

Centripetal Force

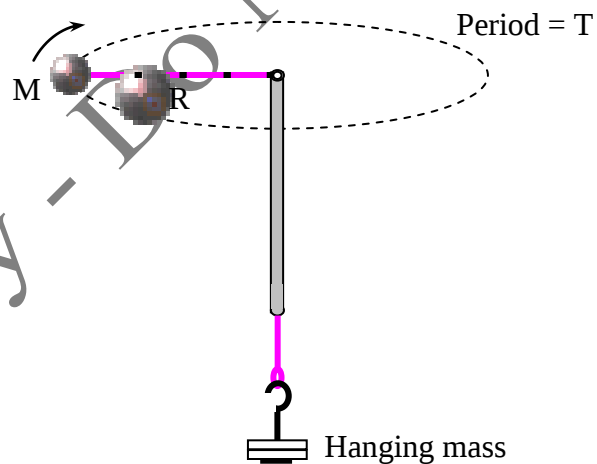
Name: _____

PES 1150 Prelab Questions

Lab Station: 005

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- What is the direction of the force acting on the spinning mass M as it is swung in a circle above your head?



Start with a free body diagram for the mass at the bottom. Draw in the force due to the acceleration due to gravity and the tension on the string. Assume that the bend at the top is effectively working as a frictionless pulley (simply changing the direction of the Tension Force Vector).

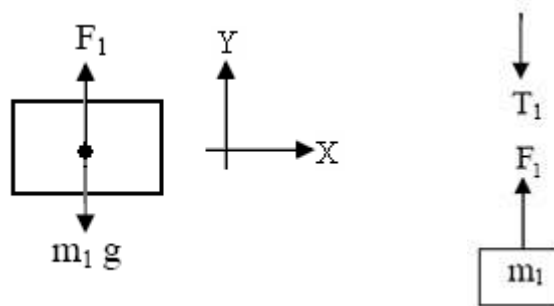


Figure 1: Free Body Diagram (left) and Application of Newton's 3rd Law (right) for the Hanging Mass.

Next, use Newton's Third Law to draw in the tension at the spinning mass, M. You should then be able to then draw the force (to be explicit this is the centrifugal force) acting on the spinning mass, M.

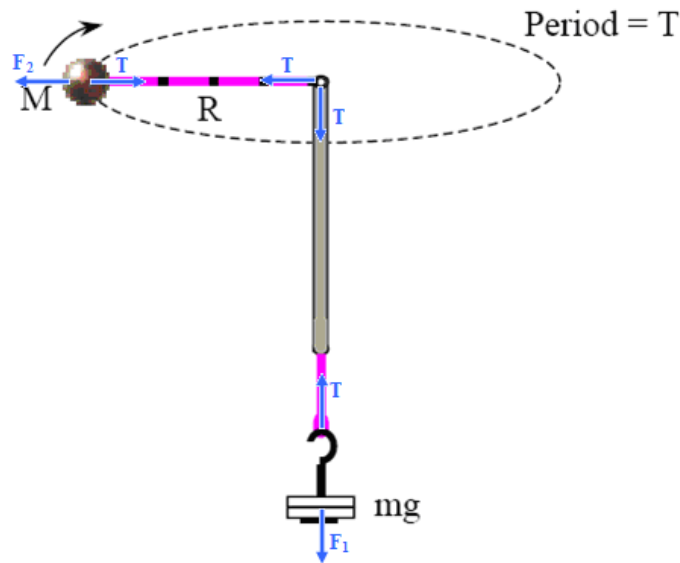


Figure 2: Explicit Forces Operating over the System.

Hence, the direction of the force acting on the spinning mass, M, is pointing outward from the center of the circle created by its spinning motion. This is typically written as “r-hat” (the radial positive direction).

Thus, writing the centrifugal force in its vector representation would look like the following:

$$\vec{F}_{centrifugal} = \vec{F}_2 = \frac{Mv^2}{R} \hat{r}$$

- What provides this force?

First, re-examine the figure above, and consider what would occur if the spinning mass was not spinning (i.e., the system was perfectly static). Since there is nothing against which the tension force at mass M can act (e.g., someone holding the mass, or attaching it to a wall, etc... [think lab 4]) F₂ would be zero, and R would shorten under the tension force in the string (this tension force is called a centripetal force).

