

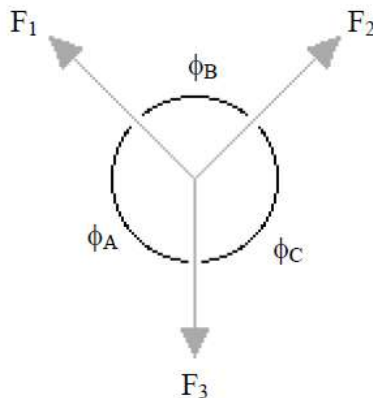
Experiment #5 Newton's Laws II Pre-lab Hints

The following are some hints for this pre-lab, since a few of these questions can be a little difficult. Note that these are not necessarily the answers to the questions, but merely a means to get to the answer. The hints are written in RED below the question for which the hint is given.

**** NOTE:** For this lab it is **crucial** to have very clear drawings. It will help to be consistent through your sketches and keep standard notations and labels. It is also helpful to keep the explanations well labeled and clear of clutter (e.g., it is easy to leave out a negative sign or not keep the directions or magnitudes straight – so be **especially** careful). The problems provided in this pre-lab are quite long when completely written out. Unlike previous pre-labs it is imperative that you explicitly write everything out for this problem (i.e., you can't just write down an answer). A guide for solving these problems have been provided below – make sure you don't leave anything important out! {I guess what I'm trying to say is, usually my pre-lab examples are relatively long due to the fact that I include extra information for your sake of learning; however, in this pre-lab it's long – and everything is needed. ☺}**

- Use the following values and the information in your manual to test Newton's Laws.

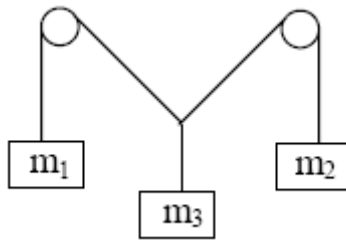
$$\begin{array}{ll} \phi_A = 117^\circ & m_1 = 12.6 \text{ g} \\ \phi_B = 140^\circ & m_2 = 11.5 \text{ g} \\ \phi_C = 103^\circ & m_3 = 8.3 \text{ g} \end{array}$$



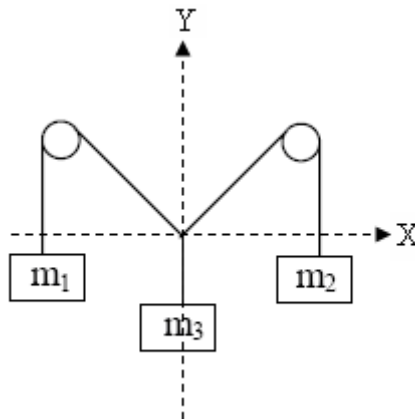
**** Notice that in the pre-lab posted on the website there already exists an inconsistency in labeling. The labeling of Phi between the question and the description in the “text” are not the same. For consistency purposes, we will change the sub-numbers of Phi to sub-letters (per this prelab). ****

Angle [deg]	Mass [grams]
$\Phi_A = 117^\circ$	$m_1 = 12.6 \text{ g}$
$\Phi_B = 140^\circ$	$m_2 = 11.5 \text{ g}$
$\Phi_C = 103^\circ$	$m_3 = 8.3 \text{ g}$

From the information in the manual, the set up for the lab is as follows:



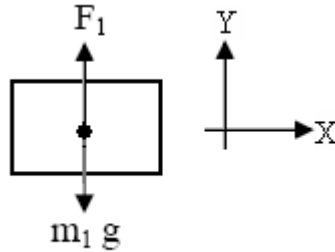
First, we need to establish a coordinate frame for this system. The one I chose is shown below. I suggest that you use the same one. ☺



We can use Newton's Laws to find all the theoretical forces acting in this X/Y-coordinate frame and then use the values provided above to plug into the resultant equations and see if the theory holds. (Note that for this lab we will assume that the two pulleys are frictionless and the connective strings are weightless. [This will of course introduce some minor error to our results, since in reality there are no “truly” frictionless pulleys.]

