetic energy is "lost"? Where did it go?

62. A new event, shown in Figure P10.62, has been proposed for the Winter Olympics. An athlete will sprint 100 m, starting from rest, then leap onto a 20 kg bobsled. The person and bobsled will then slide down a 50-m-long ice-covered ramp, sloped at 20°, and into a spring with a carefully calibrated spring constant of 2000 N/m. The athlete who compresses the spring the farthest wins the gold medal. Lisa, whose mass is 40 kg, has been training for this event. She can reach a maximum speed of 12 m/s in the 100 m dash.

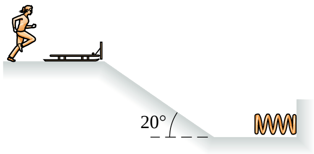


FIGURE P10.62

   a. How far will Lisa compress the spring?
   b. The Olympic committee has very exact specifications about the shape and angle of the ramp. Is this necessary? If the committee asks your opinion, what factors about the ramp will you tell them are important?

63. A 20 g ball is fired horizontally with initial speed v_i toward a 100 g ball that is hanging motionless from a 1.0-m-long string. The balls undergo a head-on, perfectly elastic collision, after which the 100 g ball swings out to a maximum angle $\theta_{max} = 50°$. What was v_i?

64. A 70 kg human sprinter can accelerate from rest to 10 m/s in 3.0 s. During the same interval, a 30 kg greyhound can accelerate from rest to 20 m/s. Compute (a) the change in kinetic energy and (b) the average power output for each.

65. A 50 g ball of clay traveling at speed v_i hits and sticks to a 1.0 kg block sitting at rest on a frictionless surface.
   a. What is the speed of the block after the collision?
   b. Show that the mechanical energy is *not* conserved in this collision. What percentage of the ball's initial kinetic energy is "lost"? Where did this kinetic energy go?

66. A package of mass m is released from rest at a warehouse loading dock and slides down a 3.0-m-high frictionless chute to a waiting truck. Unfortunately, the truck driver went on a break without having removed the previous package, of mass $2m$, from the bottom of the chute as shown in Figure P10.66.
   a. Suppose the packages stick together. What is their common speed after the collision?
   b. Suppose the collision between the packages is perfectly elastic. To what height does the package of mass m rebound?

FIGURE P10.66

67. A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration.
   a. What is the magnitude of the horizontal force acting on the sprinter?
   b. What is the sprinter's average power output during the first 2.0 s of his run?
   c. What is the sprinter's average power output during the final 2.0 s?

68. Bob can throw a 500 g rock with a speed of 30 m/s. He moves his hand forward 1.0 m while doing so.
INT
   a. How much force, assumed to be constant, does Bob apply to the rock?
   b. How much work does Bob do on the rock?

69. A 2.0 hp electric motor on a water well pumps water from 10 m below the surface. The density of water is 1.0 kg per L. How many liters of water can the motor pump in 1 hr?

70. The human heart has to pump the average adult's 6.0 L of blood through the body every minute. The heart must do work to overcome frictional forces that resist the blood flow. The average blood pressure is 1.3×10^4 N/m^2.
BIO
   a. Compute the work done moving the 6.0 L of blood completely through the body, assuming the blood pressure always takes its average value.
   b. What power output must the heart have to do this task once a minute?

   Hint: When the heart contracts, it applies force to the blood. Pressure is just force/area, so we can write work = (pressure)(area)(distance). But (area)(distance) is just the blood volume passing through the heart.

Passage Problems

Tennis Ball Testing

A tennis ball bouncing on a hard surface compresses and then rebounds. The details of the rebound are specified in tennis regulations. Tennis balls, to be acceptable for tournament play, must have a mass of 57.5 g. When dropped from a height of 2.5 m onto a concrete surface, a ball must rebound to a height of 1.4 m. During impact, the ball compresses by approximately 6 mm.