We can then definitively state that the outward pointing centrifugal force is provided by the circular (or centripetal) acceleration of the spinning mass, M. The circular acceleration is defined as:

$$\vec{a}_{\text{circular}} = -\frac{v^2}{R} \hat{r}$$

[Note: If we compare the direction of the circular acceleration vector to the generated centrifugal force vector, we notice they are aligned in opposite directions. The following figure shows the orientation of the Centrifugal Force, Centripetal Acceleration, and Velocity vectors around one “orbit” of the spinning mass.]

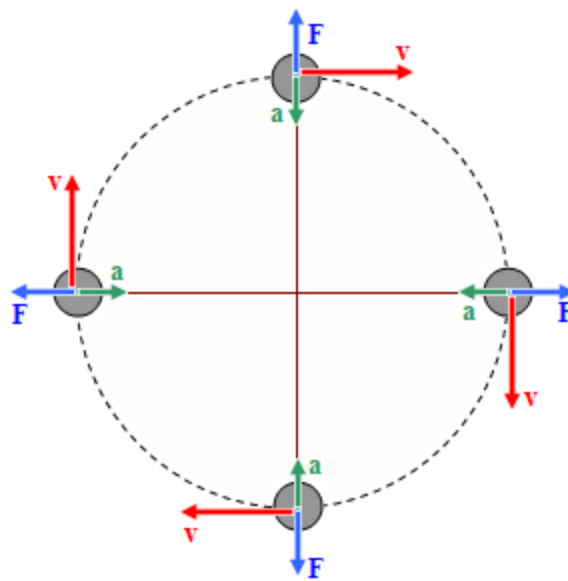


Figure 3: Top/Down View of Comparative Vectors of Centrifugal Force (blue), Centripetal Acceleration (green), and Velocity (red) Operating over the Duration of One Orbit.

- What would be the centripetal force acting on a spinning mass of 130g circling at a radius of 15 cm with angular velocity of 12.5 rad/s ?

Using the values provided by the question, we can assign the following values:

$$v_{\text{Ang}} = 12.5 \text{ rad/sec} = v_{\text{Lin}}/R$$

$$M = 130 \text{ g} = 0.130 \text{ kg}$$

$$R = 15 \text{ cm} = 0.15 \text{ m}$$

Notice, however that we are given angular velocity – and not linear velocity, so we should calculate the linear velocity, first:

$$v_{linear} = v_{angular} R = \left(12.5 \frac{rad}{s} \right) (0.15 \text{ m}) = 1.875 \frac{m}{s}$$

Next, we know the definition of the centripetal force is simply given as:

$$F = -\frac{Mv_{linear}^2}{R} = \text{centripetal force}$$

By re-examining the figure from question 1, we can see that the forces are equal due to the connection of the tensile forces through the string. Hence solving directly for the centripetal force we find:

$$F = -\frac{Mv^2}{R} = -\frac{(0.130 \text{ kg}) \left(1.875 \frac{m}{s} \right)^2}{(0.15 \text{ m})}$$

$$F = -3.046875 \text{ N}$$

- Describe the motion of the spinning mass if the string would break.

The string's tension must provide the necessary centripetal force to move the ball in a circle. If the string breaks, the ball will move off in a straight line (effectively following its velocity vector). The straight line motion in the absence of the constraining force is an example of Newton's first law, due to inertia. The example shown below presumes that no other net forces are acting, such as horizontal motion, on a frictionless surface.

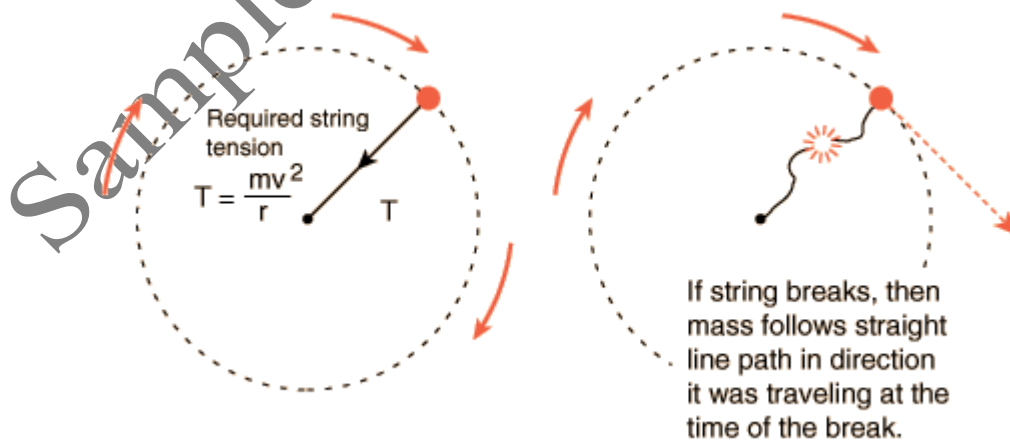


Figure 4: Top/Down View of Motion of Spinning Mass with Intact and Broken Strings.