First, whenever we are doing a problem with Newton's Laws, we must **ALWAYS DRAW A FREE BODY DIAGRAM** for each of the components.

Free Body Diagram for Mass 1



According to Newton, for a static system:

$$\sum F = 0$$

That is to say: The sum of the forces in a free body diagram must add up to zero. Since there are no forces in the x-direction, that one is easy. ☺

For the sum of the forces in the x-direction:

$$\sum F_x = 0$$

$$0 = 0$$

For the sum of the forces in the y-direction:

$$\sum F_y = 0$$

$$(+ F_1) + (- m_1 g) = 0$$

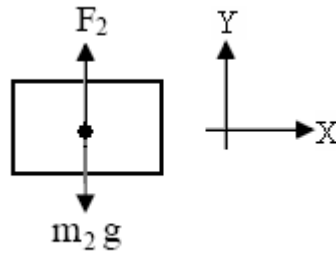
** Here notice that we used the + sign for forces that are pointing up (in the positive y-hat direction) and used the – sign for forces that are pointing down (in the negative y-hat direction). **

$$F_1 - m_1 g = 0$$

$$\boxed{F_1 = m_1 g}$$

[Let's call this Equation # 1.]

Free Body Diagram for Mass 2



For the sum of the forces in the x-direction:

$$\sum F_x = 0$$

$$0 = 0$$

For the sum of the forces in the y-direction:

$$\sum F_y = 0$$

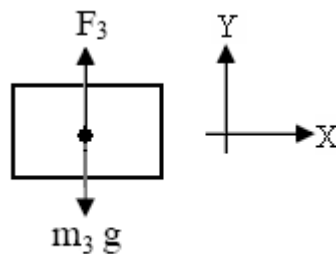
$$(+F_2) + (-m_2 g) = 0$$

$$F_2 - m_2 g = 0$$

$$\boxed{F_2 = m_2 g}$$

[Let's call this Equation # 2.]

Free Body Diagram for Mass 3



For the sum of the forces in the x-direction:

$$\sum F_x = 0$$

$$0 = 0$$

For the sum of the forces in the y-direction:

$$\sum F_y = 0$$

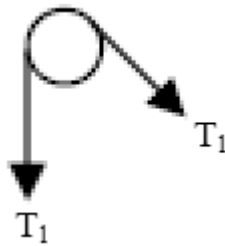
$$(+ F_3) + (- m_3 g) = 0$$

$$F_3 - m_3 g = 0$$

$$\boxed{F_3 = m_3 g}$$

[Let's call this Equation # 3.]

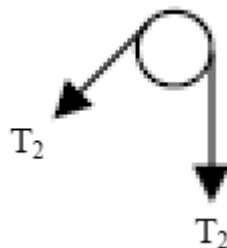
Free Body Diagram for Pulley 1



Since we are considering the pulley to be frictionless, the effect of the pulley just changes the direction of the tension vector – not the magnitude.

$$\boxed{T_1 = T_1}$$

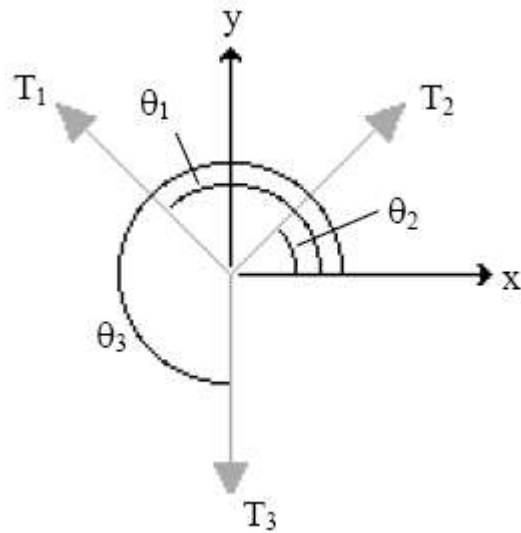
Free Body Diagram for Pulley 2



Again, since we are considering the pulley to be frictionless, the effect of the pulley just changes the direction of the tension vector – not the magnitude.