71. How fast is the ball moving when it hits the concrete surface? (Ignore air resistance.)
   A. 5 m/s B. 7 m/s C. 25 m/s D. 50 m/s

72. If the ball accelerates uniformly when it hits the floor, what is its approximate acceleration as it comes to rest before rebounding?
   A. 1000 m/s^2 B. 2000 m/s^2 C. 3000 m/s^2 D. 4000 m/s^2

73. The ball's kinetic energy just after the bounce is less than just before the bounce. In what form does this lost energy end up?
   A. Elastic potential energy
   B. Gravitational potential energy
   C. Thermal energy
   D. Rotational kinetic energy

74. By what percent does the kinetic energy decrease?
   A. 35% B. 45% C. 55% D. 65%

75. When a tennis ball bounces from a racket, the ball loses approximately 30% of its kinetic energy to thermal energy. A ball that hits a racket at a speed of 10 m/s will rebound with approximately what speed?
   A. 8.5 m/s B. 7.0 m/s C. 4.5 m/s D. 3.0 m/s

Work and Power in Cycling

When you ride a bicycle at constant speed, almost all of the energy you expend goes into the work you do against the drag force of the air. In this problem, assume that *all* of the energy expended goes into working against drag. As we saw in Section 5.7, the drag force on an object is approximately proportional to the square of its speed with respect to the air. For this problem, assume that $F \propto v^2$ exactly and that the air is motionless with respect to the ground unless noted otherwise. Suppose a cyclist and her bicycle have a combined mass of 60 kg and she is cycling along at a speed of 5 m/s.

76. If the drag force on the cyclist is 10 N, how much energy does she use in cycling 1 km?
   A. 6 kJ B. 10 kJ C. 50 kJ D. 100 kJ

77. Under these conditions, how much power does she expend as she cycles?
   A. 10 W B. 50 W C. 100 W D. 200 W

78. If she doubles her speed to 10 m/s, how much energy does she use in cycling 1 km?
   A. 20 kJ B. 40 kJ C. 400 kJ D. 400 kJ

79. How much power does she expend when cycling at that speed?
   A. 100 W B. 200 W C. 400 W D. 1000 W

80. Upon reducing her speed back down to 5 m/s, she hits a headwind of 5 m/s. How much power is she expending now?
   A. 100 W B. 200 W C. 500 W D. 1000 W

STOP TO THINK ANSWERS

Stop to Think 10.1: D. Since the child slides at a constant speed, his kinetic energy doesn't change. But his gravitational potential energy at the top of the slide decreases as he descends, and is transformed into thermal energy in the slide and his bottom.

Stop to Think 10.2: C. $W = Fd\cos\theta$. The 10 N force at 90° does no work at all. $\cos 60° = \frac{1}{2}$, so the 8 N force does less work than the 6 N force.

Stop to Think 10.3: B > D > A = C. $K = (1/2)mv^2$. Using the given masses and velocities, we find $K_A = 2.0$ J, $K_B = 4.5$ J, $K_C = 2.0$ J, $K_D = 4.0$ J.

Stop to Think 10.4: $(U_g)_3 > (U_g)_2 = (U_g)_4 > (U_g)_1$. Gravitational potential energy depends only on height, not speed.

Stop to Think 10.5: C. U_s depends on x^2, so doubling the compression increases U_s by a factor of 4. All the potential energy is converted to kinetic energy, so K increases by a factor of 4. But K depends on v^2, so v increases by only a factor of $\sqrt{4} = 2$.

Stop to Think 10.6: $P_B > P_A = P_C > P_D$. The power here is the rate at which each runner's internal chemical energy is converted into gravitational potential energy. The change in gravitational potential energy is $mg\Delta y$, so the power is $mg\Delta y/\Delta t$. For runner A, the ratio $m\Delta y/\Delta t$ equal $(80\text{ kg})(10\text{ m})/(10\text{ s}) = 80\text{ kg}\cdot\text{m/s}$. For C, it's the same. For B, it's $100\text{ kg}\cdot\text{m/s}$, while for D the ratio is $64\text{ kg}\cdot\text{m/s}$.