Re-examine figure 3 above, notice that the centripetal acceleration causes the velocity to change direction over the duration of one orbit.

- When a car travels in a circular path what provides the force necessary to keep it moving in a circle?

Friction provides the force necessary to keep the car moving in a circular path.

- A rider in the car may claim to be “pushed outwards”. Discuss.

To explain this problem fully, it is necessary to re-emphasize the difference between a centrifugal and centripetal force.

Any motion in a curved path represents accelerated motion, and requires a real force directed toward the center of curvature of the path. This force is called the centripetal force, which means "center seeking" force.

The pseudo or fictitious centrifugal force appears when a rotating reference frame is used for analyzing the system. The centrifugal force is exerted on all objects, and directed away from the axis of rotation (see figure 3). The reactive fictitious centrifugal force is the reaction to the real centripetal force. This is equal and opposite in magnitude to the centripetal force, directed away from the center of rotation, and is exerted by the rotating object upon the object which exerts the centripetal force.

As a car swerves around a wide counter-clockwise corner, the passenger's body pushes against the outer edge of the car. This is the reactive fictitious centrifugal force, which is called a reaction force because it results from passive interaction with the car which actively pushes against the body. (Effectively, the passenger's inertia wants the person to travel in a straight line, due to Newton's 1st law; however, the car is colliding with the person.)

Using a reference frame which is fixed relative to the car (a model which those inside the car will often find natural) and while ignoring its rotation, it looks like an external force is pulling the passenger out of the car. This is the fictitious centrifugal force, so called because it is not an actual force exerted by some other object, only the passenger's inertia.

Whereas the centripetal force is seen as a force which must be applied by an external agent to force an object to move in a curved path, the centrifugal force is an "effective force" which is invoked to explain the behavior of objects from a frame of reference which is rotating.

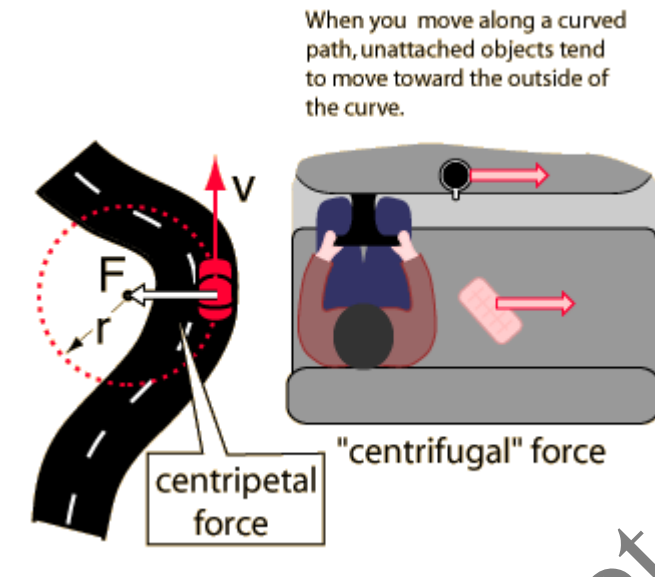


Figure 5: Centripetal Force Operating on a Vehicle (left) and Centrifugal Force Operating on Objects inside the Vehicle

Notice in the figure above, the driver of a car on a curve is in a rotating reference frame and he could invoke a "centrifugal" force to explain why his coffee cup and the carton of eggs he has on the seat beside him tend to slide sideways. The friction of the seat or dashboard may not be sufficient to accelerate these objects in the curved path.