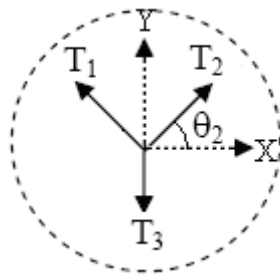
$$\boxed{T_2 = T_2}$$

Center of System Free Body Diagram

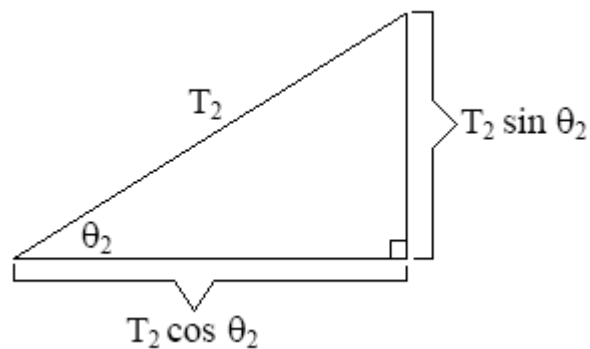


Notice that the tensions in this picture do not lie along the coordinate axis. We need to break tension 1 and tension 2 into their components so we can use the sum of the forces.

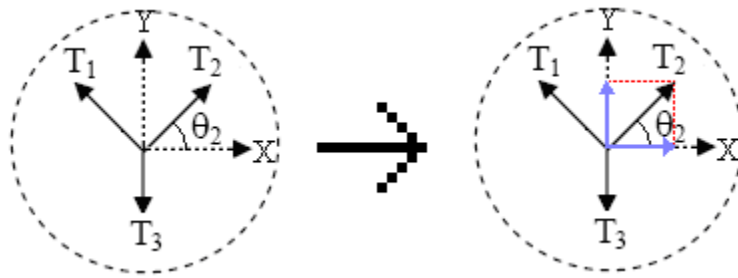
Consider the following for breaking up the tension 2 vector.



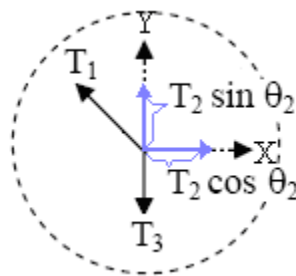
If we focus on simply the coordinate axis and the trigonometric triangle formed by using tension 2 as the hypotenuse, we get the following figure:



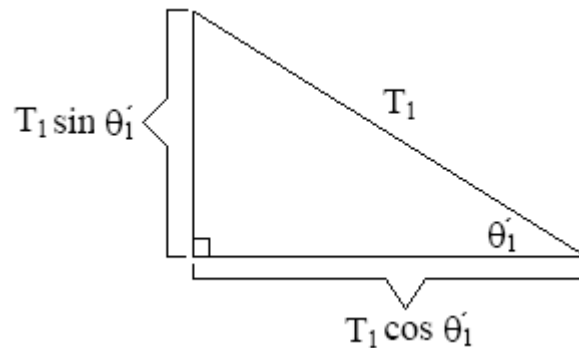
Thus:



Or



If we perform the same analysis for tension 1, as we did for tension 2 we would find that (** NOTE that this is where it tends to get confusing. Students tend to forget a negative sign here, or leave it off all together. Be especially careful with this part – and reread the solution a few times if you need to get a clear view as to what is occurring. **):



Where we used the co-angle theta prime (Refer back to the figure with the angles of Theta drawn in if this is confusing. The figure is in the Center of System Free Body Diagram section.):

$$\theta_1' = 180^\circ - \theta_1$$

Note from trig that the following identities hold:

$$A \sin(180^\circ - \omega) = A \sin(\omega)$$

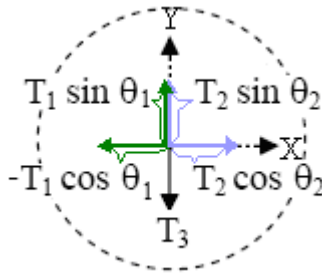
$$A \cos(180^\circ - \omega) = -A \cos(\omega)$$

Thus for our case:

$$T_1 \sin(\theta'_1) = T_1 \sin(180^\circ - \theta_1) = T_1 \sin(\theta_1)$$

$$T_1 \cos(\theta'_1) = T_1 \cos(180^\circ - \theta_1) = -T_1 \cos(\theta_1)$$

So, drawing this out explicitly, we have the following:



For the sum of the forces in the x-direction (**WARNING! BE CAREFUL!**):

$$\sum F_x = 0$$

$$(T_2 \cos(\theta_2)) + (-(-T_1 \cos(\theta_1))) = 0$$

$$(T_2 \cos(\theta_2)) + (T_1 \cos(\theta_1)) = 0$$

$$\boxed{T_1 \cos(\theta_1) = -T_2 \cos(\theta_2)}$$

[Let's call this Equation # 4.]

For the sum of the forces in the y-direction:

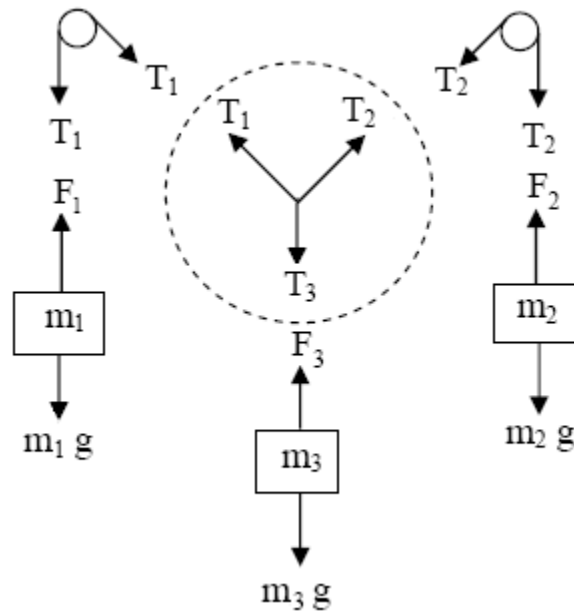
$$\sum F_y = 0$$

$$(T_1 \sin(\theta_1)) + (T_2 \sin(\theta_2)) + (-T_3) = 0$$

$$\boxed{(T_1 \sin(\theta_1)) + (T_2 \sin(\theta_2)) = T_3}$$

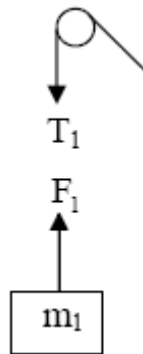
[Let's call this Equation # 5.]

Now we can consider the entire system with all the components included. This is shown below.



From the above diagram, it is easy to see the following correlations:

Connection at Mass 1 and Pully 1

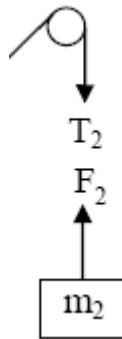


The opposite forces have to balance out. Thus:

$$F_1 = T_1$$

[Let's call this Relationship A.]

Connection at Mass 2 and Pulley 2

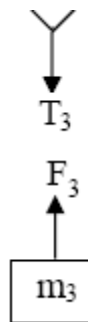


Again, the opposite forces have to balance out. Thus:

$$F_2 = T_2$$

[Let's call this Relationship B.]

Connection at Mass 3 and Center of the System



Again, the opposite forces have to balance out. Thus:

$$F_3 = T_3$$

[Let's call this Relationship C.]

Putting it all together

Let's use everything we found thus far to combine the equations and relationships together. Using Relationships A – C and Equations # 4 and #5, re-write Equations #4 and #5 as follows:

Equation #4 (Forces in the x-direction):

$$F_1 \cos(\theta_1) = -F_2 \cos(\theta_2)$$

[Let's call this Equation # 4a.]

Equation #5 (Forces in the y-direction):

$$(F_1 \sin(\theta_1)) + (F_2 \sin(\theta_2)) = F_3$$

[Let's call this Equation # 5a.]