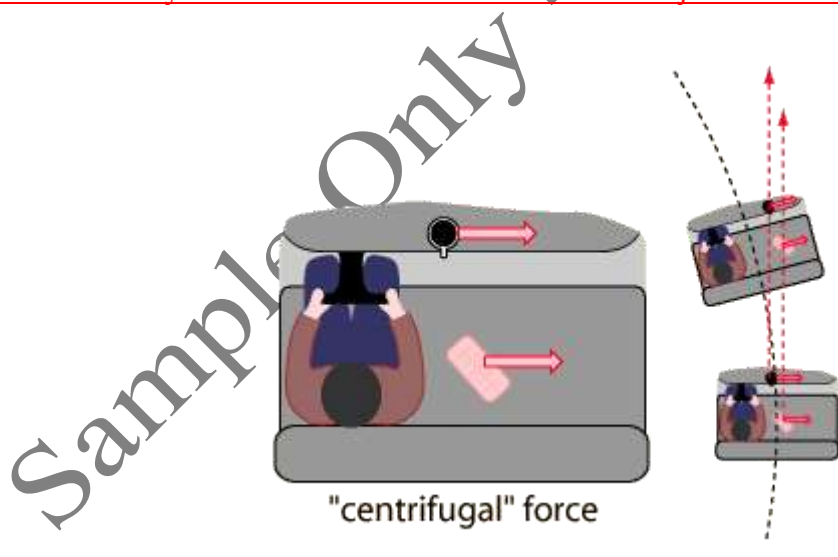


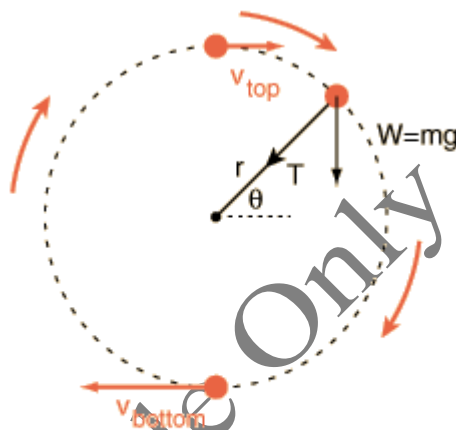
Figure 6: Centrifugal Force Operating on Objects inside a Vehicle and their Motion in an Inertial Frame (a straight line)

A person in a hovering helicopter above the car could describe the movement of the cup and the egg carton as just going straight while the car travels in a curved path. This is similar to the broken string example (see above).

The centrifugal force is a useful concept when the most convenient reference frame is one which is moving in a curved path, and therefore experiencing a centripetal acceleration. Since the car above will be experiencing a centripetal acceleration v^2/r , then an object of mass m on the seat will require a force mv^2/r toward the center of the circle to stay at the same spot on the seat. From the reference frame of a person in the car, there seems to be an outward centrifugal force mv^2/r acting to move the mass radially outward. In practical descriptive terms, you would say that your carton of eggs is more likely to slide outward if you have a higher speed around the curve (the velocity squared factor) and more likely to slide outward if you go around a sharper curve (the inverse dependence upon r).

[Note: Re-read the answers for the previous questions and see if this all makes sense now! ☺]

- If you swing a bucket full of water over your head at a great enough speed, the water will not spill out of the bucket. Why does this happen?



$$\text{Required centripetal force} = \frac{mv^2}{r}$$

At the top, the minimum speed to maintain a circle is the speed which gives a downward centripetal acceleration $a = g$. So the minimum speed at the top is given by

$$v_{top}(\text{minimum}) = \sqrt{gr}$$

Conservation of energy relates the velocities at the top and bottom of the circle:

$$\frac{1}{2}mv_{top}^2 + mg2r = \frac{1}{2}mv_{bottom}^2$$

Substitution gives the relationship between the tensions at top and bottom:

$$T_{bottom} = T_{top} + 6mg$$

The motion of a mass on a string in a vertical circle includes a number of mechanical concepts. It must satisfy the constraints of centripetal force to remain in a circle, and must satisfy the demands of conservation of energy as gravitational potential energy is converted to kinetic energy when the mass moves downward. The velocity must increase as the mass moves downward from the top of the circle, subject to the constraints stated.

For a mass moving in a vertical circle of radius $r =$ m,

if we presume that the string stays taut, then the minimum speed for the mass at the top of the circle is (for $g = 9.8 \text{ m/s}^2$)

$$v_{top}(\text{minimum}) = \sqrt{gr} = \boxed{} \text{ m/s}$$

This is the condition for "weightlessness" in any curved motion in a vertical plane. For any velocity above this minimum, we can use conservation of energy to relate the velocity at the bottom of the circle to the velocity at the top.

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