If we use Equation #1, #2 and #3, Equations #4a and #5a can further be written as:

Equation #4a (Forces in the x-direction):

$$m_1 g \cos(\theta_1) = -m_2 g \cos(\theta_2)$$

[Let's call this Equation # 4b.]

Equation #5a (Forces in the y-direction):

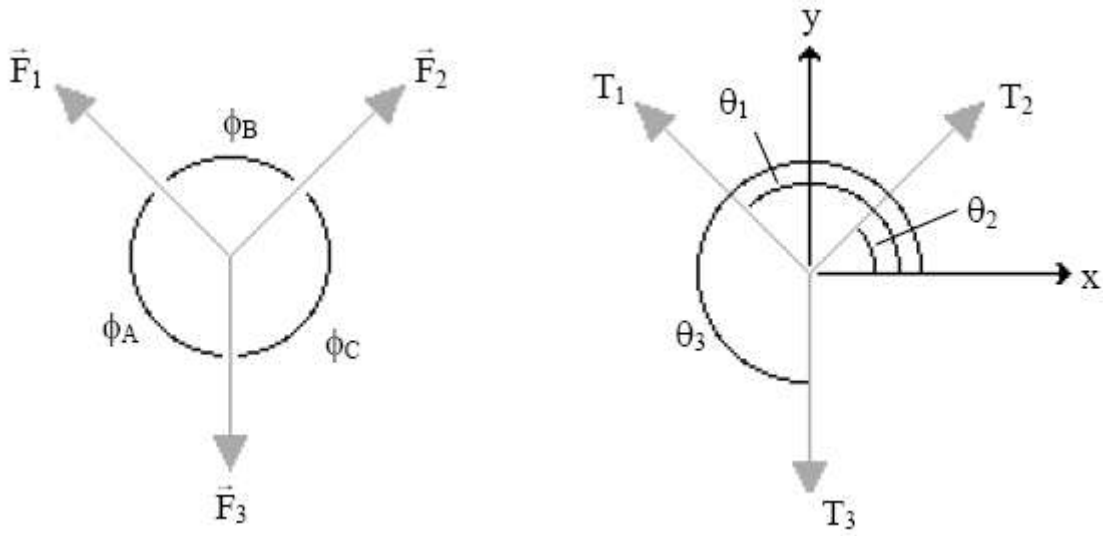
$$(m_1 g \sin(\theta_1)) + (m_2 g \sin(\theta_2)) = m_3 g$$

[Let's call this Equation # 5b.]

We will use equations # 4b and # 5b, plug in the values, and see if the equalities hold for each of these two equations. So, if Newton's Laws are true, the value on the left side of the equations should be identical to the number on the right side of the equations. Notice there is one problem, still. Equations # 4b and # 5b are in terms of Theta 1 and Theta 2, and we were given the values of Phi A, Phi B and Phi C. So, we need a means of finding Theta 1 and Theta 2 from the measured angles Phi A, Phi B and Phi C.

** Note that we don't need to find Theta 3 since this is not used in the equations to prove Newton's Laws. **

Finding Theta from the Measured Angles Phi



By comparison it is easy to compare the angles of Phi to the angles of Theta.

$$\theta_3 = 270^\circ$$

$$\theta_2 = \phi_C - 90^\circ$$

$$\theta_1 = \phi_B + \theta_2 = \phi_B + \phi_C - 90^\circ$$

If we plug these into Equation # 4b and # 5b, we get the following (Notice now that these equations are completely in terms of masses and phi – exactly what we wanted.):

Equation #4b (Forces in the x-direction):

$$m_1 g \cos(\theta_1) = -m_2 g \cos(\theta_2)$$

$$m_1 g \cos(\phi_B + \phi_C - 90^\circ) = -m_2 g \cos(\phi_C - 90^\circ)$$

Equation #5b (Forces in the y-direction):

$$(m_1 g \sin(\theta_1)) + (m_2 g \sin(\theta_2)) = m_3 g$$

$$(m_1 g \sin(\phi_B + \phi_C - 90^\circ)) + (m_2 g \sin(\phi_C - 90^\circ)) = m_3 g$$

**** These are what we will use to test Newton's Laws! ****

For forces in the x-direction:

$$m_1 g \cos(\phi_B + \phi_C - 90^\circ) = -m_2 g \cos(\phi_C - 90^\circ)$$

For forces in the y-direction:

$$(m_1 g \sin(\phi_B + \phi_C - 90^\circ)) + (m_2 g \sin(\phi_C - 90^\circ)) = m_3 g$$

Test Newton's Laws from Measurements (FINALLY!)

We are given (or in the case of the lab will measure using a scale and protractor) the following measurements of the angles and masses of the system. Using the two equations above for the total forces in the x-direction and the total forces in the y-direction, we can test Newton's Laws.

Angle [deg]	Mass [grams]
$\Phi_A = 117^\circ$	$m_1 = 12.6 \text{ g}$
$\Phi_B = 140^\circ$	$m_2 = 11.5 \text{ g}$
$\Phi_C = 103^\circ$	$m_3 = 8.3 \text{ g}$

First, we need to get the masses into kilograms, so when we multiply it by an acceleration (namely the acceleration due to gravity) we will have units of Newtons.

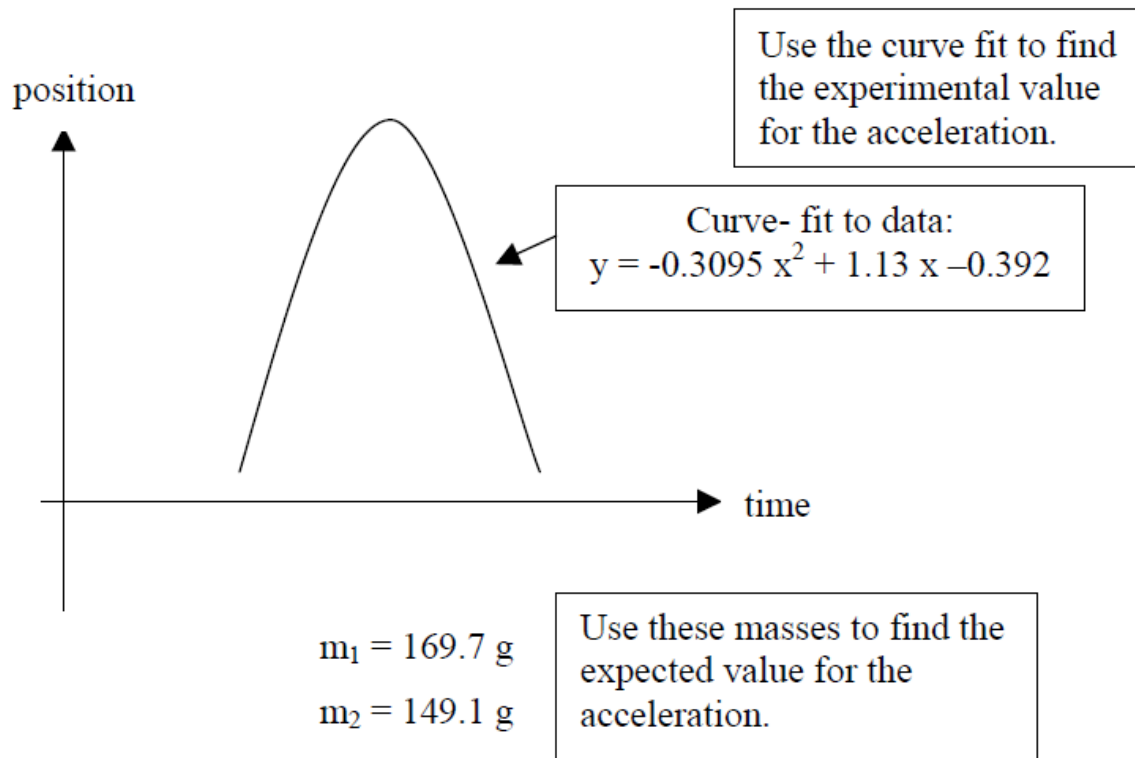
1. Plug the values into the equation "For forces in the x-direction" and see if both sides of the equal side come out to be "relatively equal".

With consideration of reasonable error introduced by human measurements, angle and mass rounding, the fact that the strings are not massless, and the fact that the pulleys are not frictionless – these values should be almost *identical*. If they are with a range of 0.1-ish Newtons, call it "close enough".

2. Plug the values into the equation "For forces in the y-direction" and see if both sides of the equal side come out to be "relatively equal".

Again, with consideration of reasonable error introduced by human measurements, angle and mass rounding, the fact that the strings are not massless, and the fact that the pulleys are not frictionless – determine if these values are almost *identical*.

- Find the measured acceleration from this experimental data. Compare the measured value to the expected value. How close are they?



**** We should be pretty good at this by now! **** Using the “Curve – fit to data”, we have the following best fit line:

$$y = \left(-0.3095 \frac{m}{s^2} \right) t^2 + \left(1.13 \frac{m}{s} \right) t + (-0.392 m)$$

If we compare the equation for kinematic 2D motion for the y-position with the found trendline, we can easily see by evaluation that the following constants correlate directly:

$$y = \frac{1}{2} a_y t^2 + v_{o,y} t + y_o$$

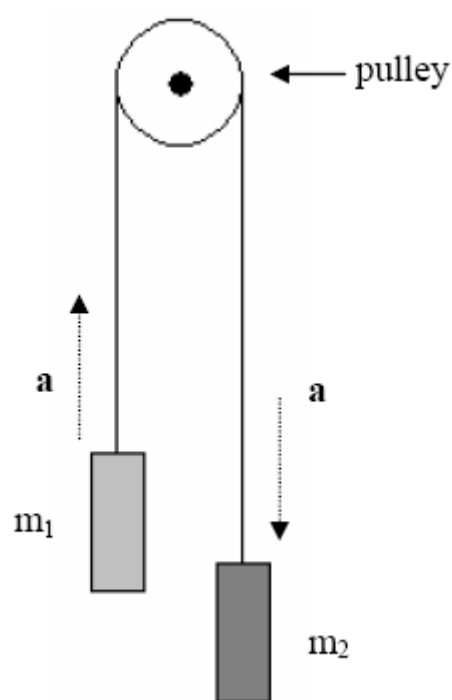
$$y = \left(-0.3095 \frac{m}{s^2} \right) t^2 + \left(1.13 \frac{m}{s} \right) t + (-0.392 m)$$

Use this relationship to find the “measured acceleration” is:

Finding Expected Acceleration from Experimental Data using Masses and Newton's Laws:

We can calculate the “theoretical (expected) acceleration” using the measurement of masses for an Atwood Machine. We were given the following masses:

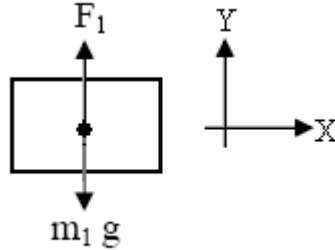
Masses [g]
$m_1 = 169.7 \text{ g}$
$m_2 = 149.1 \text{ g}$



Again, we can use Newton's Laws to find all the theoretical forces acting in this X/Y-coordinate frame. (Note that for this lab we will assume that the two pulleys are frictionless and the connective strings are weightless. [Again, this will of course introduce some minor error to our result.]) {Note that we can tell right away that since mass 1 is larger than mass 2, that the resultant calculated acceleration should be negative!}

Again, first, whenever we are doing a problem with Newton's Laws, we must *ALWAYS DRAW A FREE BODY DIAGRAM* for each of the components.

Free Body Diagram for Mass 1



According to Newton, for a dynamic system:

$$\sum F = ma$$

For the sum of the forces in the x-direction:

$$\sum F_x = m_1 a_x$$

$$0 = m_1 a_x$$

And since the mass is not zero, then the acceleration must be:

$$\boxed{a_x = 0}$$

For the sum of the forces in the y-direction:

$$\sum F_y = m_1 a_y$$

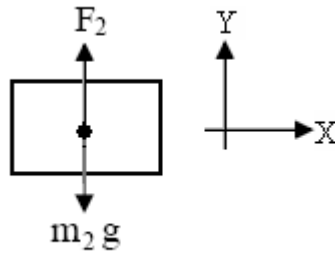
$$(+ F_1) + (- m_1 g) = m_1 a_y$$

$$\boxed{F_1 - m_1 g = m_1 a_y}$$

[Let's call this Equation # 1.]

Notice that the acceleration on the right hand side of the equation is positive, because in our drawing we drew the acceleration over mass 1 in the positive y direction.

Free Body Diagram for Mass 2



For the sum of the forces in the x-direction:

$$\sum F_x = m_2 a_x$$

$$0 = m_2 a_x$$

And again, since the mass is not zero, then the acceleration must be:

$$\boxed{a_x = 0}$$

For the sum of the forces in the y-direction:

$$\sum F_y = -m_2 a_y$$

$$(+F_2) + (-m_2 g) = -m_2 a_y$$

$$\boxed{F_2 - m_2 g = -m_2 a_y}$$

[Let's call this Equation # 2.]

Notice that the acceleration on the right hand side of the equation is negative, because in our drawing we drew the acceleration over mass 2 in the negative y direction.

Free Body Diagram for Pully



Since we are considering the pulley to be frictionless, the effect of the pulley just changes the direction of the tension vector – not the magnitude.

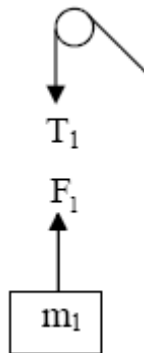
$$T_1 = T_2$$

Let's call these both just T for a generic tension.

$$T_1 = T_2 = T$$

[Let's call this Relationship A.]

Connection at Mass 1 and Pully

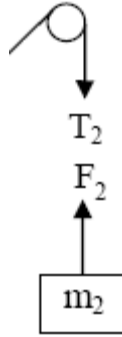


The opposite forces have to balance out. Thus:

$$F_1 = T_1$$

[Let's call this Relationship B.]

Connection at Mass 2 and Pully



Again, the opposite forces have to balance out. Thus:

$$F_2 = T_2$$

[Let's call this Relationship C.]

Putting it all together

Let's use everything we found thus far to combine the equations and relationships together. Using Relationships A – C and Equations # 1 and # 2, we can re-write Equations #1 and #2 as follows:

Equation #1 (Forces in the y-direction for mass 1):

$$T - m_1 g = m_1 a_y$$

[Let's call this Equation # 1a.]

Equation #2 (Forces in the y-direction for mass 2):

$$T - m_2 g = -m_2 a_y$$

[Let's call this Equation # 2a.]

If we solve equation # 1a for tension, and do the same for equation # 2a, we get the following:

Equation #1a (Forces in the y-direction for mass 1):

$$T = m_1 a_y + m_1 g$$

[Let's call this Equation # 1ba.]

Equation #2a (Forces in the y-direction for mass 2):

$$T = -m_2 a_y + m_2 g$$

[Let's call this Equation # 2ba.]

Now if we set these equal tensions equal to each other (that is – set equation # 1b equal to equation # 2b):

$$m_1 a_y + m_1 g = -m_2 a_y + m_2 g$$

Now solve for the acceleration in the y-direction:

$$m_1 a_y + m_2 a_y = m_2 g - m_1 g$$

$$a_y (m_1 + m_2) = g(m_2 - m_1)$$

$$a_y = \frac{(m_2 - m_1)}{(m_1 + m_2)} g$$

Use the measurements of the masses of the system. Using the equation above for the acceleration in the y-direction of a dynamic system, we can calculate the expected acceleration.

Notice that the units of the masses will cancel out (so we don't have to convert these to kilograms – but you can if you *really* want to.)

Compare the measured value to the expected value:

Using the percent difference, we can compare the “measured” acceleration with the “expected” acceleration.

$$\% \text{ difference} = \frac{|a_{\text{Expected}}| - |a_{\text{Measured}}|}{|a_{\text{Expected}}|} \times 100\%$$